Deep Learning on Meshes and Point Clouds

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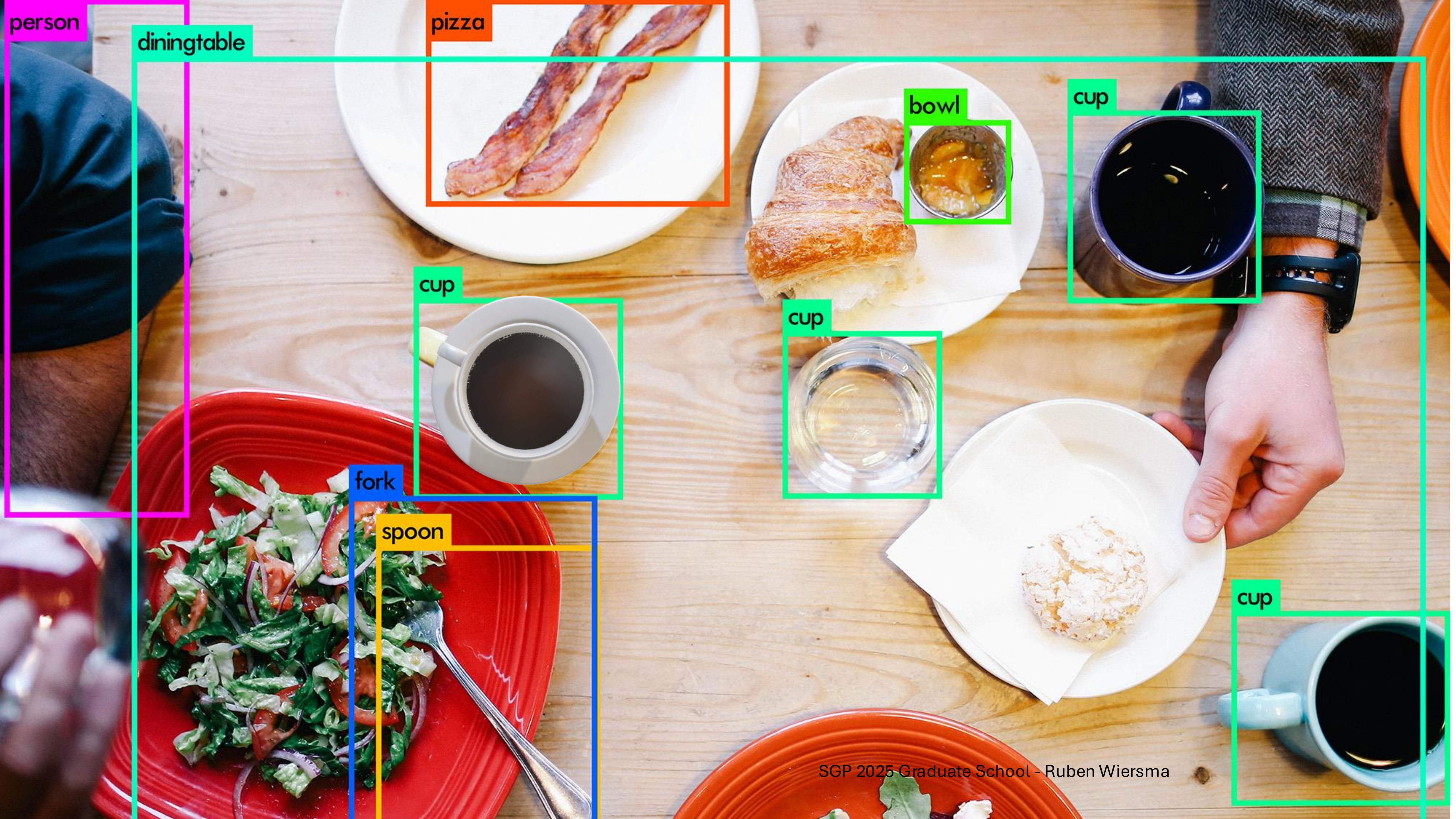
SGP 2025 Graduate School, Bilbao

Who are you?

Goal

- Provide a ‘map’ of deep learning on 3D shapes
- Outcome
  - Applying deep learning to 3D tasks
  - Developing deep learning techniques for 3D tasks
- Audience
  - Some familiarity with optimization or machine learning

Why are we interested?


person
diningtable
pizza
bowl
cup
cup
cup
cup
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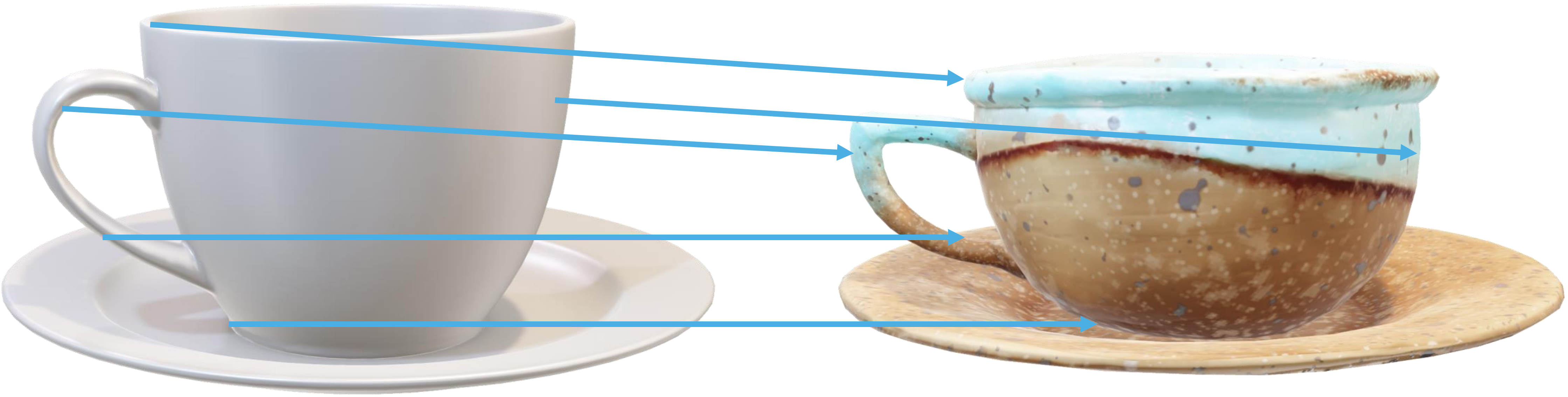
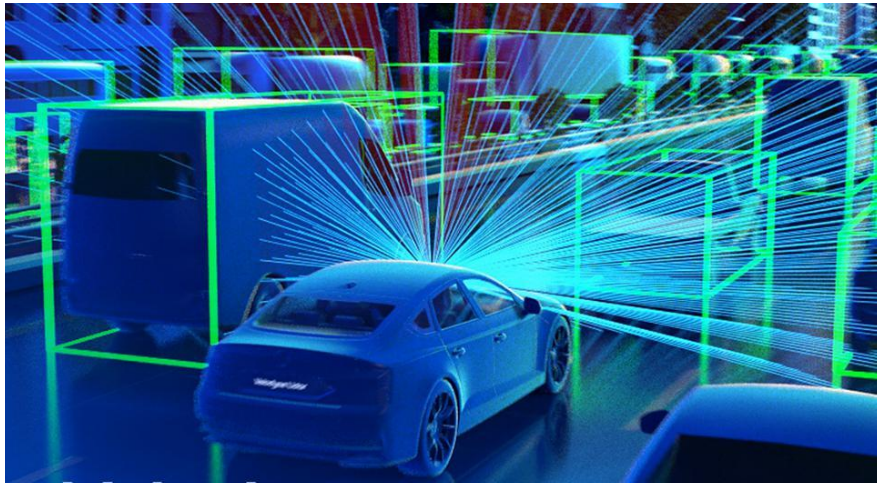
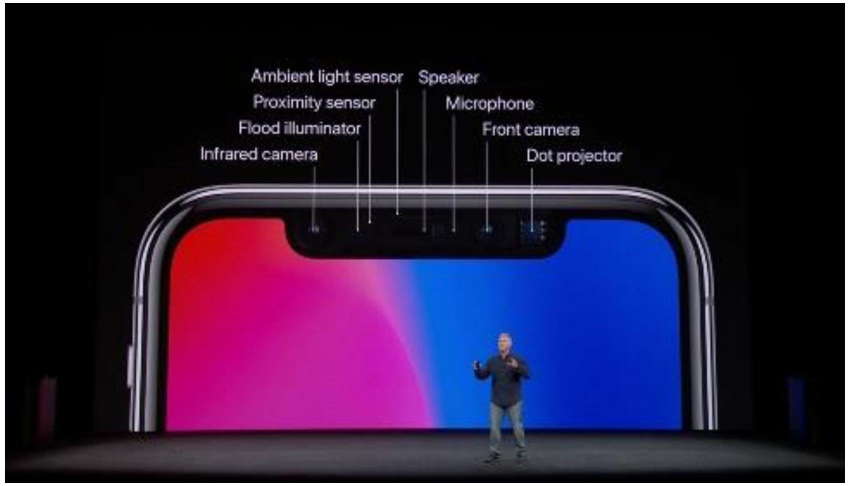
spoon


coffeecup
Cup
Handle
Plate

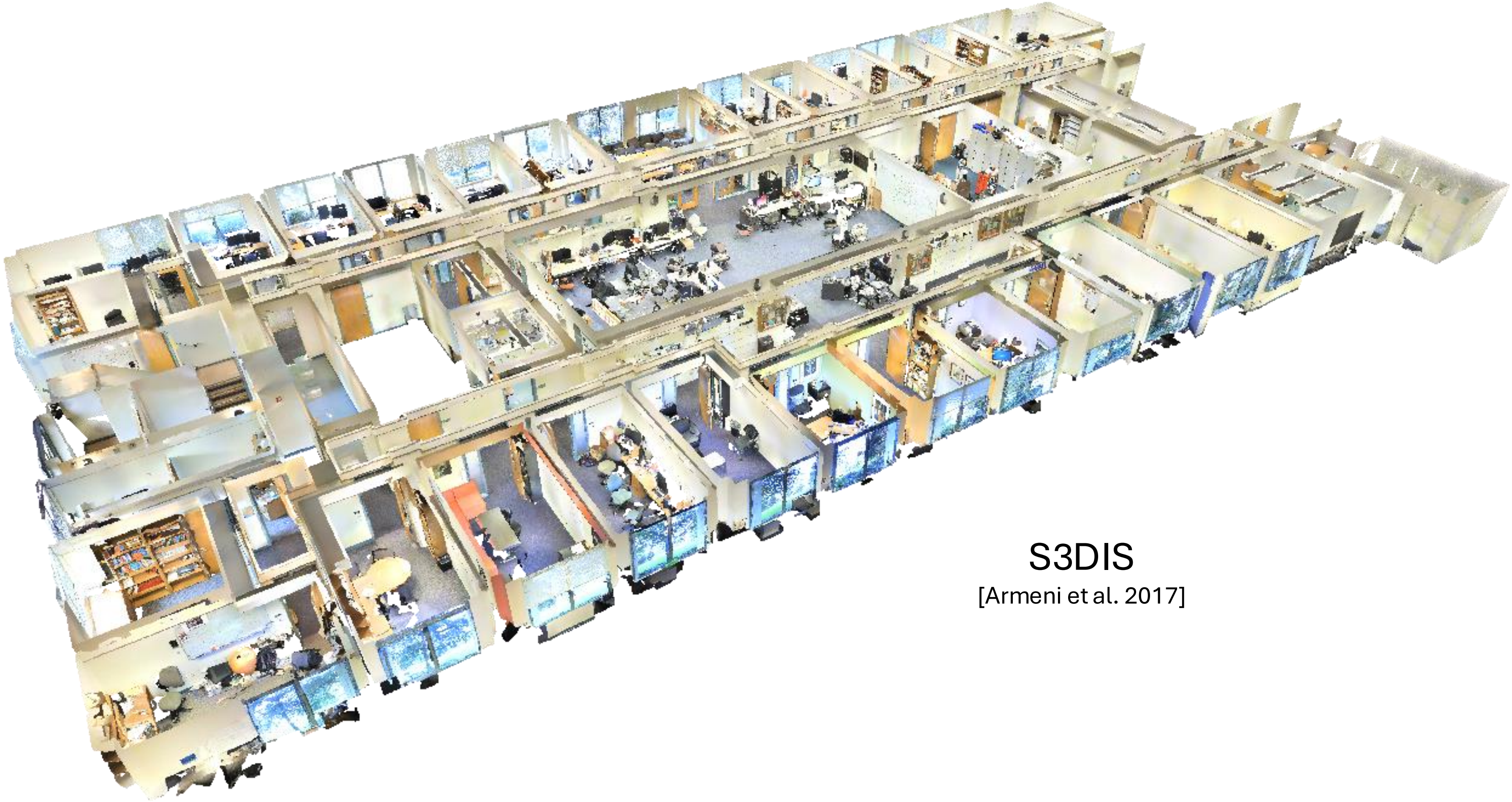





90 mm x 25 mm x 25 mm

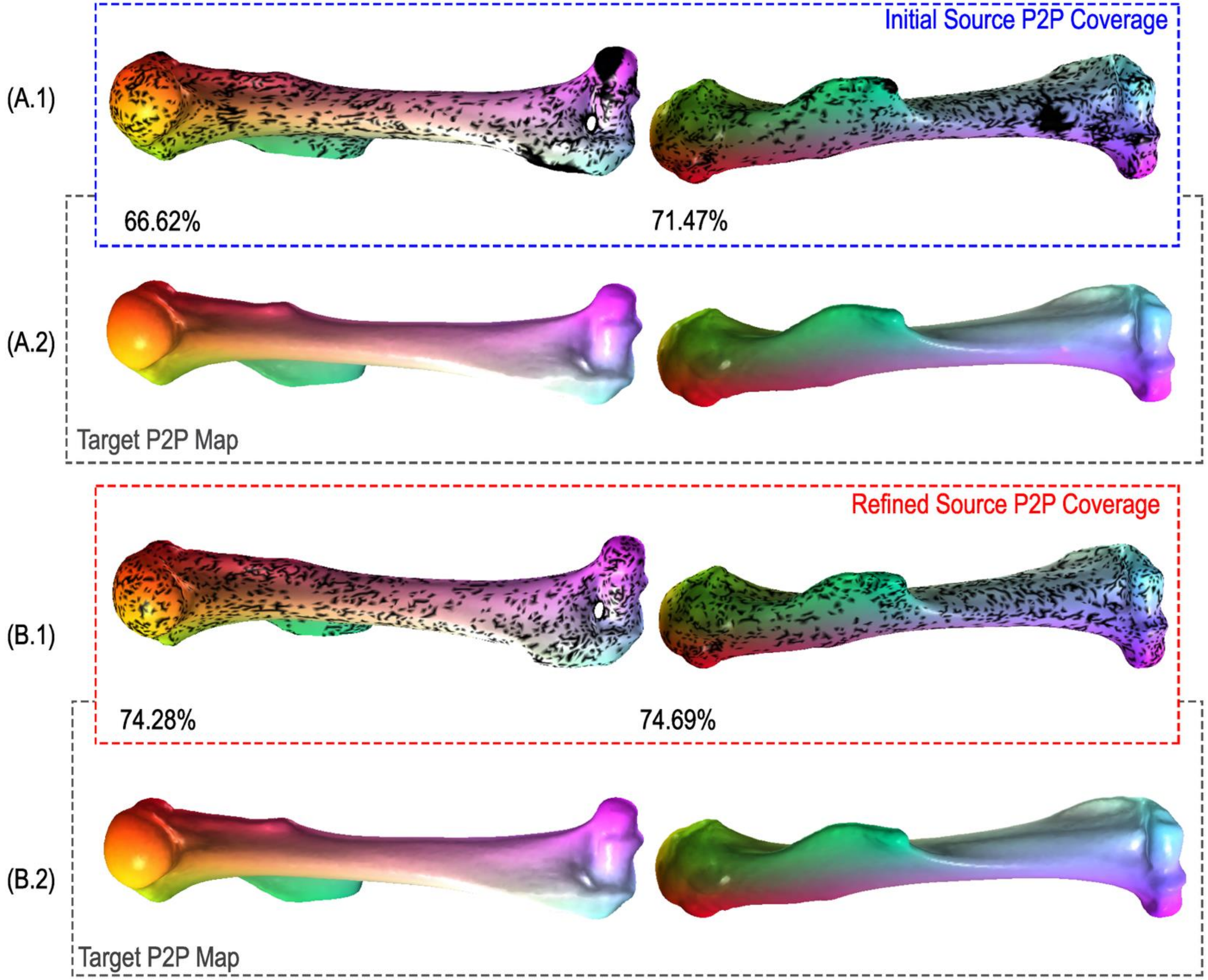

Ambient light sensor
Speaker
Proximity sensor
Microphone
Flood illuminator
Front camera
Infrared camera
Dot projector


From Bronstein

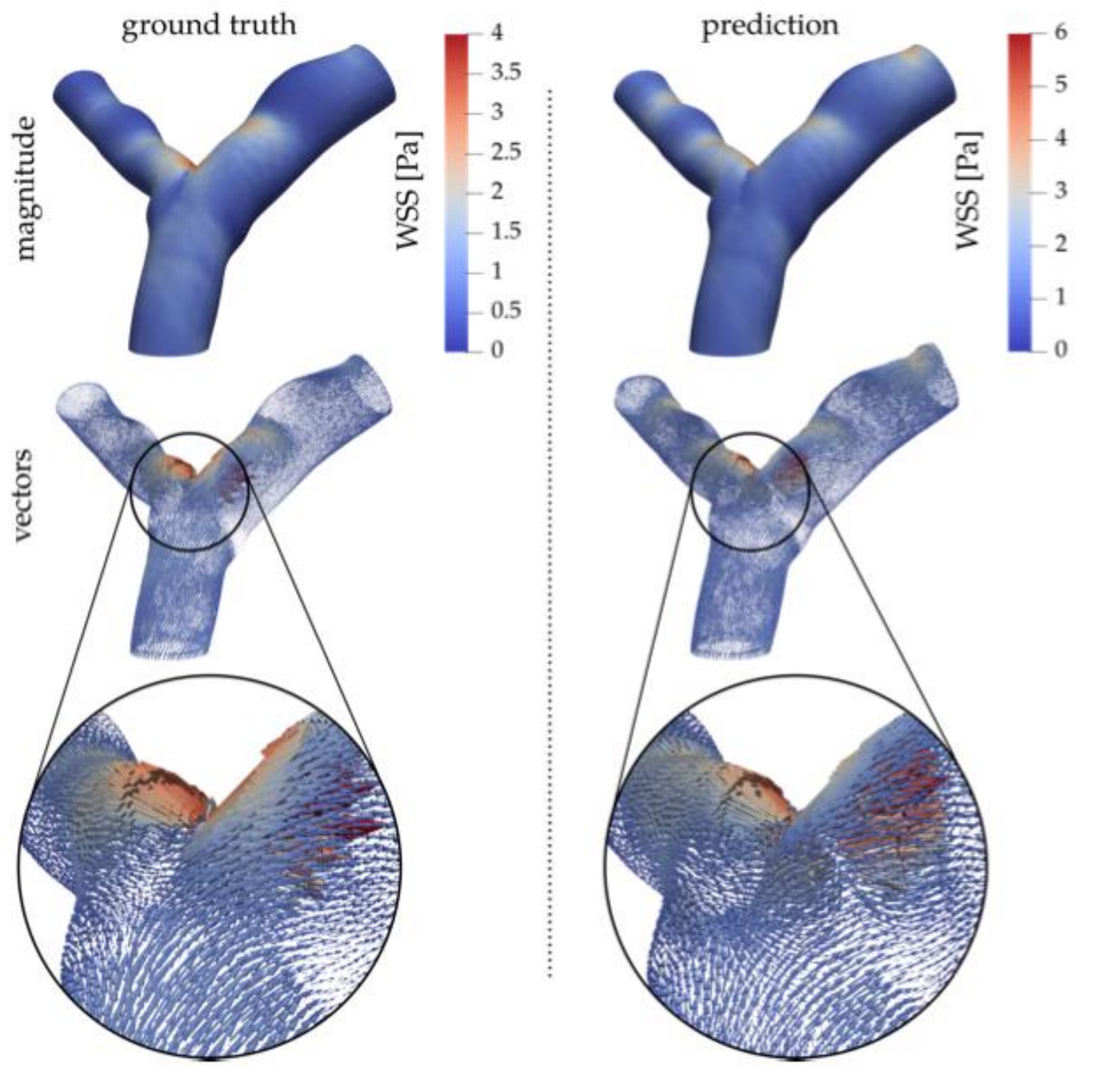


S3DIS
[Armeni et al. 2017]

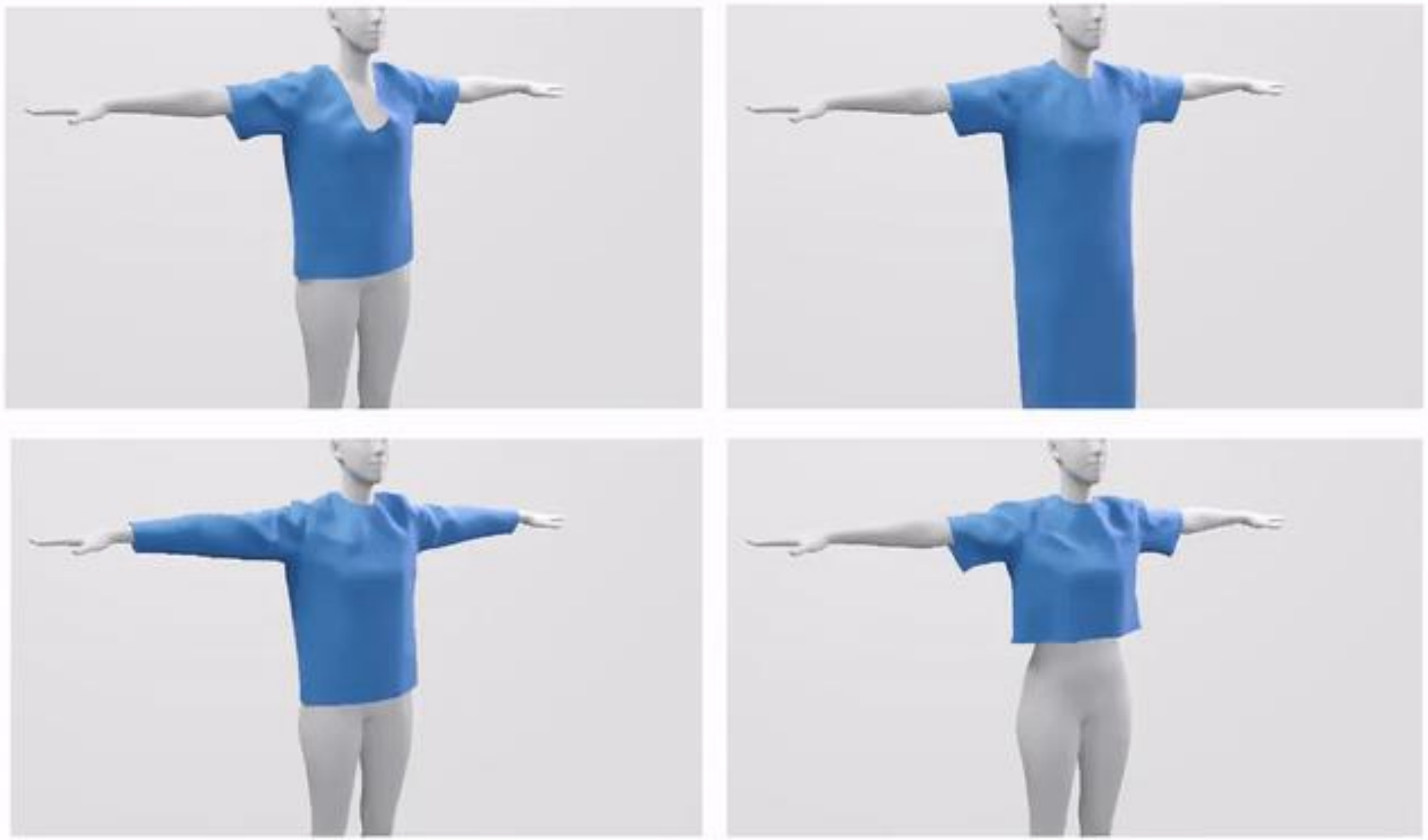
Understanding and Improving Features Learned in Deep Functional Maps, Attaiki and Ovsjanikov (2023)



Automated morphological phenotyping [...], O. Thomas et al. (2023)



Mesh Neural Networks for SE(3)-Equivariant Hemodynamics Estimation of the Artery Wall, Suk et al. (2022)

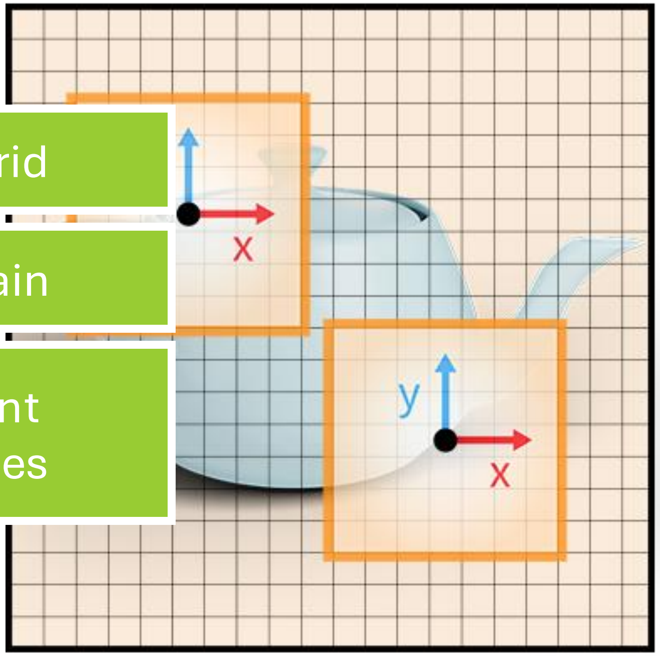


Fully Convolutional Graph Neural Networks for Parametric Virtual Try-On, Vidaurre et al. (2020)

Why are we interested?

- Classification
- Segmentation
- Registration, correspondence
- Surface reconstruction
- Speeding up classical geometry tasks (i.e., smart lookup table)
- Generative modeling

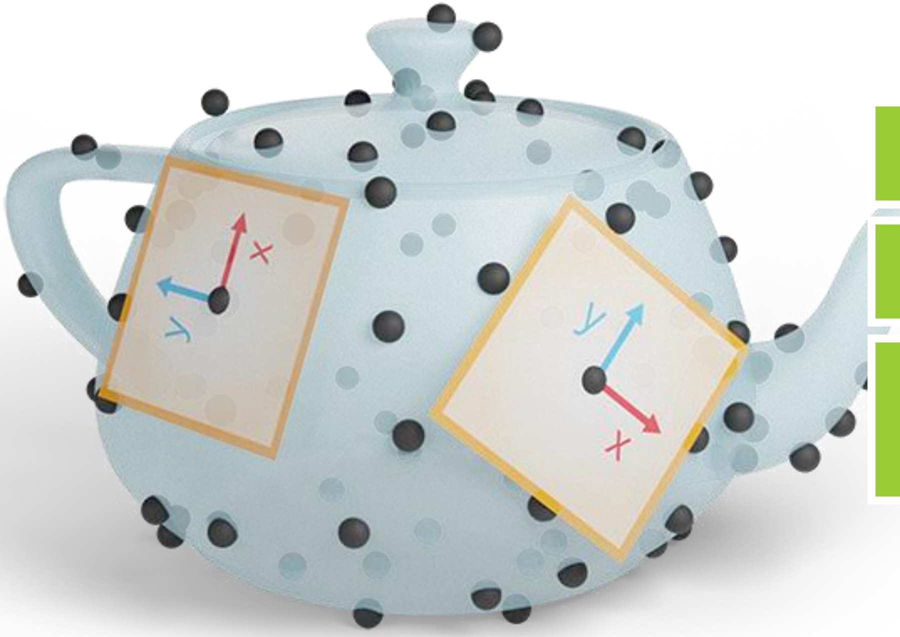
What’s the difference?



Regular grid

Flat domain

Consistent coordinates



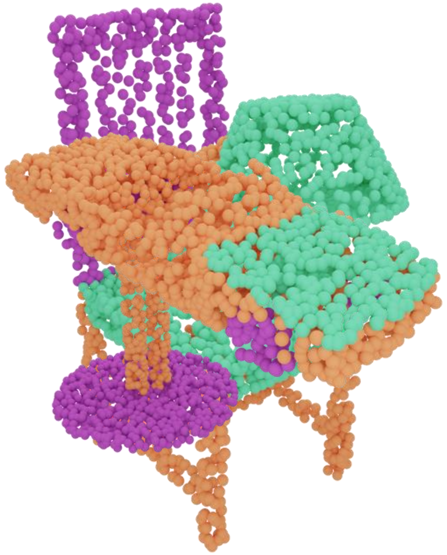
Irregular sampling

Curved domain

Local coordinates (charts)

Chapter 1: Basics

Deep learning is a machine learning method

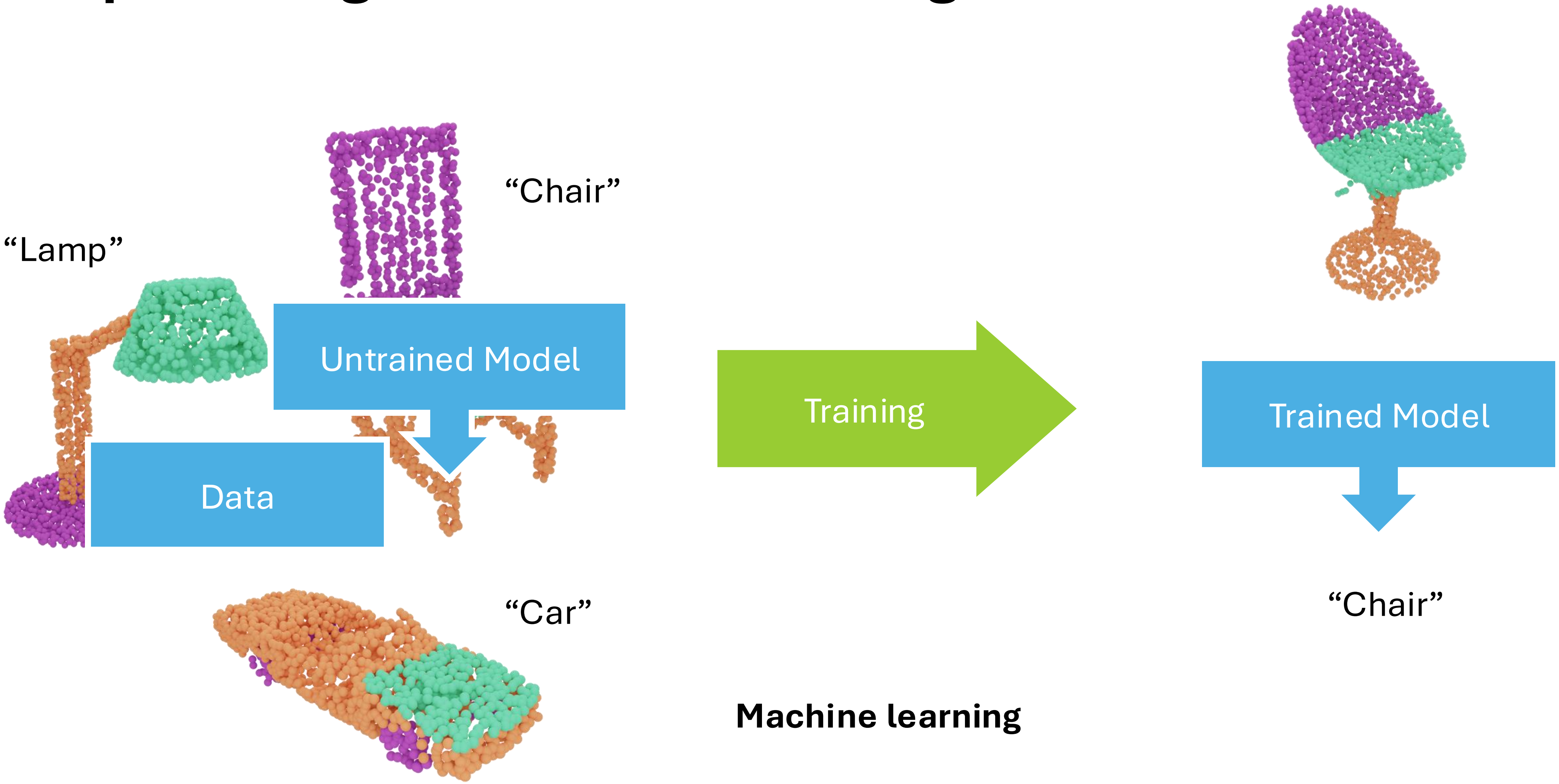


Algorithm

“Chair”
“Lamp”

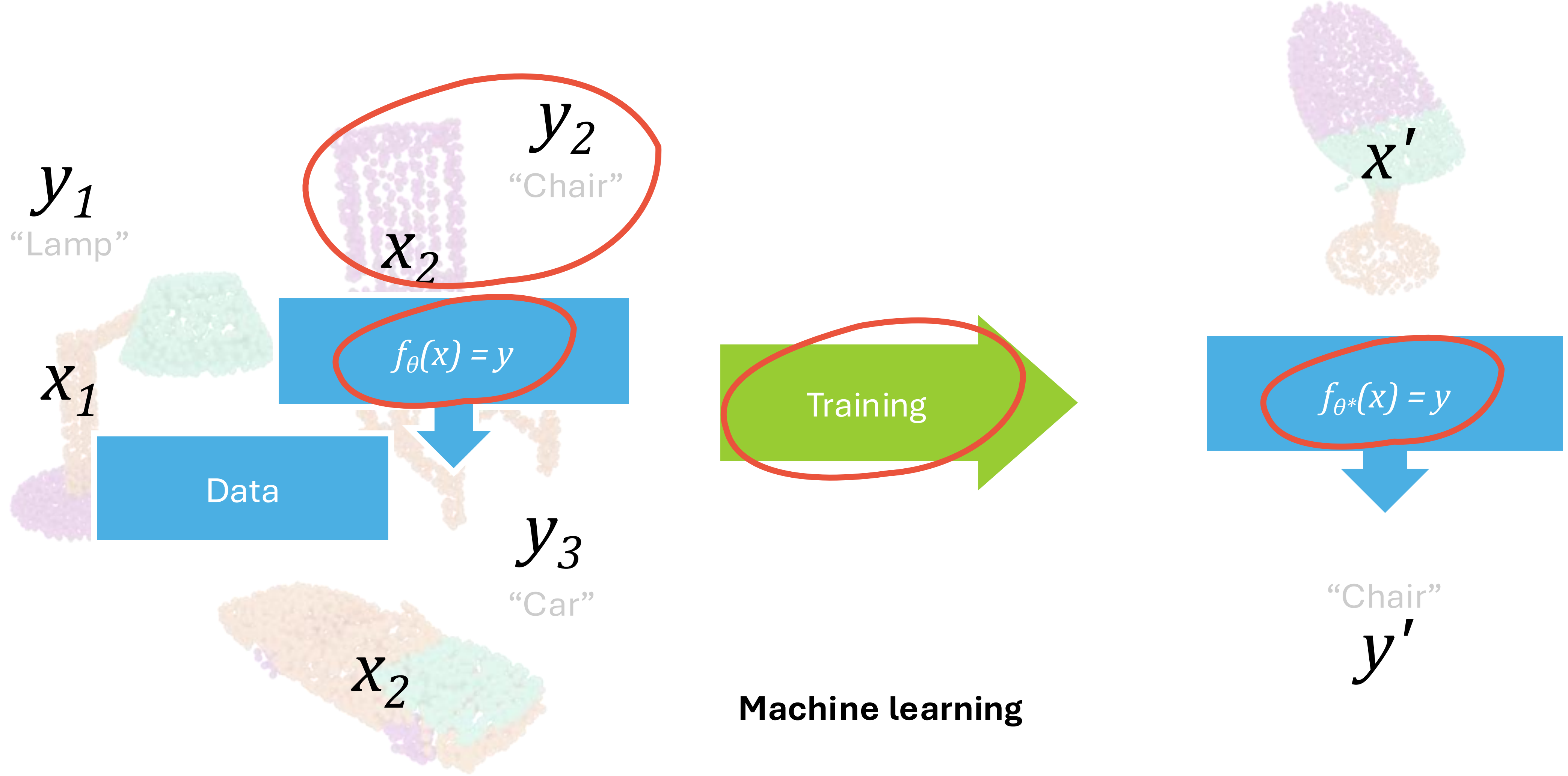
“Classic” approaches

Deep learning is a machine learning method



Machine learning

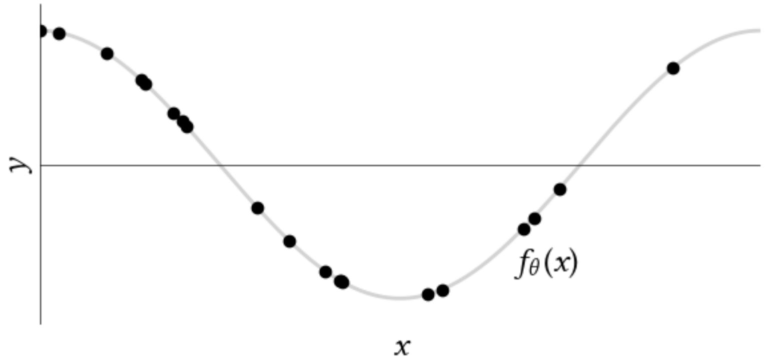
Deep learning is a machine learning method

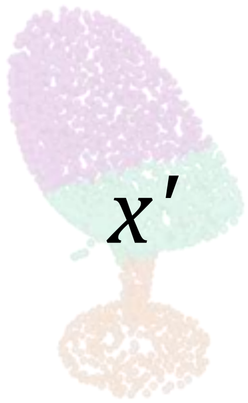


Machine learning

Machine learning basics

$f_\theta(x_i)$





$f_{\theta^*}(x) = y$

"Chair"

y'

Deep learning basics

θ are the unknowns, not x

$$\theta^* = \underbrace{\text{argmin}_\theta}_{\text{Training}} \underbrace{\sum_i^N \|\overbrace{f_\theta(x_i)}^{\text{Model}} - y_i\|_2^2}_{\text{Loss}}$$

- $f_\theta(x)$ is non-linear → numerical optimization
- θ is high-dimensional → gradient descent (instead of higher order methods)
- N is large → stochastic gradient descent (only a few x, y pairs at a time)

Characterization of deep learning

Data	Model	Optimizer	Hardware
Large datasets	Non-linear, layered	Gradient Descent	GPUs
	High # parameters	Backpropagation	
	End-to-end		

'Philosophy': Scaling (data, compute) beats algorithmic complexity

Is deep learning all you need?

Data	Model	Optimizer	Hardware
Small dataset	Non-linear, layered	Gradient Descent	GPUs
	High # parameters	Backpropagation	
	End-to-end		

Do you really need the complexity?

‘Philosophy’: Scaling (data, compute) beats algorithmic complexity

Is deep learning all you need?

Data	Model	Optimizer	Hardware
Small dataset	Linear model	Gradient Descent	GPUs

Better optimizers are available

Tip: Numerical Algorithms (Solomon)

‘Philosophy’: Scaling (data, compute) beats algorithmic complexity

“It’s all about the data” – Alexei Efros

e.g., https://www.youtube.com/watch?v=M1VHu1d4sGQ

Data	Model	Optimizer	Hardware
Large datasets	kNN		GPUs

‘Philosophy’: Scaling (data, compute) beats algorithmic complexity

Characterization of deep learning

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‘Philosophy’: Scaling (data, compute) beats algorithmic complexity

Typical models

- MLP – Multi-layer perceptron
  - The Perceptron: A Probabilistic Model For Information Storage And Organization in the Brain – Rosenblatt (**1958**)
- CNN – Convolutional Neural Network
  - Gradient-based learning applied to document recognition – Lecun et al. (**1998**)
- Transformer (Attention)
  - Attention Is All You Need – Vaswani et al. (**2017**), attention was around before that

Multi-Layer Perceptron (geometric perspective)

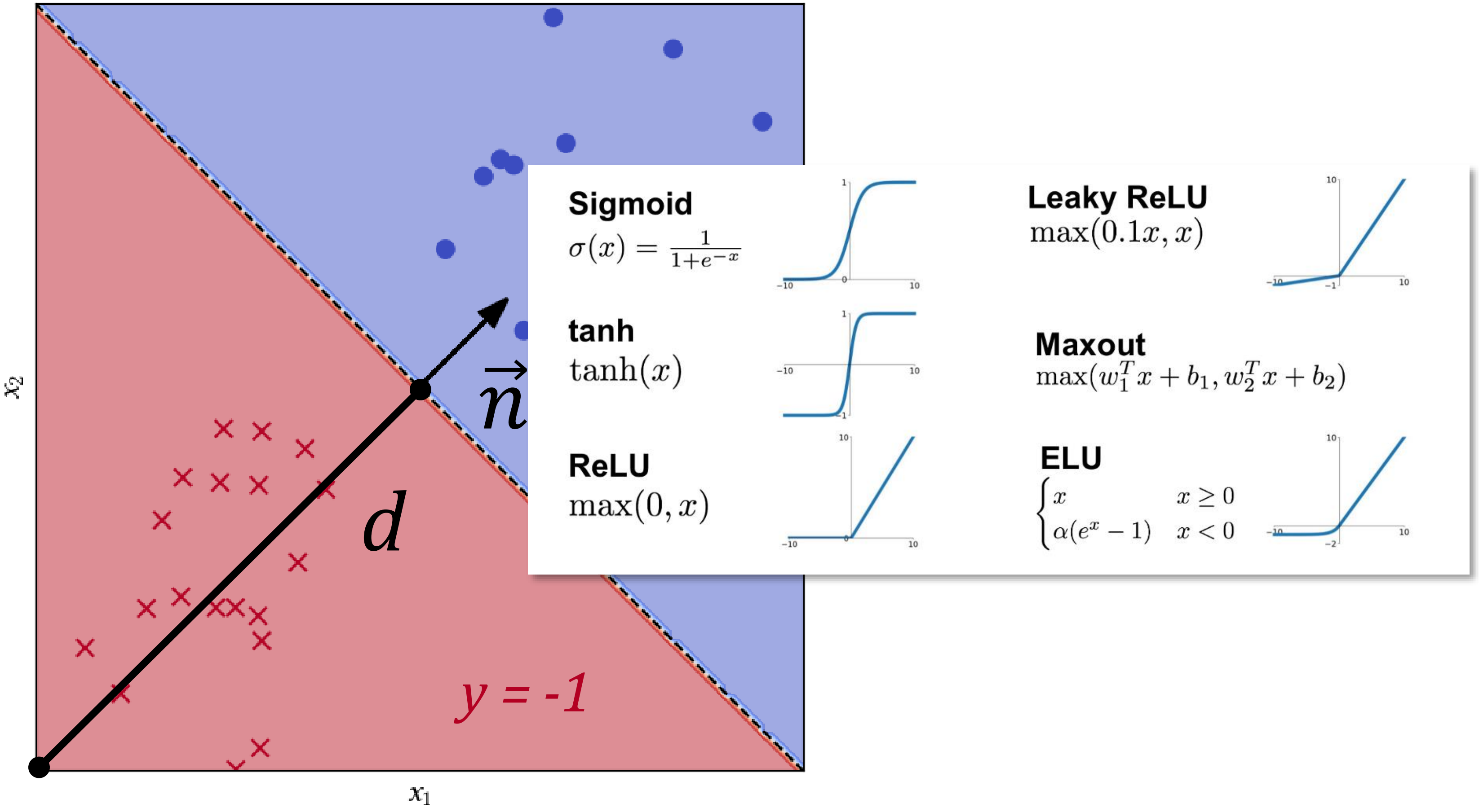
→ Separate these points

- Distance to line/plane

  $$\vec{x} \cdot \vec{n} - d$$

- Step function

  $$y = \sigma(\vec{x} \cdot \vec{n} - d)$$



https://medium.com/@shrutijadon/survey-on-activation-functions-for-deep-learning-9689331ba092

Multi-Layer Perceptron (geometric perspective)

→ Separate these points

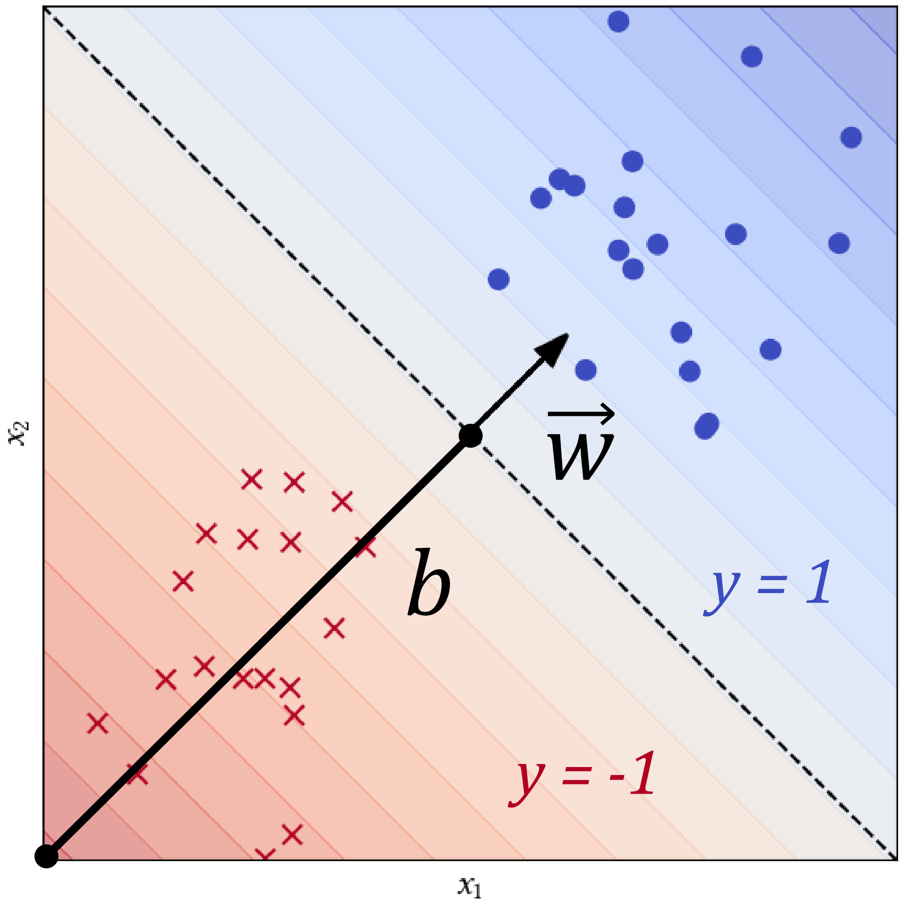
- Distance to line/plane

$$\vec{w}^T\vec{x} + b$$

- Step function

$$y = \sigma(\vec{w}^T\vec{x} + b)$$

- Weights, biases
- σ activation function
  - Non-linear!



Multi-Layer Perceptron

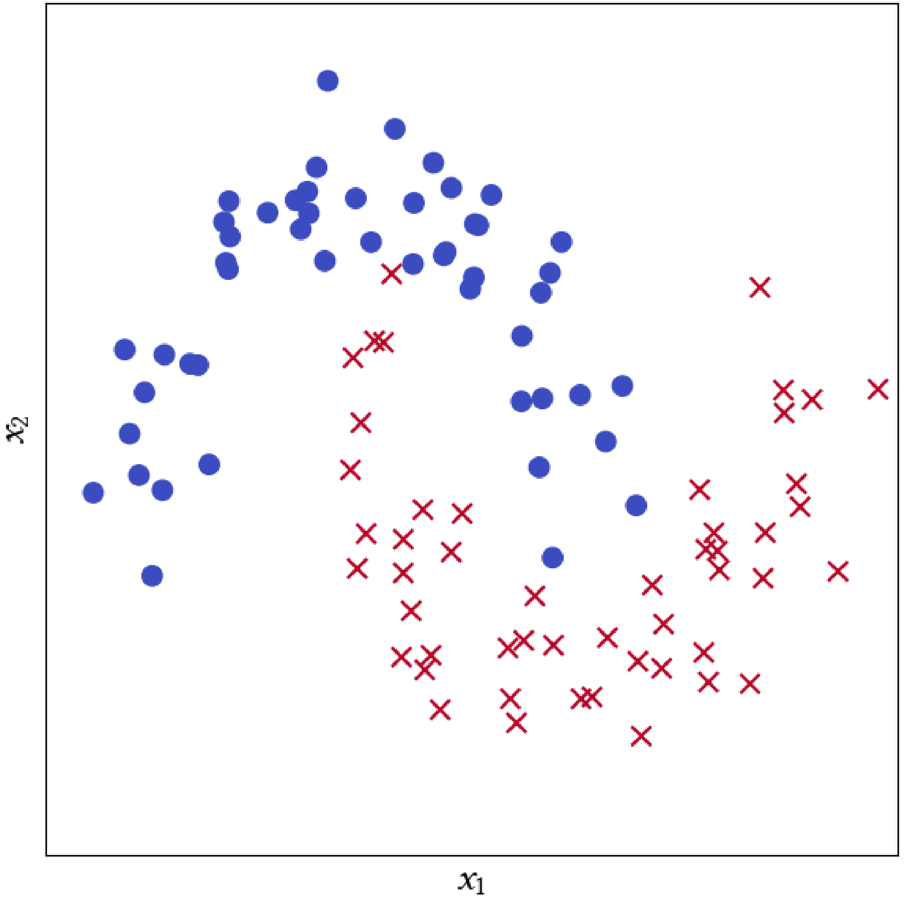
$$y = \sigma(\vec{w}^T \vec{x} + b)$$

→ Multiple outputs? | "Channels"

$$\vec{y} = \sigma(W\vec{x} + \vec{b})$$

→ Not linearly separable? | "Layers"

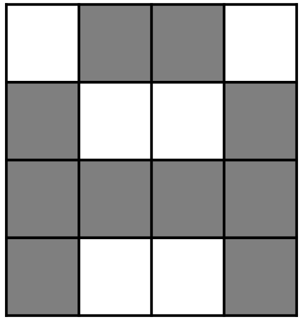
$$\vec{y} = \underbrace{\qquad\qquad\qquad}_{\text{Hidden layers}} \sigma\left(W_1\vec{x} + \vec{b}_1\right)$$



Multi-Layer Perceptron Notes

- Examples in 2D, but $\vec{x}$ can be of any dimension
- Interpretation: cutting up space with halfspaces
- "Multilayer Feedforward Networks [MLPs] are Universal Approximators"
  - Hornik, Stinchcombe and White (1989)
  - Non-linearity is necessary!
  - Compare, e.g., Fourier transform
- Higher complexity, more risk of overfitting

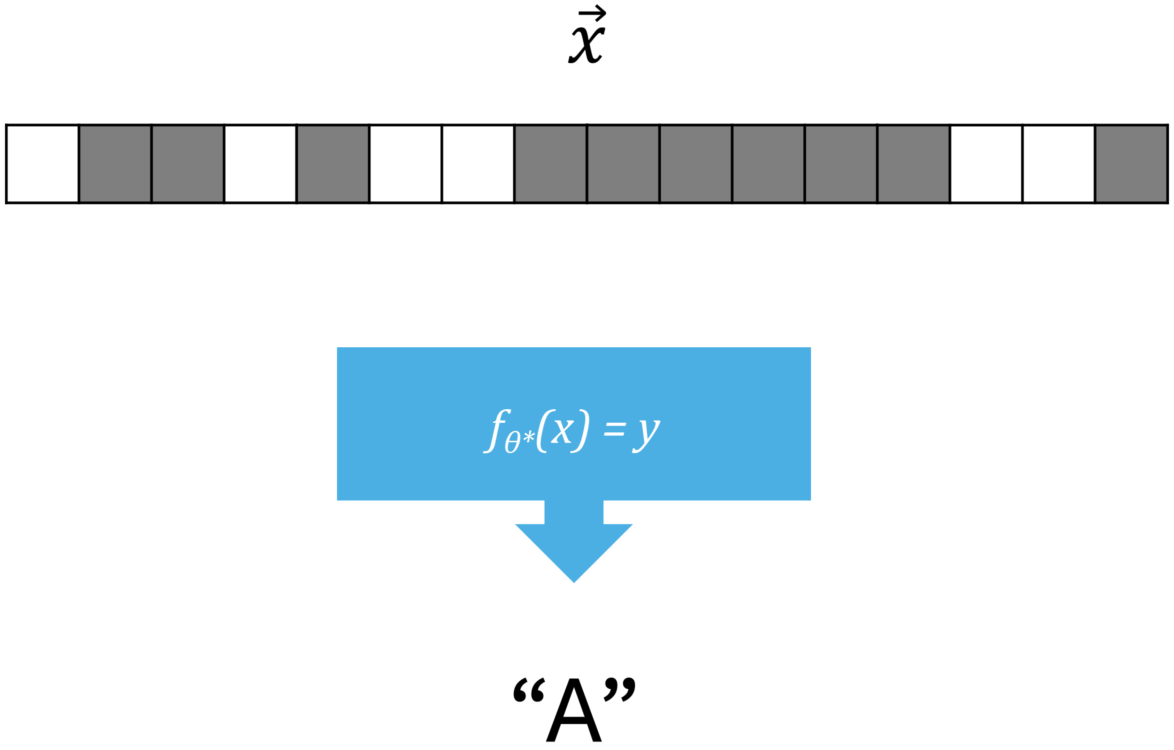
MLP on images



$f_{\theta^*}(x) = y$

“A”

MLP on images



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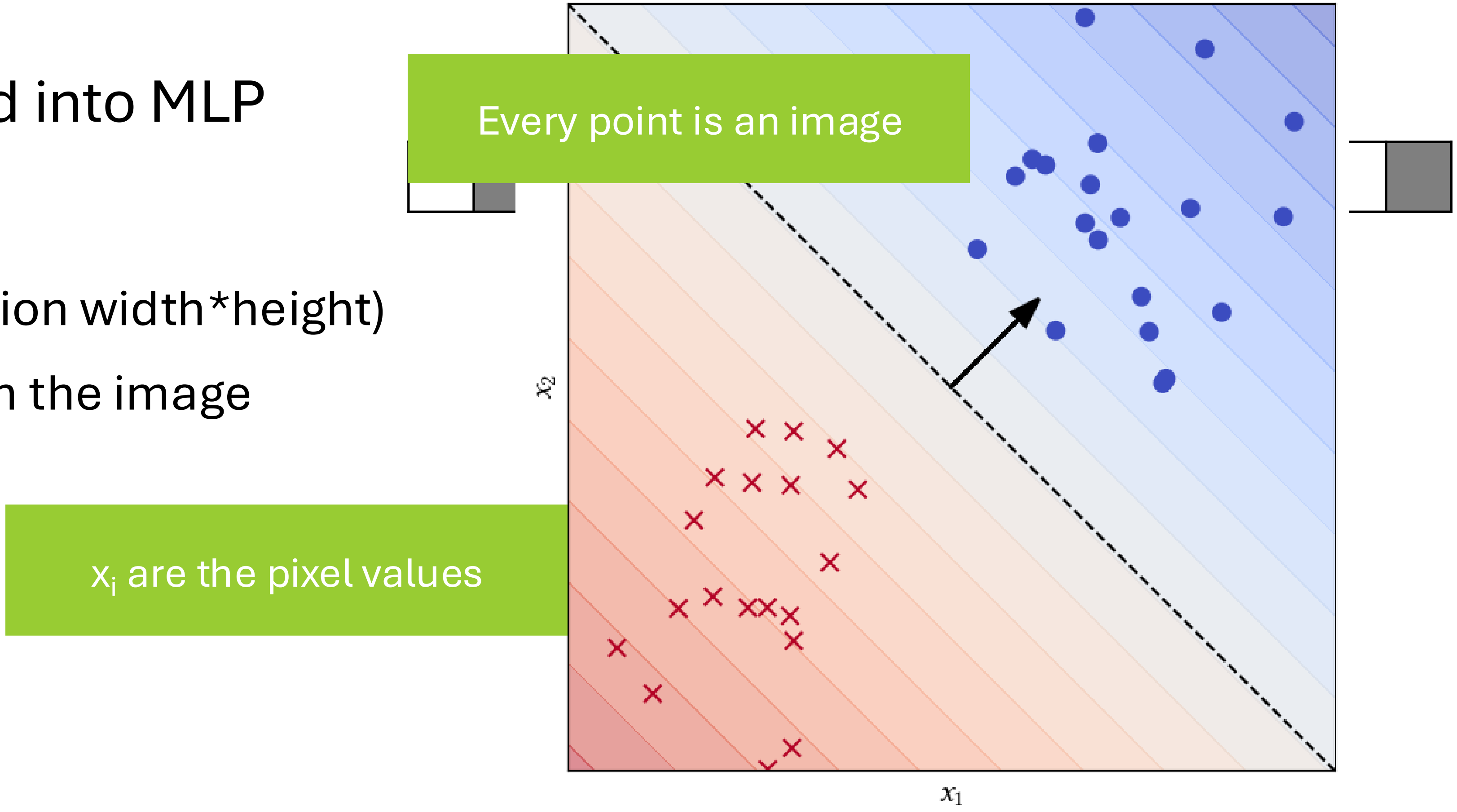
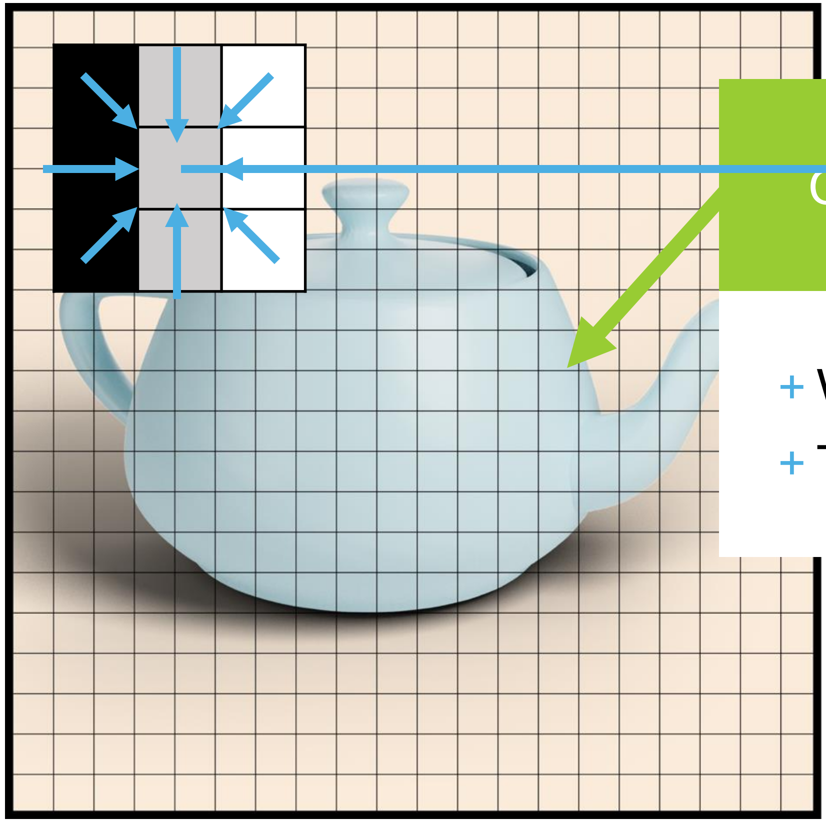
MLP on images

- Stack all pixel values, feed into MLP
- Problem?
  - Not efficient ($\vec{x}$ is of dimension width*height)
  - Patterns can be anywhere in the image

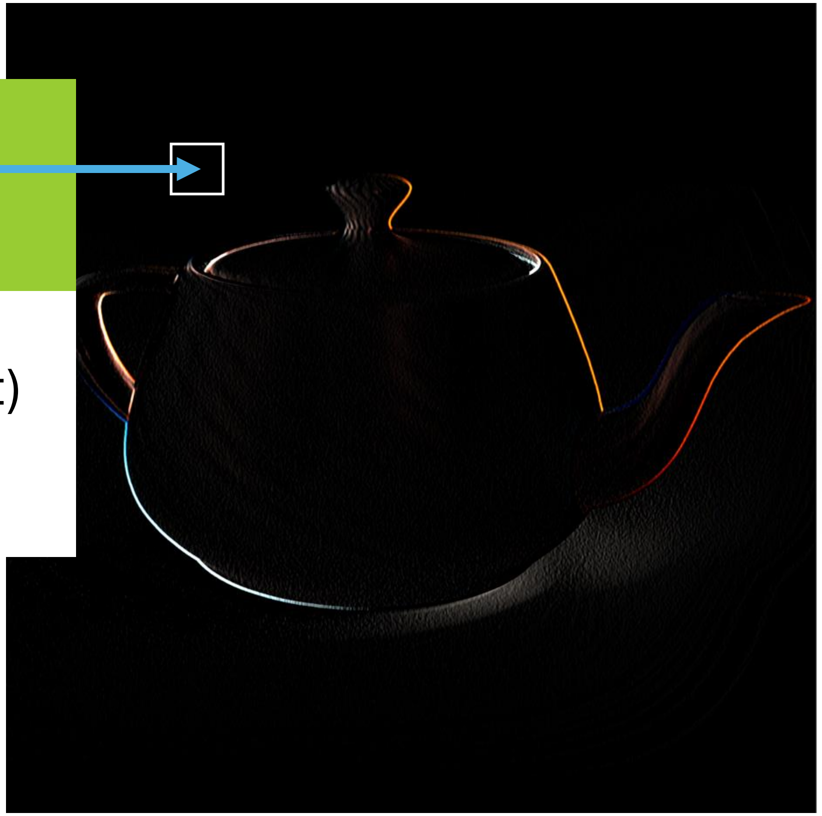


Convolutional Neural Network



Optimize kernel weights

+ Weight sharing (efficient)
+ Translation invariance



Schematics

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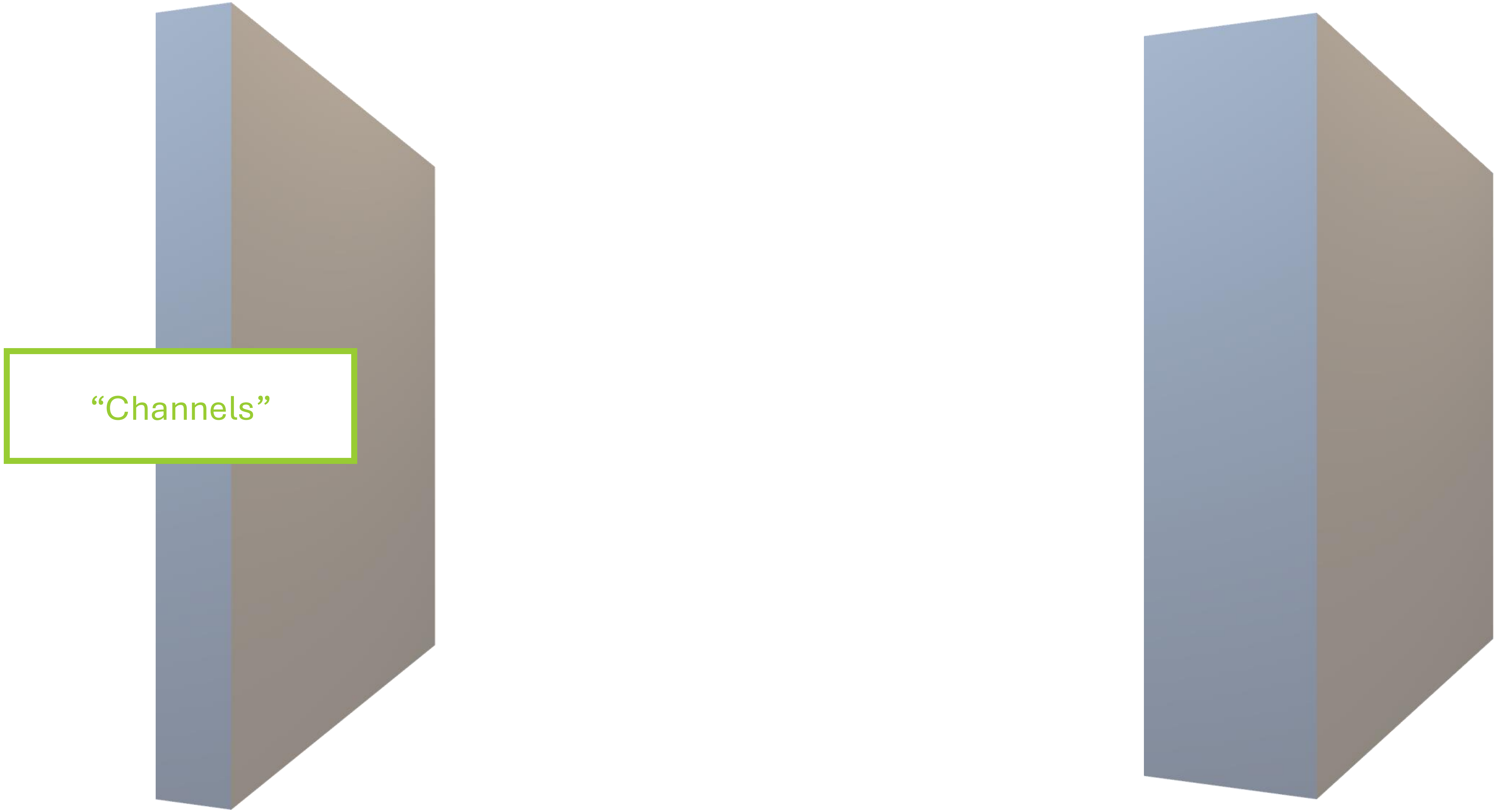
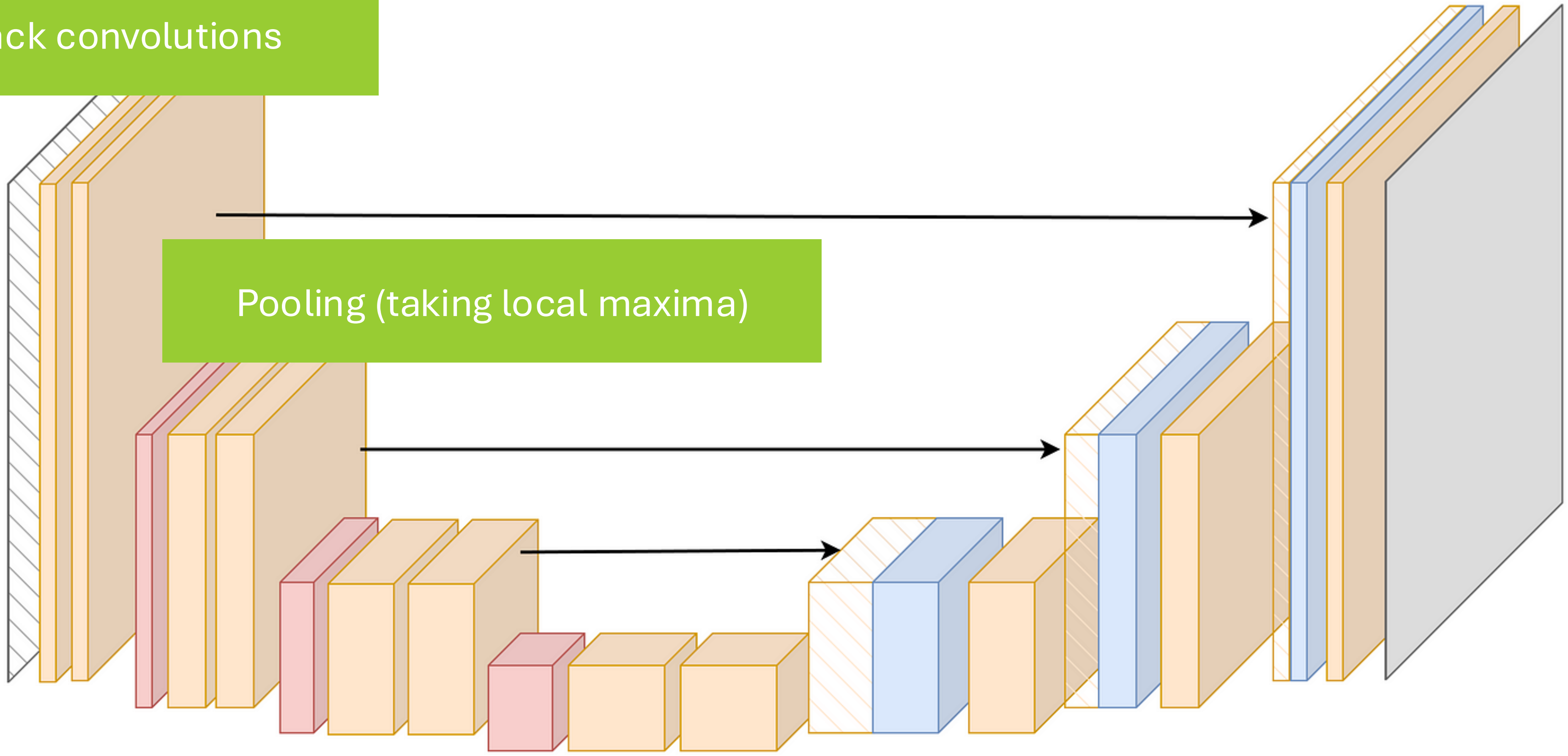
Schematics



Convolutional Neural Networks

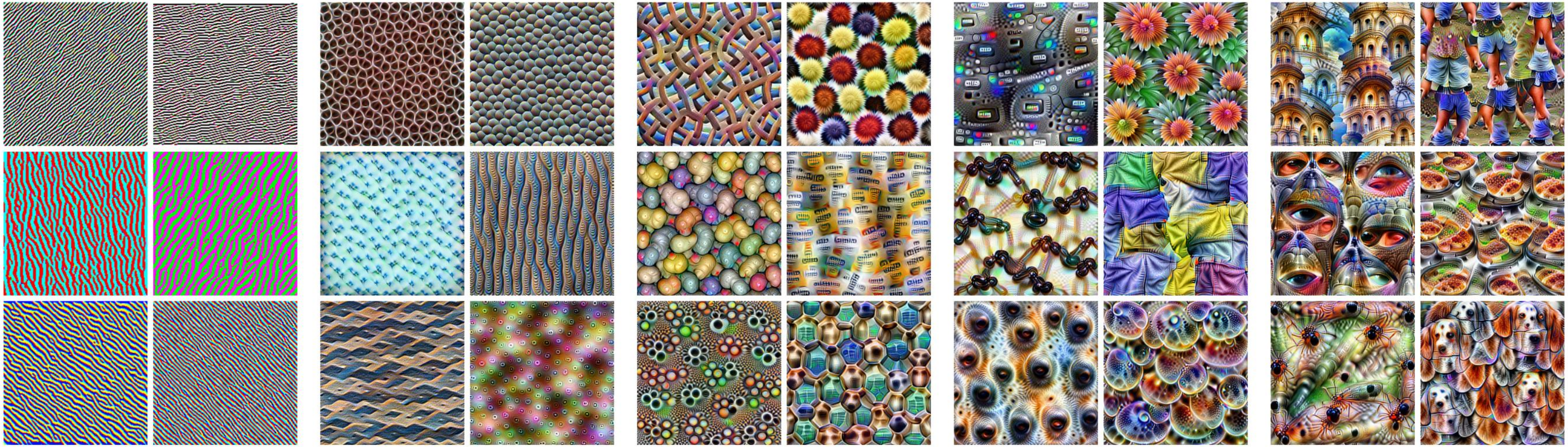
Stack convolutions

Pooling (taking local maxima)



https://medium.com/data-science/u-net-explained-understanding-its-image-segmentation-architecture-56e4842e313a

Hierarchies of Features in CNNs

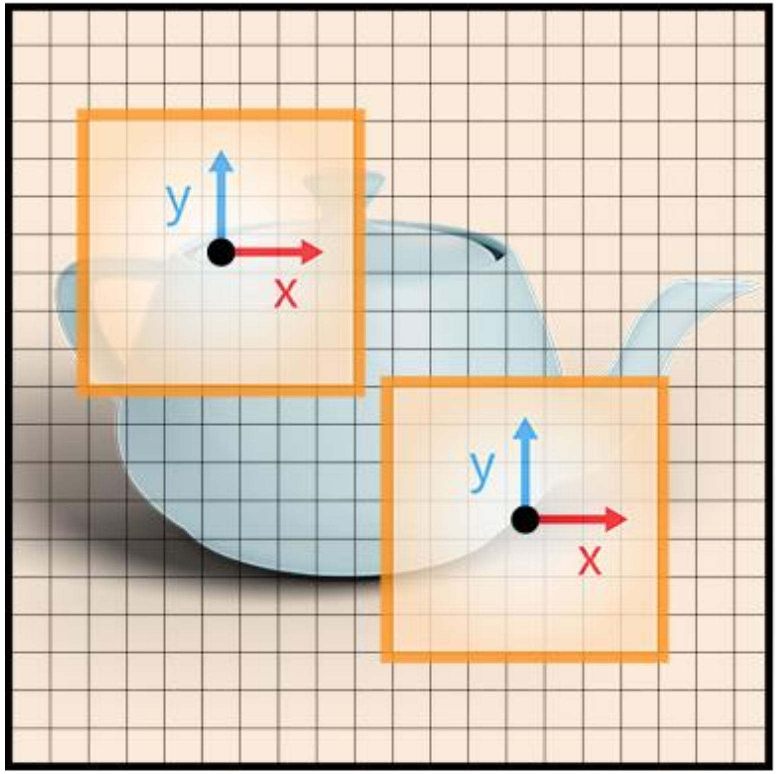


Edges (layer conv2d0) **Textures** (layer mixed3a) **Patterns** (layer mixed4a) **Parts** (layers mixed4b & mixed4c) **Objects** (layers mixed4d & mixed4e)

Feature Visualization, Olah, Mordvintsev, Schubert (2017) – https://distill.pub/2017/feature-visualization/

Convolutional Neural Networks Notes

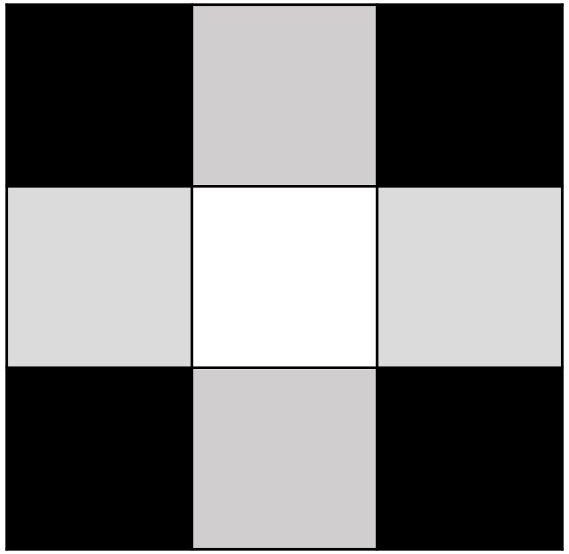
- Interpretation: MLP on patches
- Consistent pixel grid helps
  - Orientation
  - Maps to hardware well
- Downside (?) Template is fixed



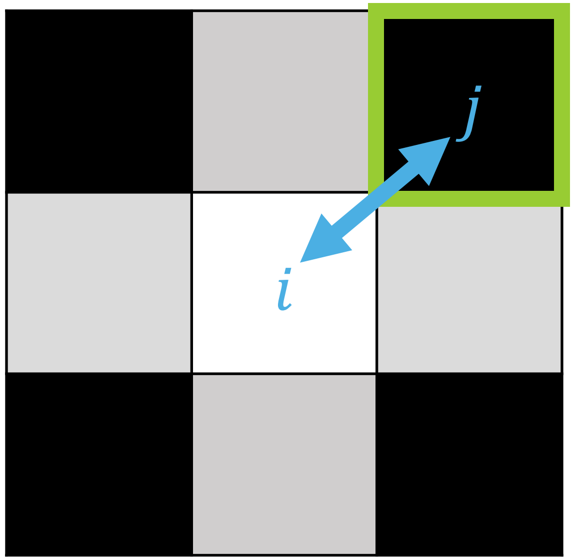
Limitation of Convolution Kernels

Kernel values based on location

Can we adapt to the signal?



Limitation of Convolution Kernels



Gaussian kernel g

$$x_i' = \sum_{j \in \mathcal{N}_i} \boxed{g\left(\underline{\|p_j - p_i\|}\right)} x_j$$

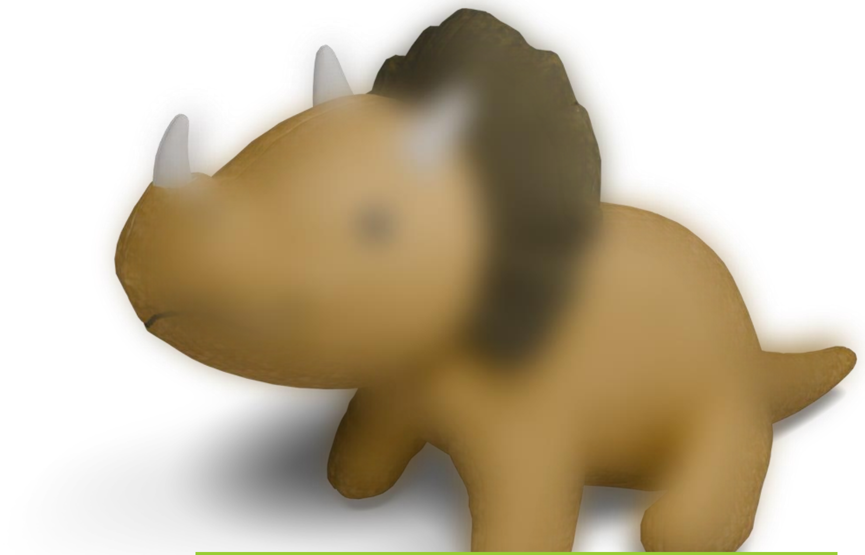
Based on distance

Denoising with Gaussian filter vs. Bilateral filter

$$x_i' = \sum_{j \in \mathcal{N}_i} g(\|p_j - p_i\|) x_j$$

$$x_i' = \sum_{j=0}^{N} \boxed{f(\|x_j - x_i\|)} g(\|p_j - p_i\|) x_j$$

Look at pixel value as well



Isotropic blurring

Preserves features

Self-attention (Transformers)

$$x_i' = \sum_{j=0}^{N} f\left(\boxed{\|x_j - x_i\|}\right) g(\|p_j - p_i\|) x_j$$

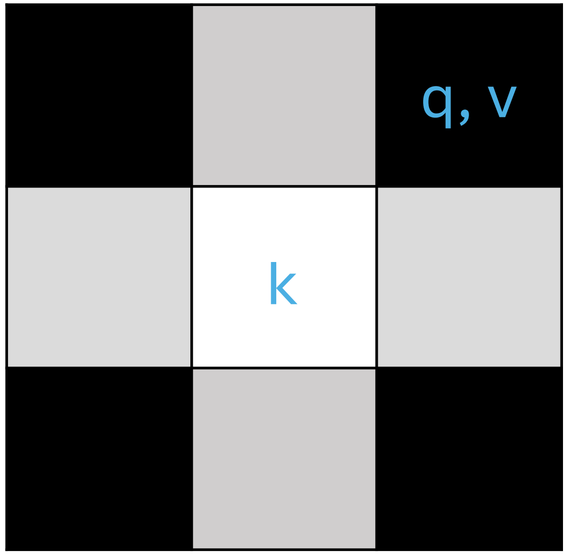
- Replace distance with cosine similarity

$$x'_i = \sum_{j=0}^{N} \boxed{\left(x_j^T x_i\right)} x_j$$

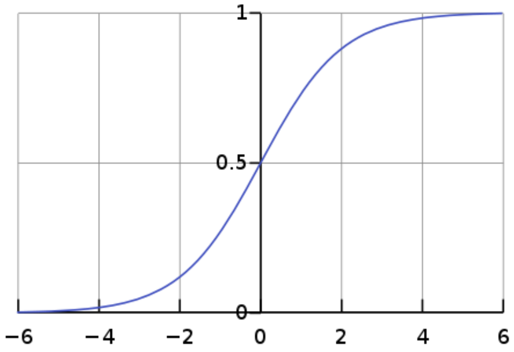
- Add flexibility with weight matrices and make sure weight 'behaves'

$$x_i' = \sum_{j=0}^{N} \mathrm{softmax}\left(\left(\mathrm{W}_q x_j\right)^T \mathrm{W}_k x_i\right) \mathrm{W}_v x_j$$

Self-attention (Transformers)



$$x_i' = \sum_{j=0}^{N} \underbrace{\text{softmax}}\Big(\underbrace{(W_q x_j)}_{\text{Query}}^T \underbrace{W_k x_i}_{\text{Key}} \Big) \underbrace{W_v x_j}_{\text{Value}}$$



Self-attention (Transformers)

$$x_i' = \sum_{j=0}^{N} \mathrm{softmax}\left(\left(\mathrm{W}_q x_j\right)^T \mathrm{W}_k x_i\right) \mathrm{W}_v x_j$$

- Transformers combine these blocks
- Positional encoding based on $p_j - p_i$

- Pro: Highly flexible, very effective (LLMs, Vision Transformers, etc.)
- Pro: ‘Global’ connections (vs. U-Net)
- Con: Computationally expensive (compared to CNN)
  - Solutions: work at coarse scale OR only apply locally

Summary

- **Multi-Layer Perceptrons (MLP)**
  - Linear combination, followed by non-linearity, repeated
  - Basic building block of Neural Networks
- **Convolutional Neural Network (CNN)**
  - Learn local kernel, convolve
  - Weight sharing, translation invariance
  - ‘Constructs’ localized features of increasing abstraction
- **Transformers**
  - Learn weights based on feature similarity
  - Highly expressive, but expensive; current SOTA for many tasks

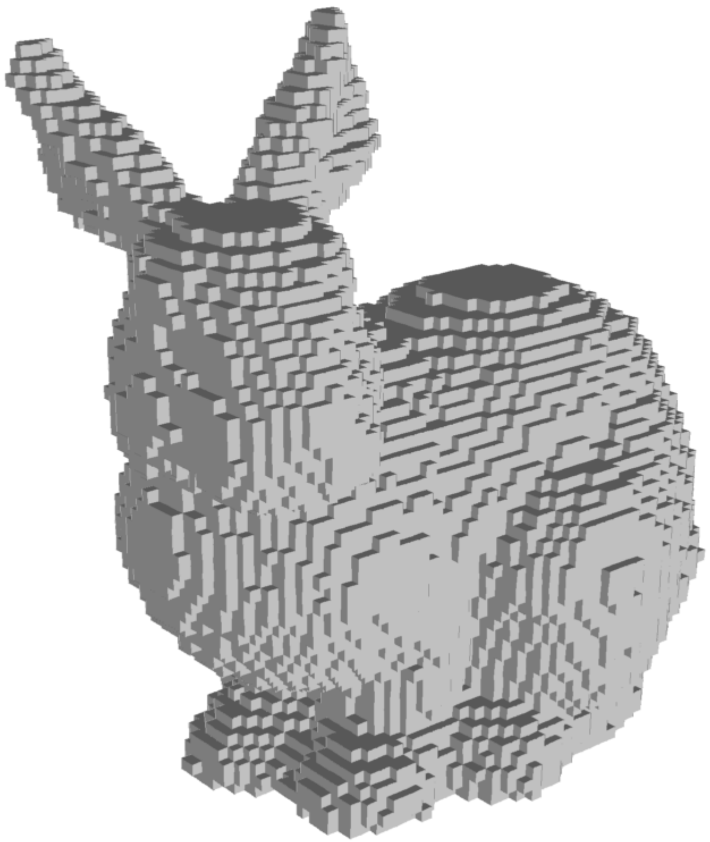
Chapter 2: 3D Data

Deep learning for 3D data

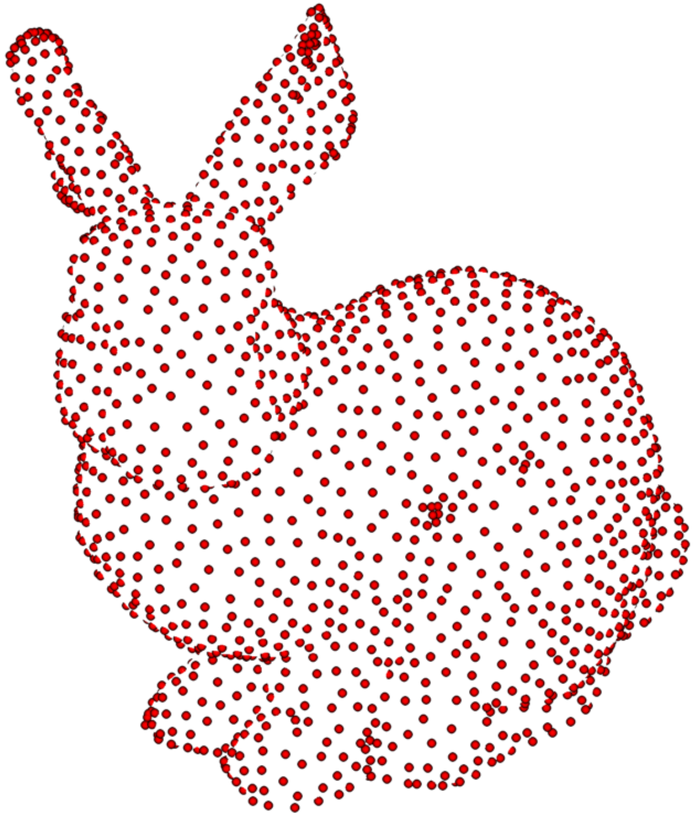
3D data	Model	Optimizer	Hardware
Large datasets	Layered, deep	Gradient Descent	GPUs
	High # parameters	Backpropagation	
	End-to-end		

'Philosophy': Scaling (data, compute) beats algorithmic complexity

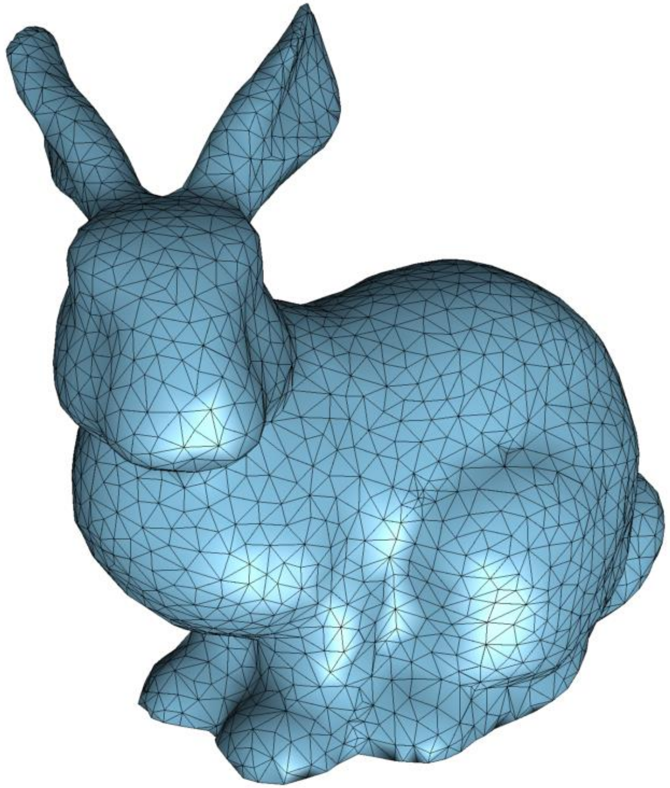
3D Data



Voxels



Point cloud



Mesh

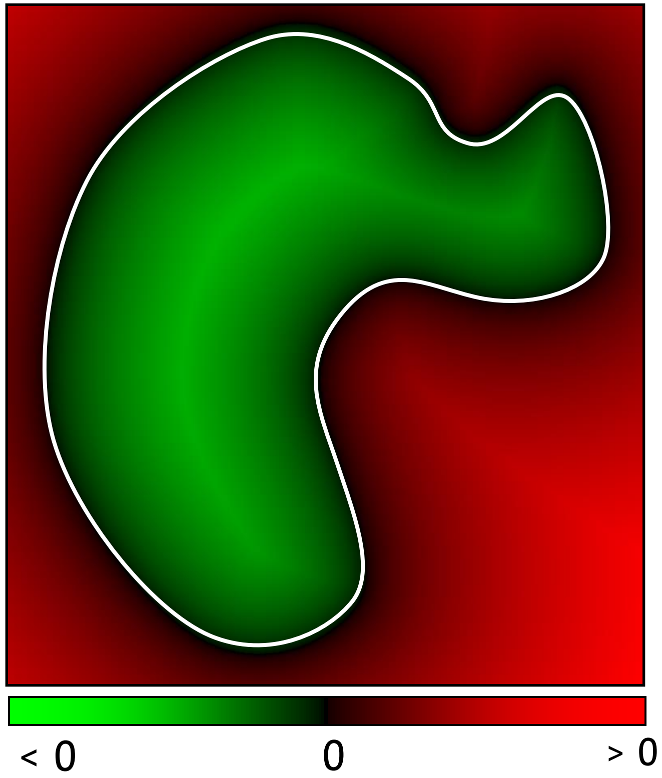
What about implicit functions?

$f: \mathbb{R}^3 \rightarrow \mathbb{R}$

Value > 0 outside shape, < 0 inside

In deep learning

- Use MLP to represent f
- Rarely used as input to MLP
  - Convert to voxels/mesh/point cloud

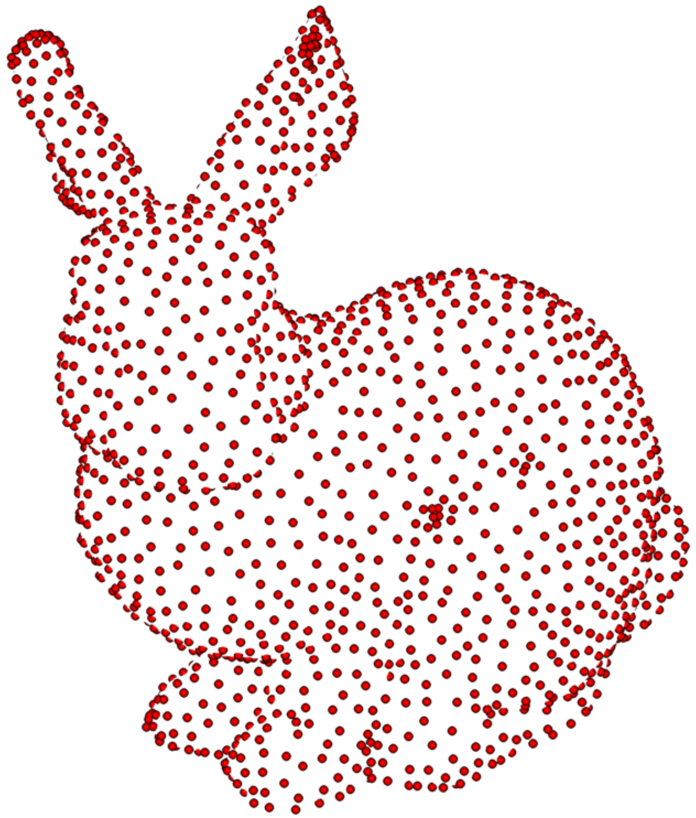


Based on slides by Olga Sorkine-Hornung

Point Cloud

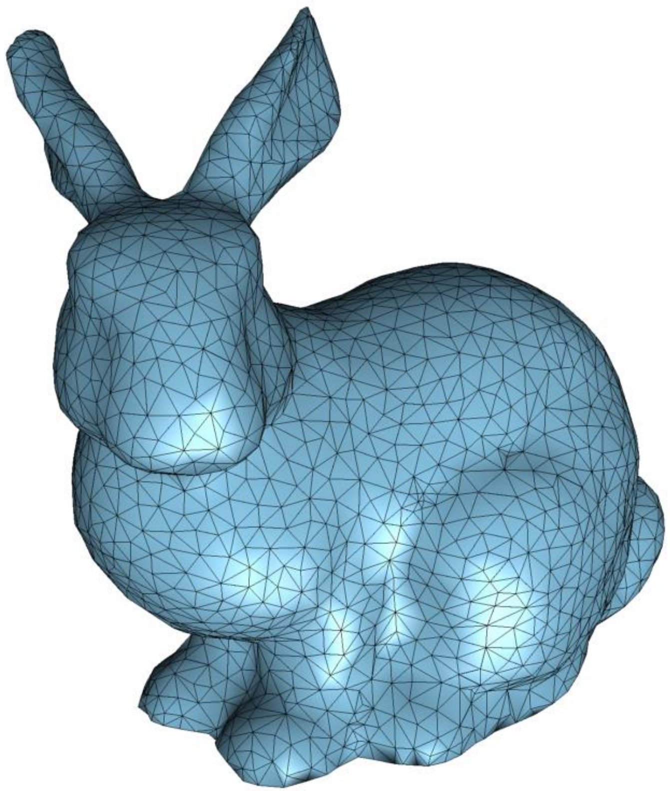
- Set of points in d-dimensional space

$$P \in \mathbb{R}^3$$



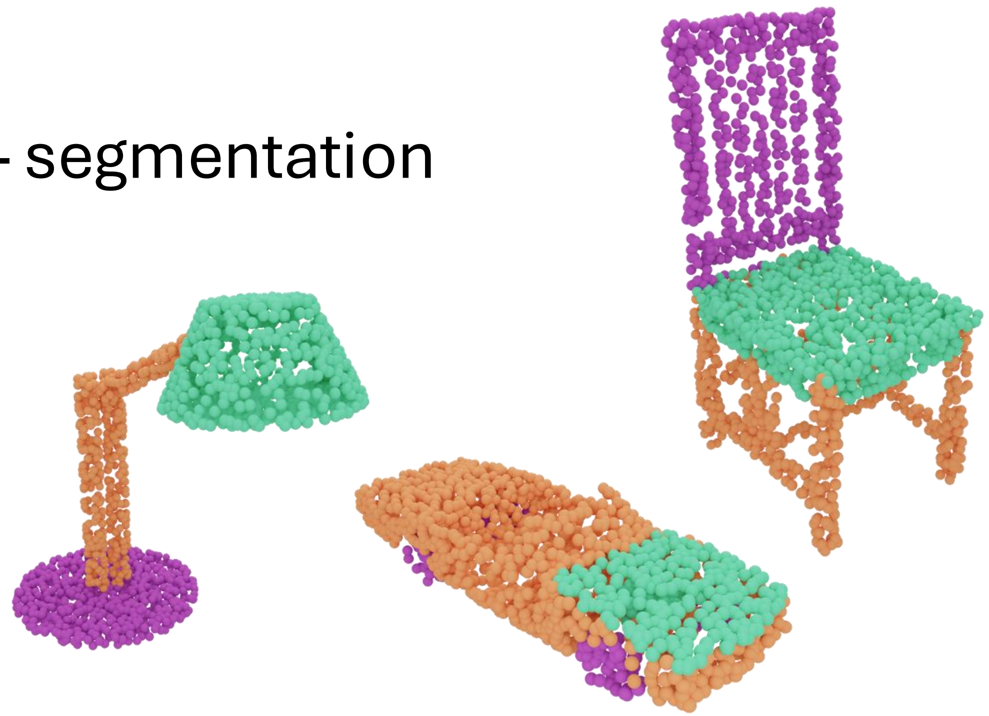
Mesh

- ‘Complements’ point cloud
  - connectivity and surface given by V, E, F
- Geometry can be on points, edges, etc.



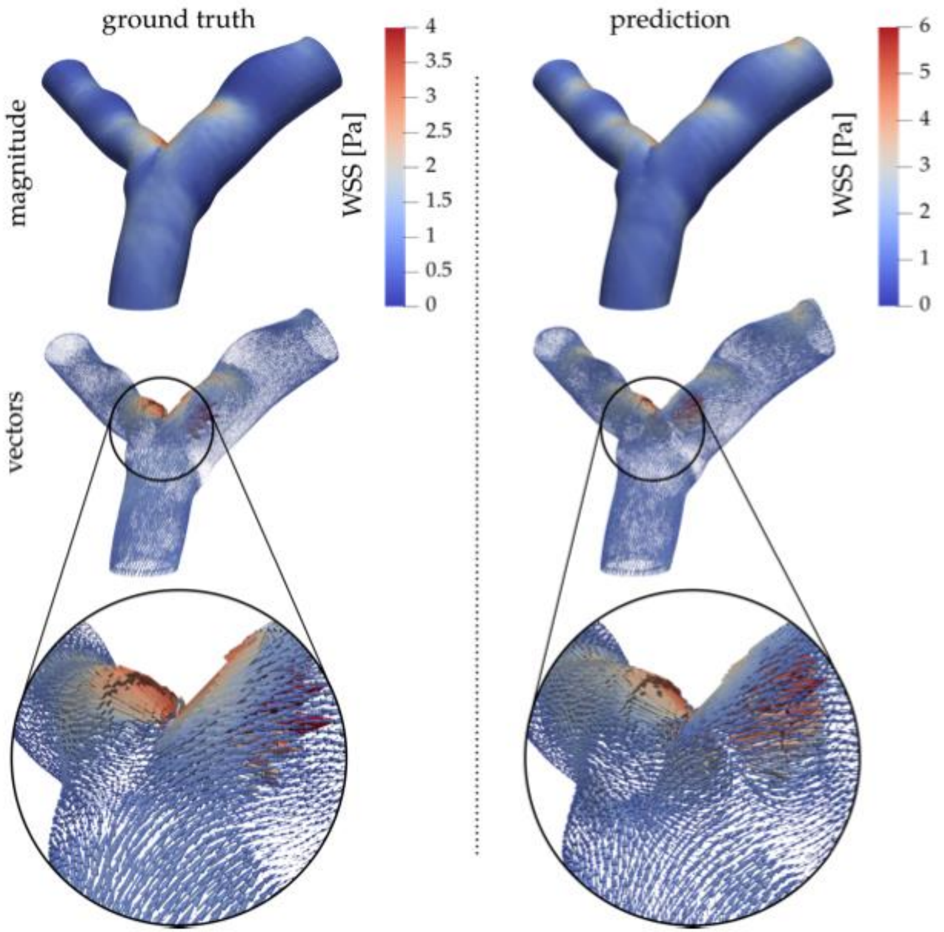
Datasets

- **ShapeNet** (meshes, point clouds) – segmentation, classification
- **ModelNet40** (meshes) – classification
- **ScanNet (v2, v3)** (RGB-D + reconstructions) – segmentation
- **S3DIS** (point clouds) – segmentation
- **Thingi10k** (meshes) – n/a
- **ABC** (meshes) – n/a
- Consider ethics, copyright!
  - E.g., https://huggingface.co/datasets/allenai/objaverse/discussions/18



More exotic inputs and outputs: simulation

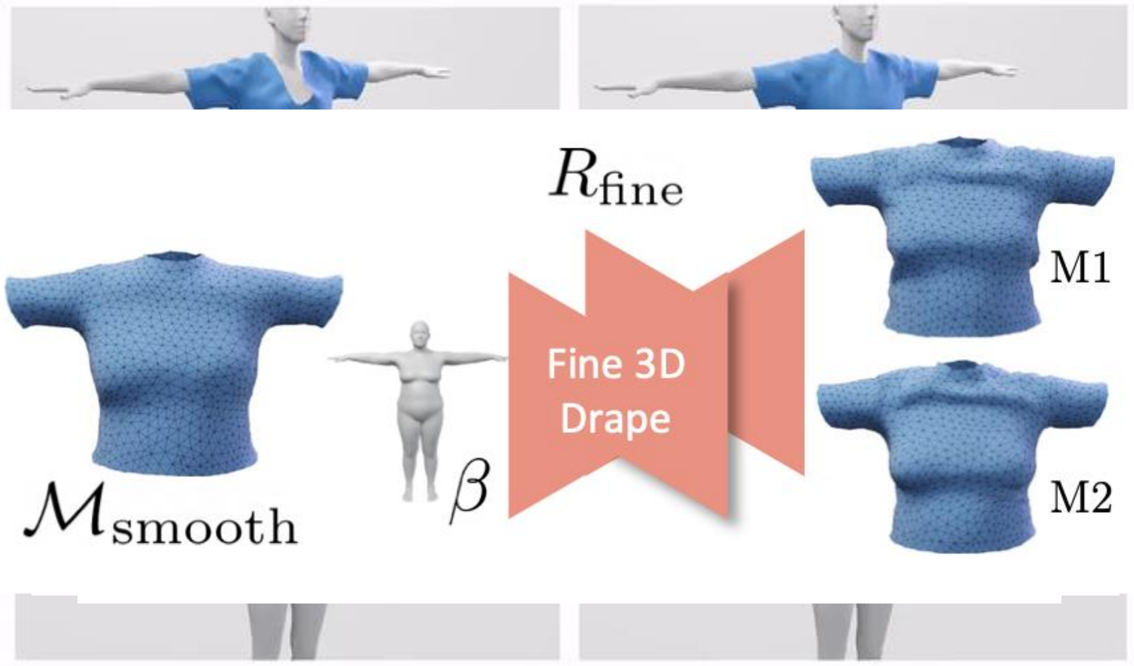
- **Input:** positions, distance to loops
- **Output:** tangent vector at each vertex



Mesh Neural Networks for SE(3)-Equivariant Hemodynamics Estimation of the Artery Wall, Suk et al. (2022)

More exotic inputs and outputs: simulation

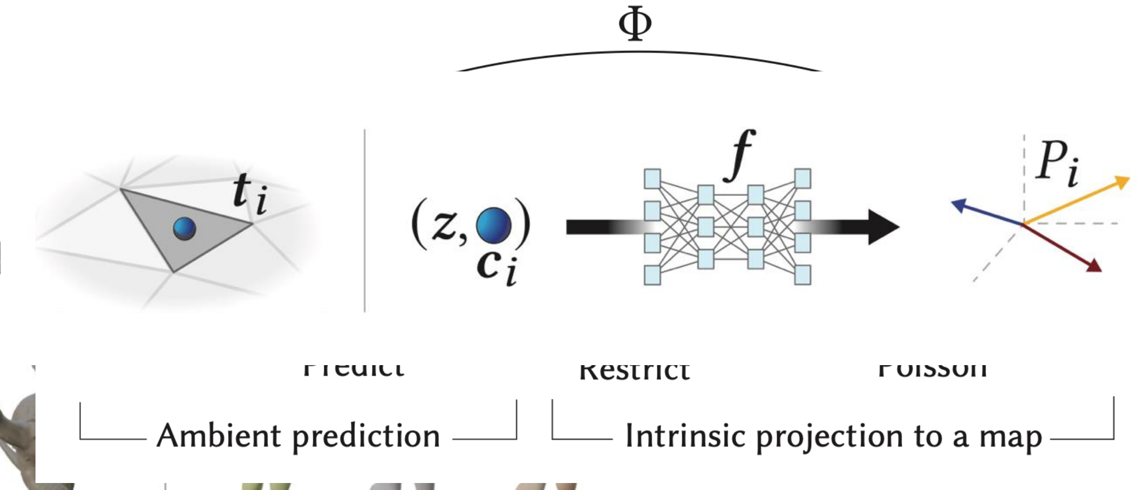
- Stage 1
  - **Input:** Garment parameters
  - **Output:** Mesh positions on mean shape
- Optimize topology
- Stage 2
  - **Input:** Mesh positions, target shape parameters
  - **Output:** Smooth mesh positions
- Stage 3
  - **Input:** Smooth mesh positions
  - **Output:** Fine mesh positions (wrinkles, etc.)

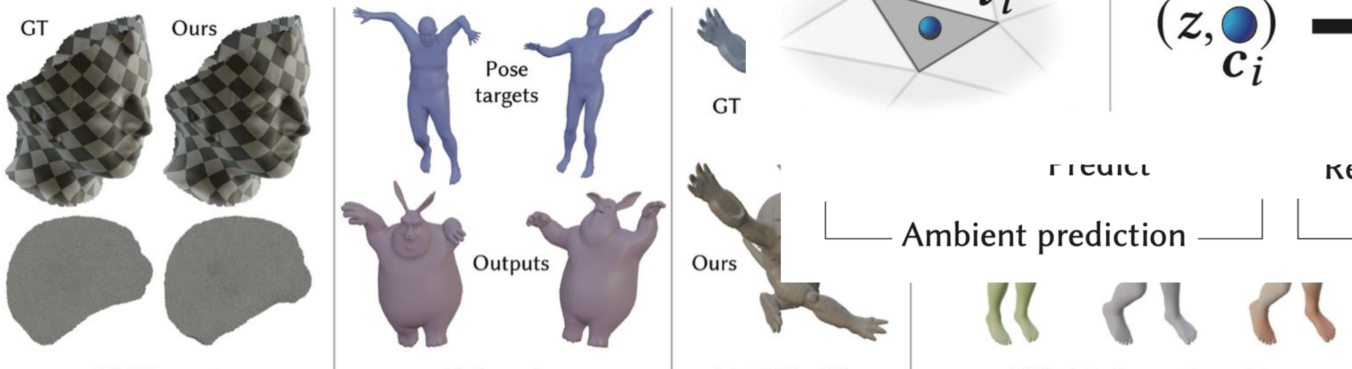


Fully Convolutional Graph Neural Networks for Parametric Virtual Try-On, Vidaurre et al. (2020)

More exotic inputs and outputs: Jacobians

- Neural Jacobian Fields
  - **Input:** Per-triangle centroid + global code
  - **Output:** Jacobian matrix
  - **Model:** MLP
- Post-proces
  - Restrict Jac
  - Solve Poiss



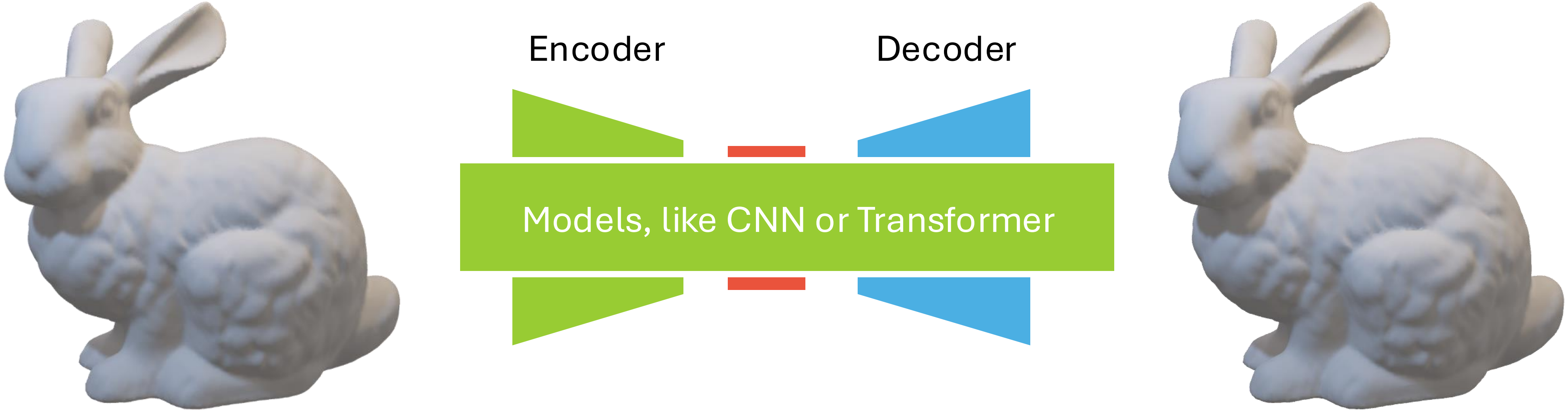


(a) UV mapping (b) Re-posing (c) ARAP editing (d) Registration and morphing

Neural Jacobian Fields: Learning Intrinsic Mappings of Arbitrary Meshes, Aigerman et al. (2022)

What if I have data, but no labels?

- Unsupervised learning
- For example: auto-encoders



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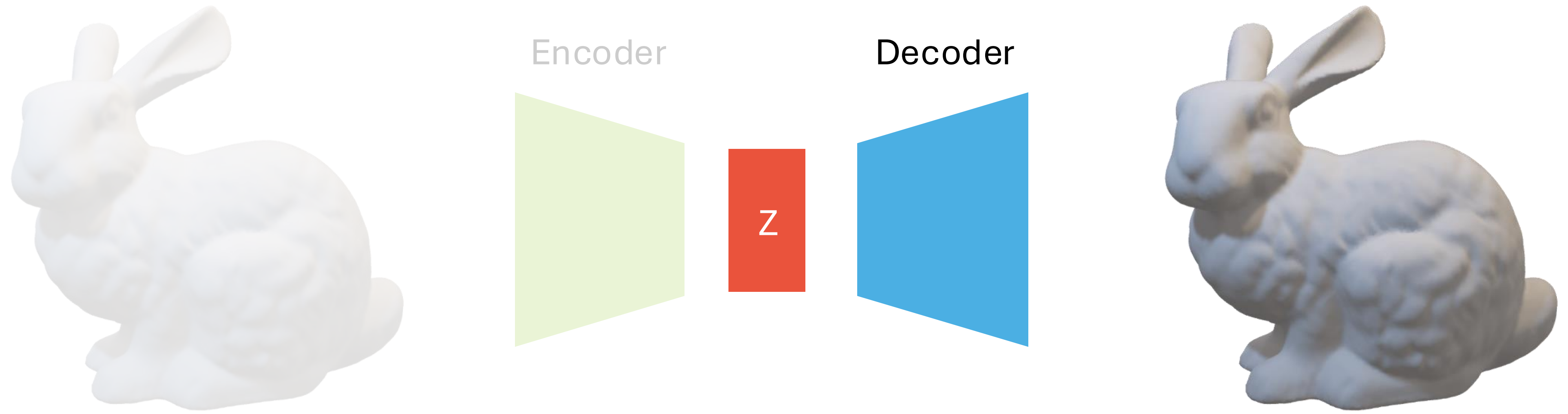
- Unsupervised learning
- For example: auto-encoders



Chapter 3:
3D Deep Learning Models

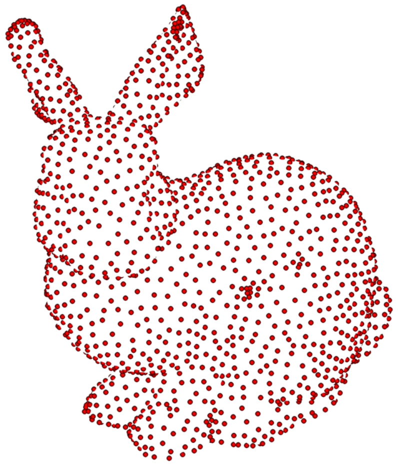
Deep learning for 3D data

3D data	Models for 3D	Optimizer	Hardware
Large datasets	Layered, deep	Gradient Descent	GPUs
	High # parameters	Backpropagation	
	End-to-end		

'Philosophy': Scaling (data, compute) beats algorithmic complexity

Let’s start simple

MLP



Stack all points?

Which order?

MLP

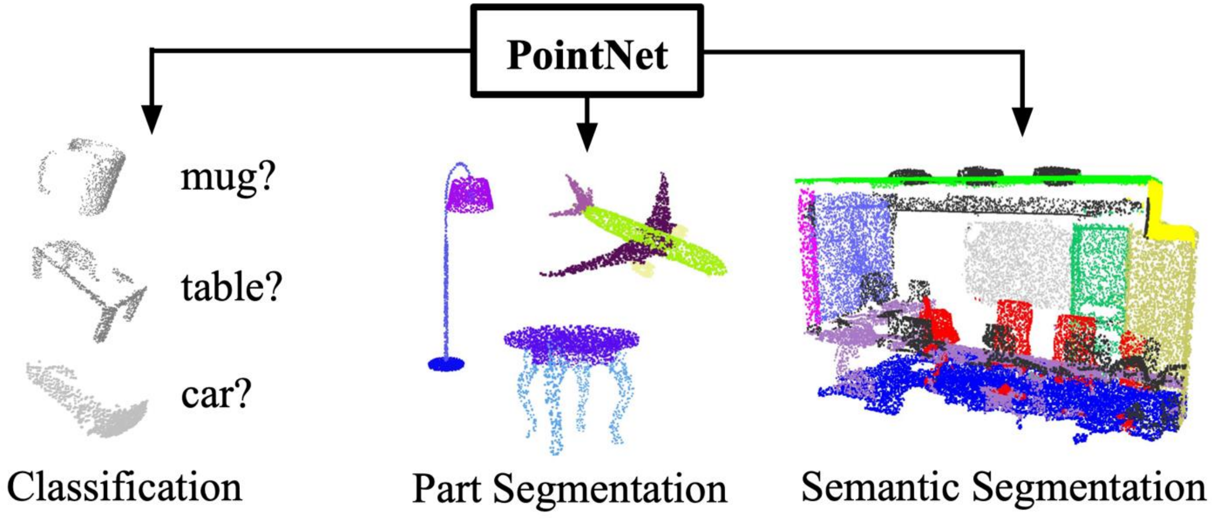
"Bunny"

PointNet

- MLP on each point
- Maximum over all points

$$y = \max_{j \in P} f_\theta(x_j)$$

Cannot learn from neighborhoods



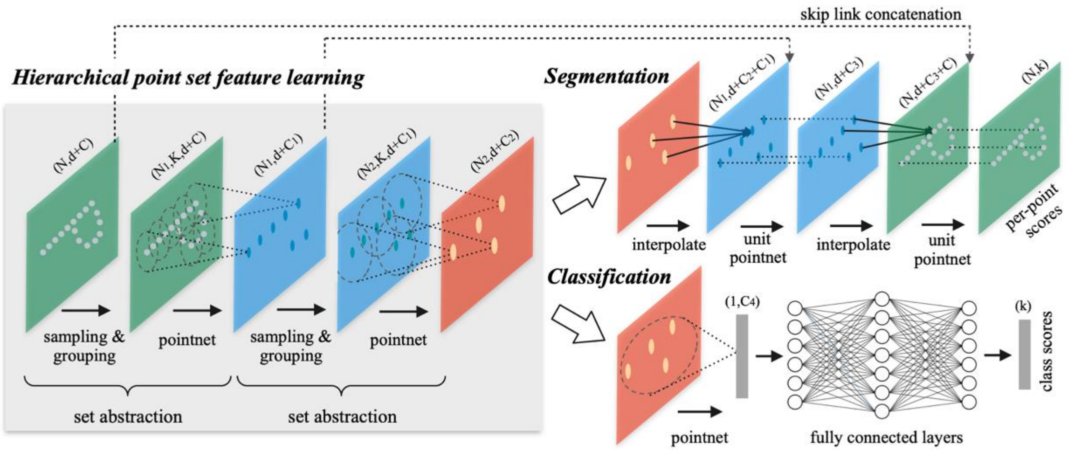
PointNet: Deep Learning on Point Sets for 3D Classification and Segmentation, Qi, Su, Mo, Guibas (2016)

PointNet++

- MLP on each point
- Maximum over neighborhood
  - kNN – homogeneous
  - Radius – more robust to sampling
    - Geodesic/Euclidean?

$$x_i' = \max_{j \in N_i} f_\theta(x_j, p_j - p_i)$$

- Hierarchies with maximum pooling (like CNN)

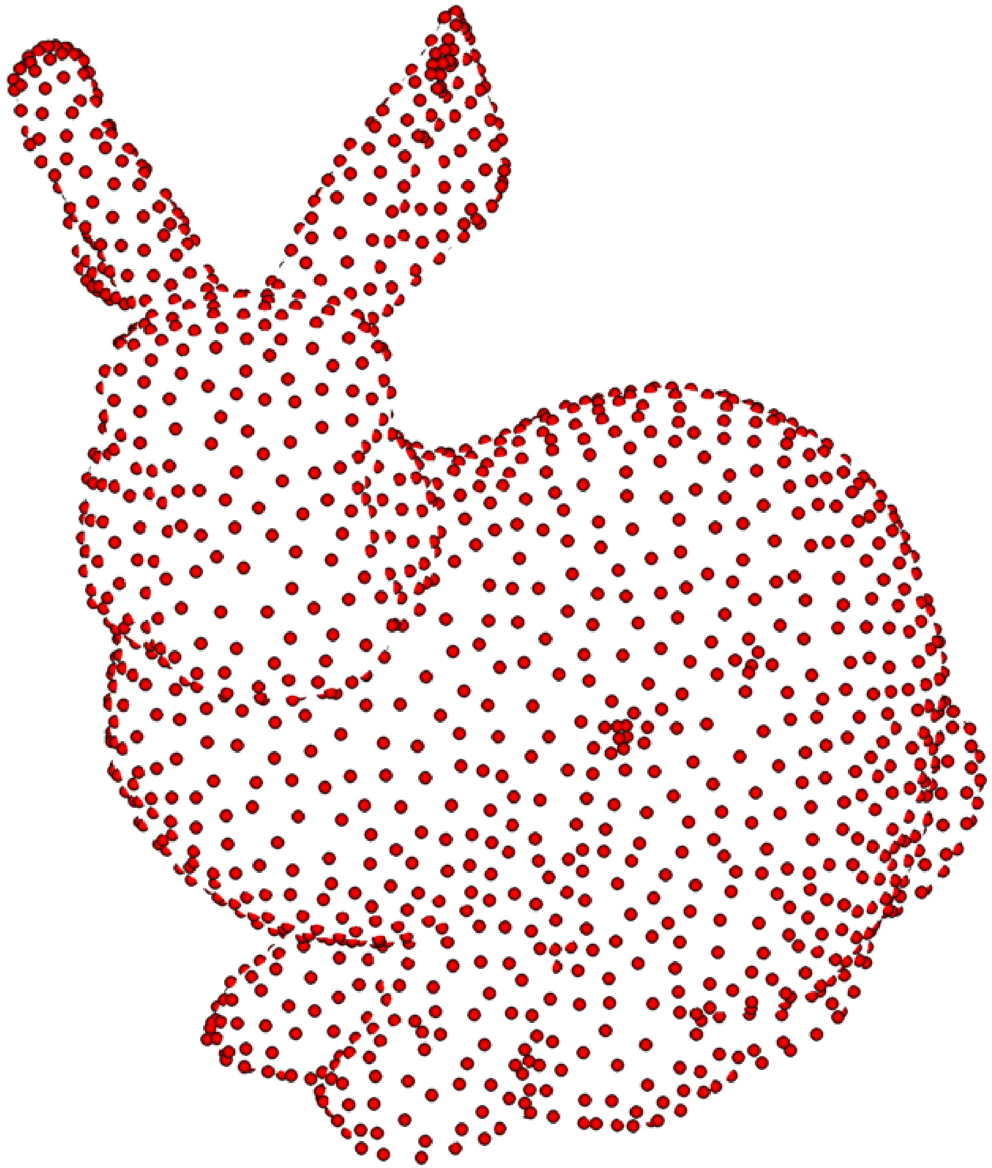


PointNet++: Deep Hierarchical Feature Learning on Point Sets in a Metric Space, Qi, Yi, Su, Guibas (2017)

Taking a step back

Surface as a graph

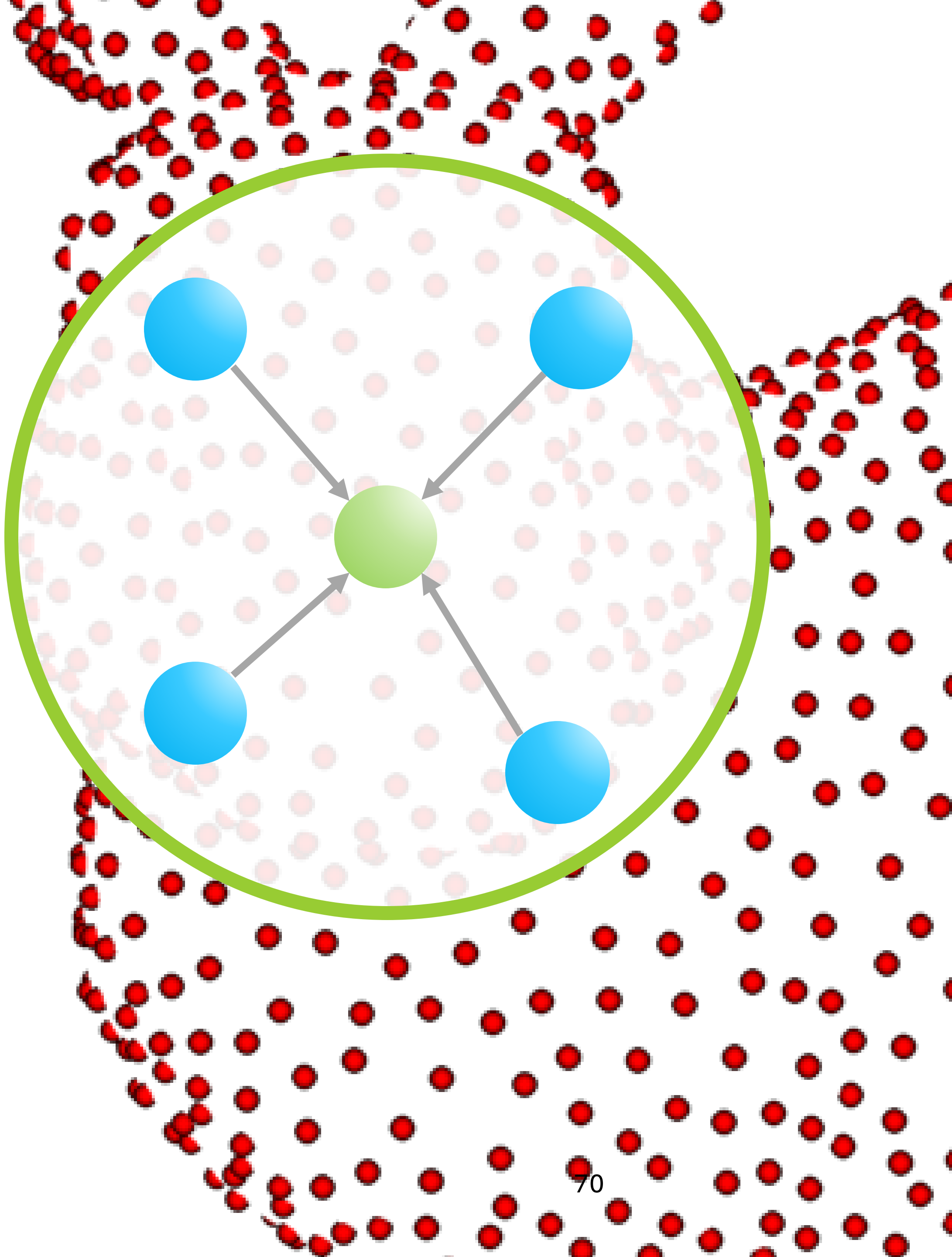
- Vertices or points are nodes



Surface as a graph

- Vertices or points are nodes
- Edges connect nearby points (radius graph, k-NN graph)
- For meshes: use mesh edges

- What can meshes help with?
  - Geodesic neighborhoods (we know connectivity)
  - Encode geometry (e.g., MeshCNN)



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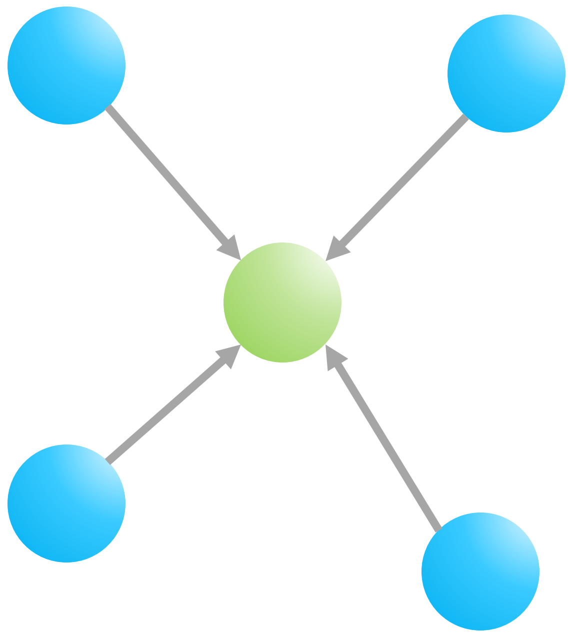
Message passing

1. Compute ‘message’ on each node
2. Aggregate messages over edges

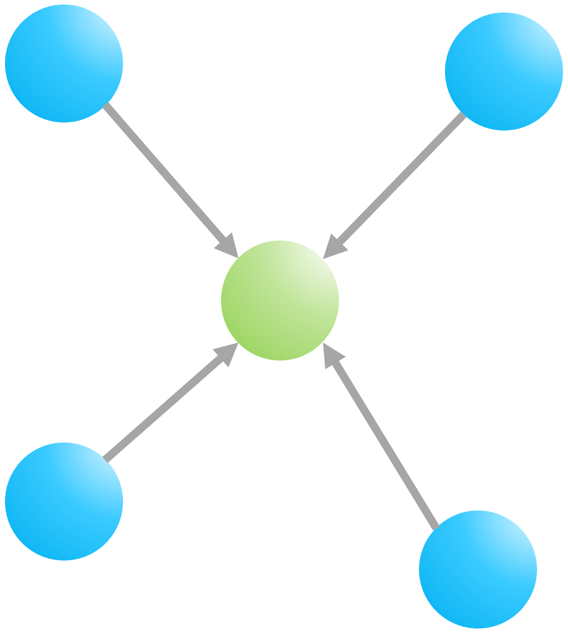


Neural Message Passing for Quantum Chemistry, Gilmer, Schoenholz, Riley, Vinayls, Dahl (2017)

https://pytorch-geometric.readthedocs.io/en/latest/tutorial/create_gnn.html

PointNet++ as Message Passing

1. **Message:** MLP on features + relative location
2. **Passing:** Maximum over neighbors



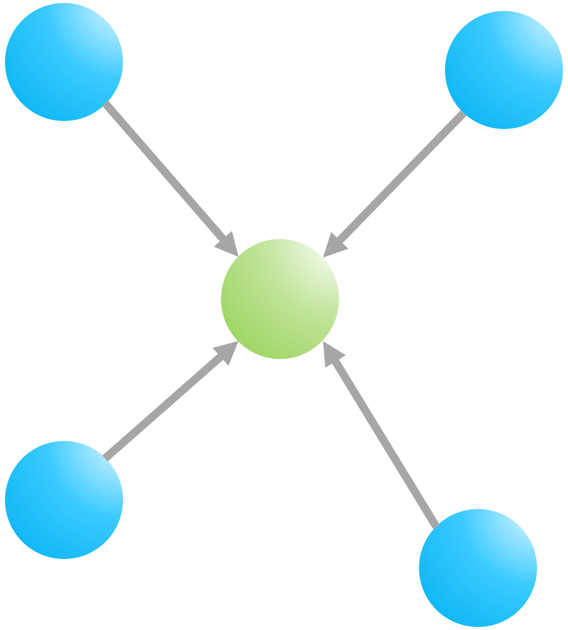
$$x_i' = \max_{j \in N_i} f_\theta(x_j, p_j - p_i)$$

PointNet++: Deep Hierarchical Feature Learning on Point Sets in a Metric Space, Qi, Yi, Su, Guibas (2017)

EdgeConv as Message Passing

1. **Message:** MLP on ‘relative features’ (edges)
2. **Passing:** Maximum over neighbors

$$x_i' = \max_{j \in N_i} f_\theta(x_i, x_j - x_i)$$

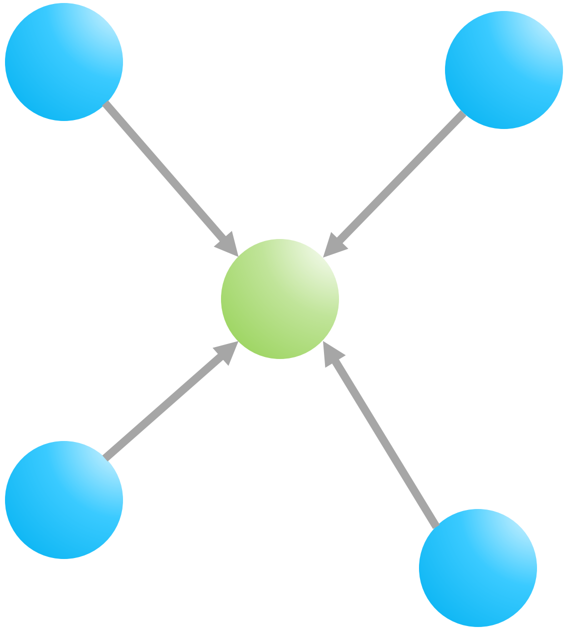


Dynamic Graph CNN for Learning on Point Clouds, Wang, Sun, Liu, Sarma, Bronstein, Solomon (2019)

GCN as Message Passing

1. **Message:** Linear transformation of features
2. **Passing:** Weight by degree, average

$$x_i' = \sigma\left(\mathrm{W}_0 x_i + \sum_{j \in N_i} \frac{1}{c_{ij}} \mathrm{W}_1 x_j\right)$$



Graph Convolutional Networks, Kipf, Welling (2016)

Laplacian in GCN and EdgeConv

- Laplacian: Sum of second derivatives
  - Discrete setting: Difference to average of neighbors

- GCN: graph Laplacian

$$x_i' = \sigma(\mathrm{W}_0 x_i + \sum_{j \in N_i} \frac{1}{c_{ij}} \mathrm{W}_1 x_j)$$

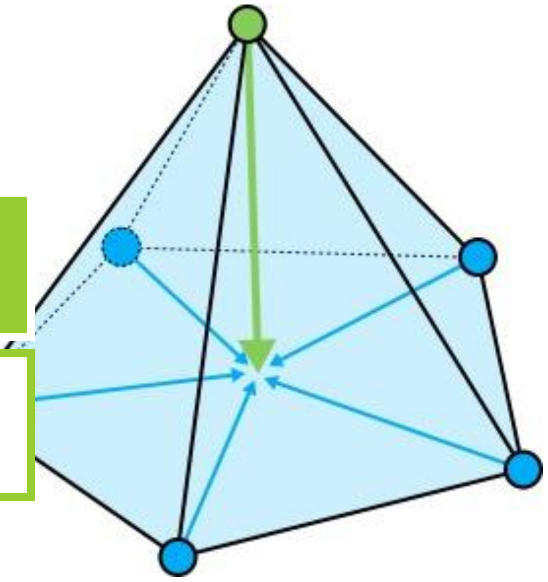
- EdgeConv

$$x_i' = \max_{j \in N_i} f_\theta(x_i, x_j - x_i)$$

Learning to Diffuse

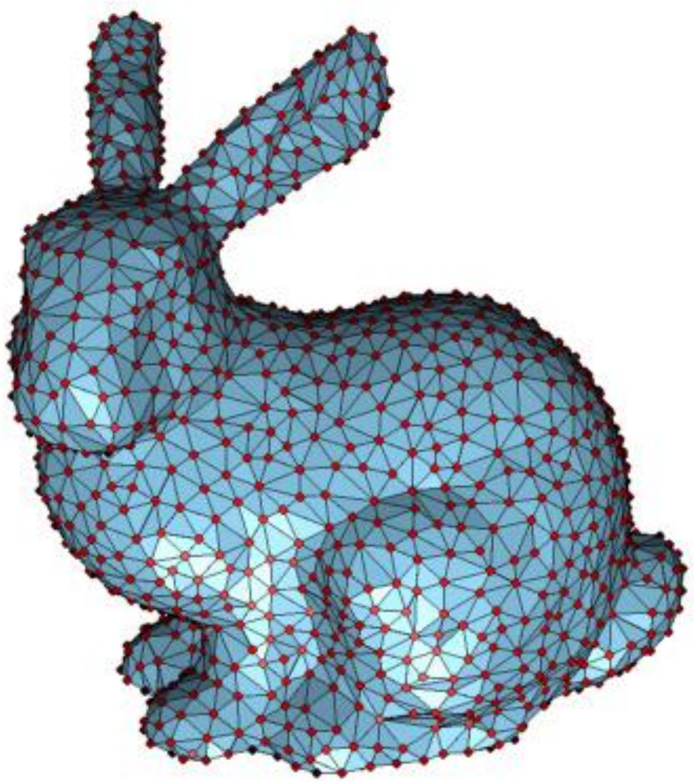
Smoothly

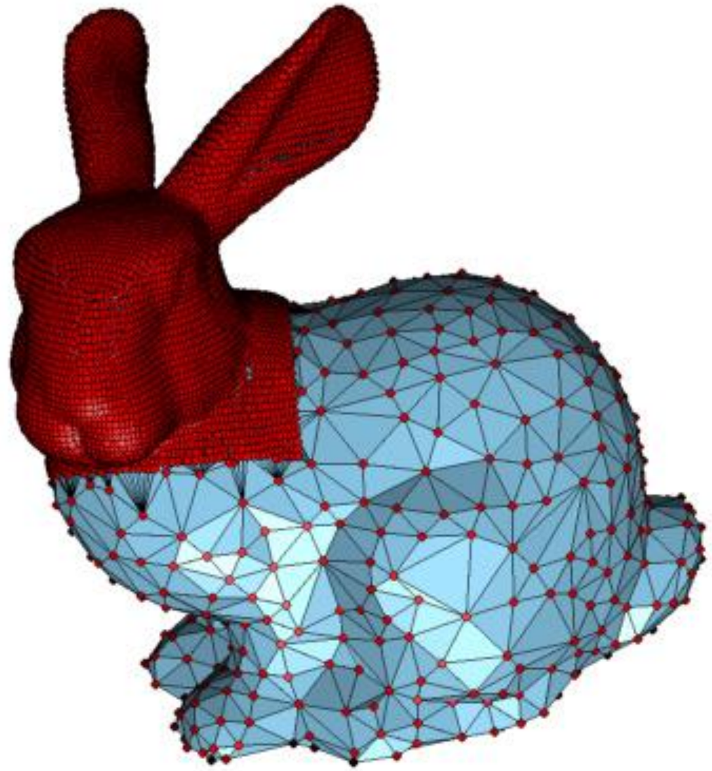
'Sharply'



More geometry

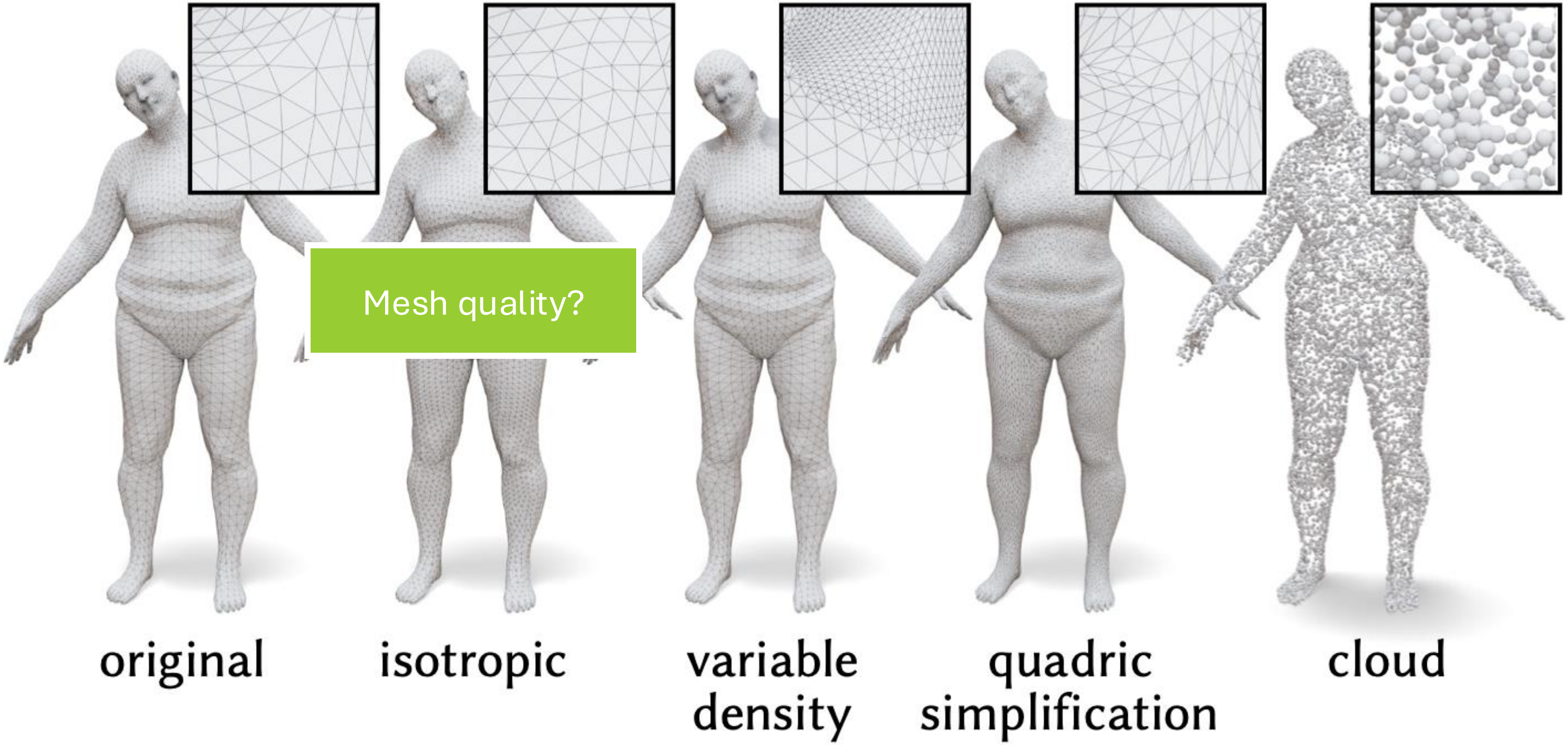
Why geometry?





Based on slides by Klaus Hildebrandt

Why geometry?

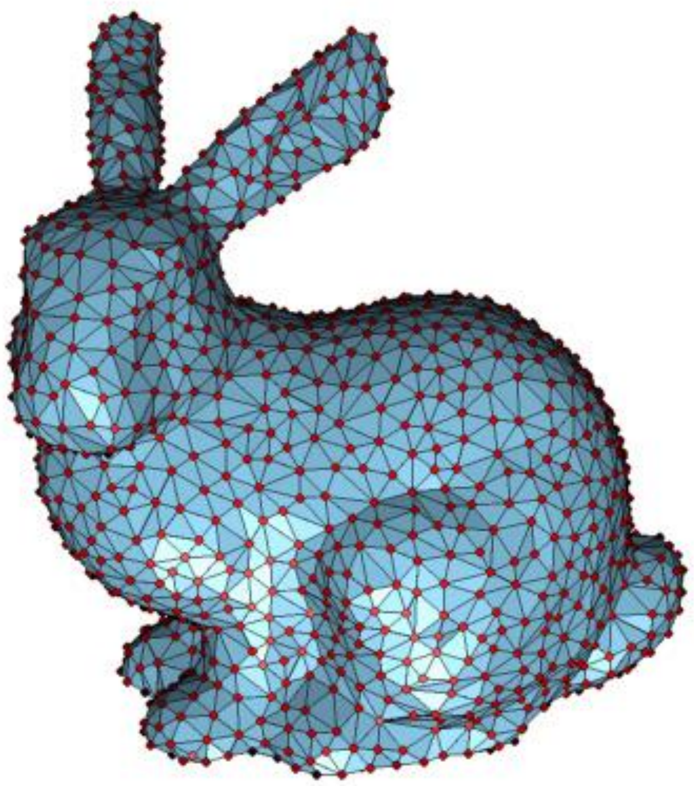


Mesh quality?

DiffusionNet: Discretization Agnostic Learning on Surfaces, Sharp et al. 2022

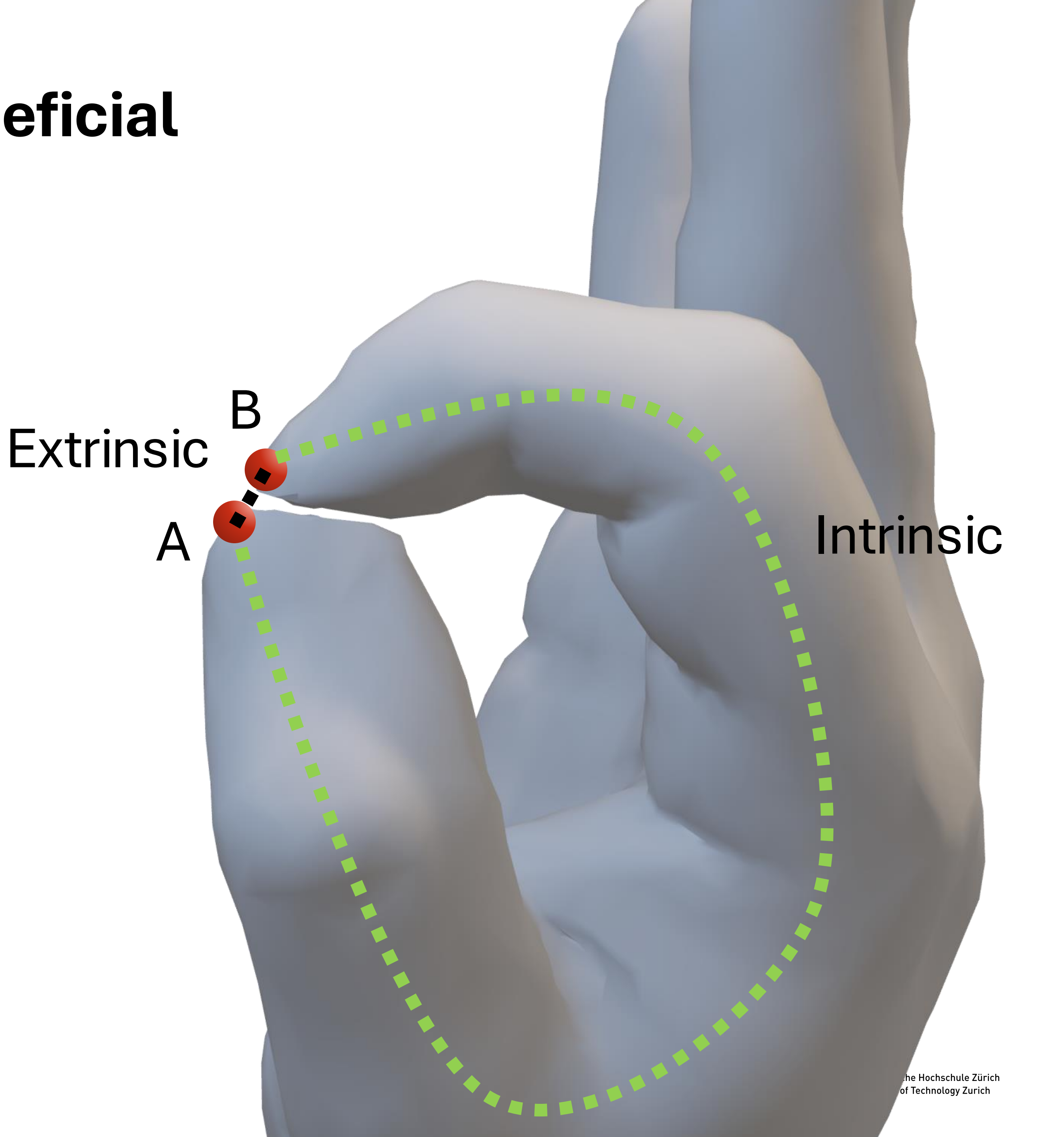
Why geometry?

- Invariances



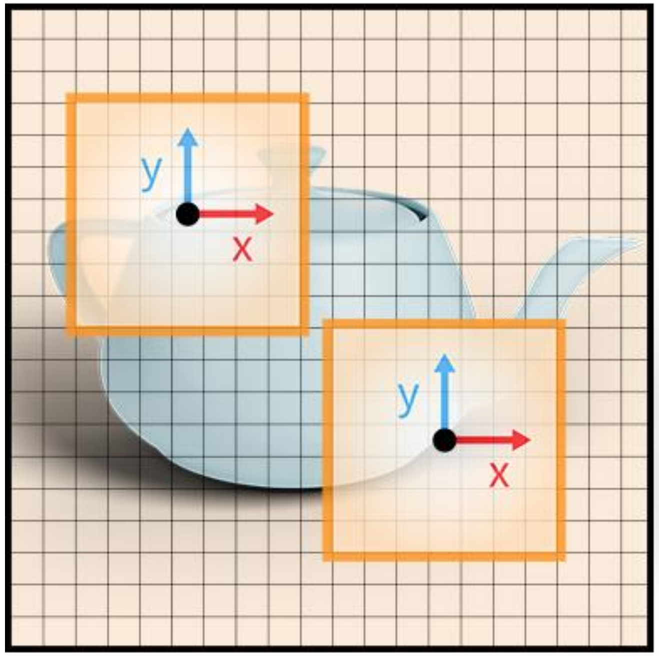
Intrinsic operations can be beneficial

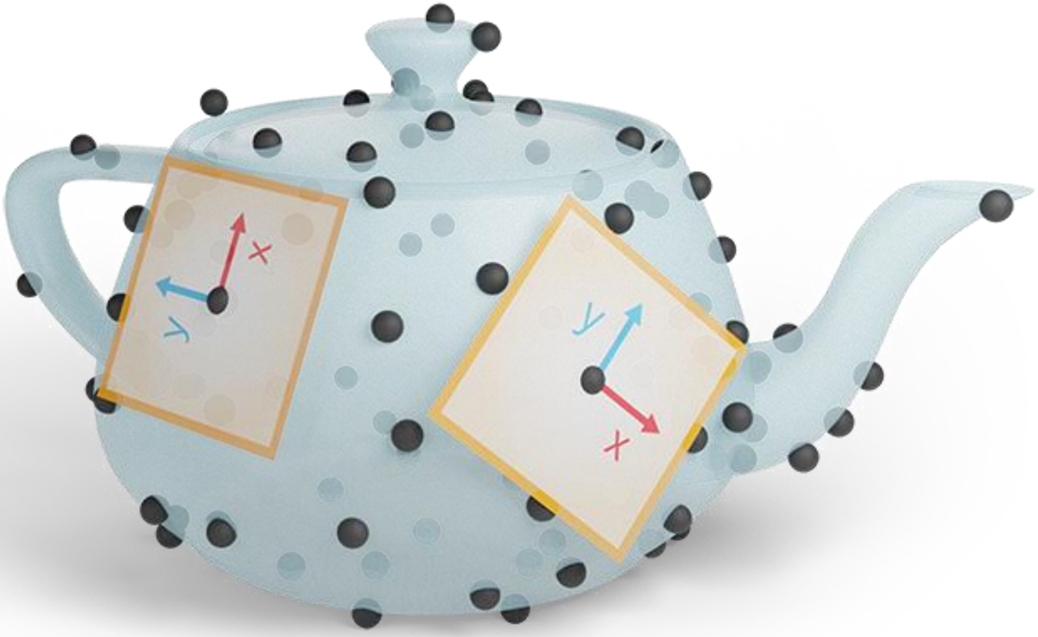
+ Robust to isometric deformations
+ 2D instead of 3D
+ No/less distortion or occlusion



Why Geometry?

- Invariances within an invariance

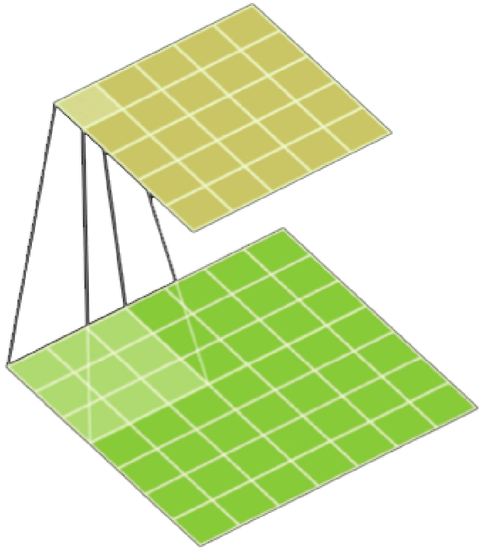


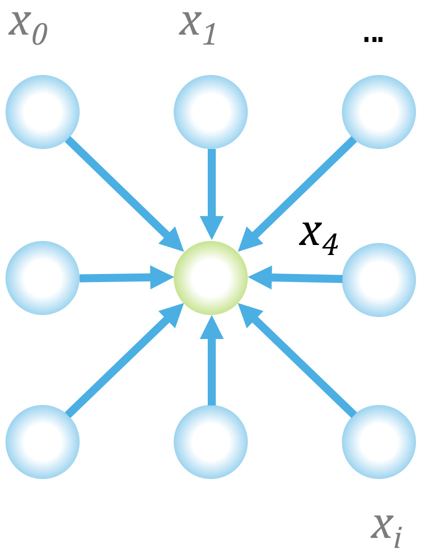


Back to CNNs

CNN on a Graph

Single CNN layer with 3x3 filter

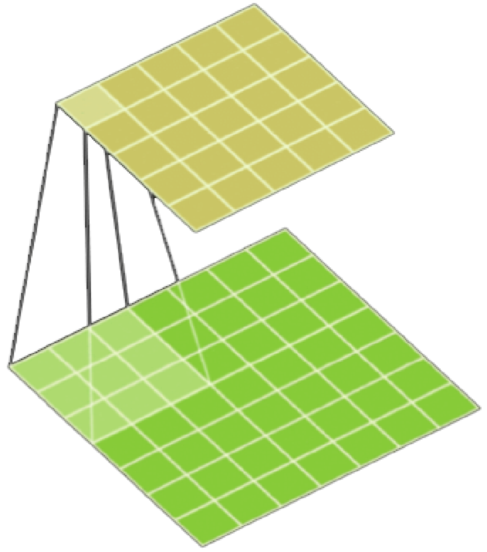


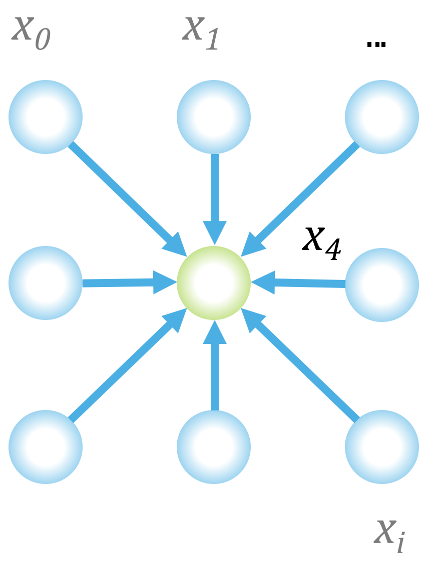


$$x'_4 = \sigma\Big(\sum_i \mathrm{W}_i x_i\Big)$$

CNN on a Graph

Single CNN layer with 3x3 filter





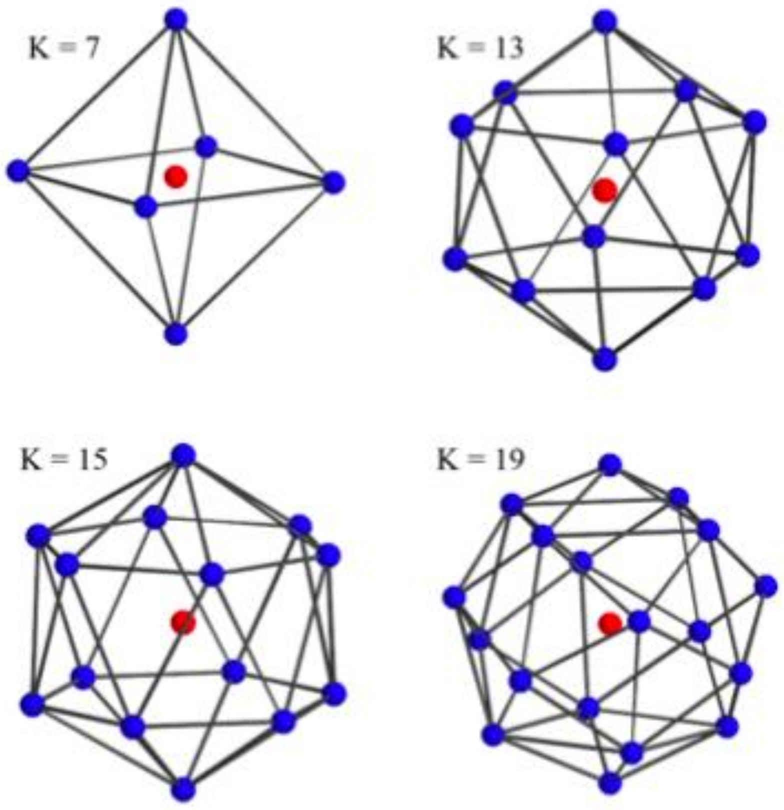
$$x_4' = \sigma\Big(\sum_i \boxed{W_i} x_i\Big)$$

Compare with GCN

$$x_i' = \sigma\Big(W_0 x_i + \sum_{j \in N_i} \frac{1}{c_{ij}} \boxed{W_1} x_j\Big)$$

CNNs for 3D

- Graph- and point based
  - GCN, PointNet++, EdgeConv
- 3D kernel (extrinsic)
  - KPConv, MinkowskiNet, SSCN

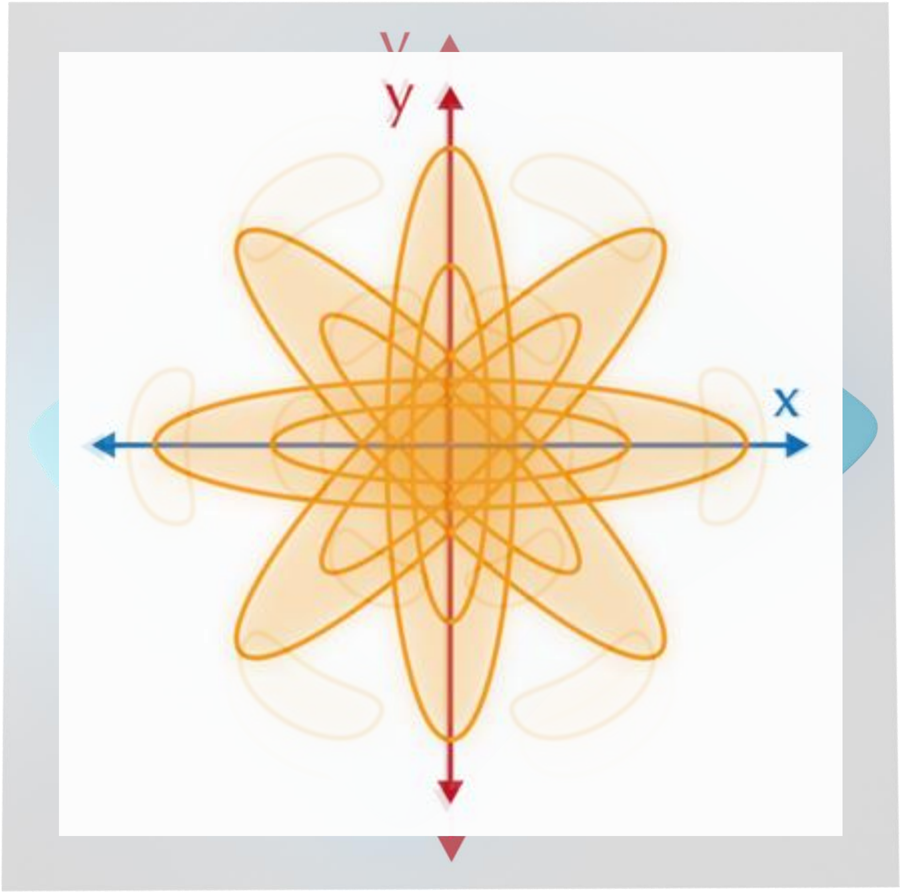


KPConv

[Thomas et al. 2019]

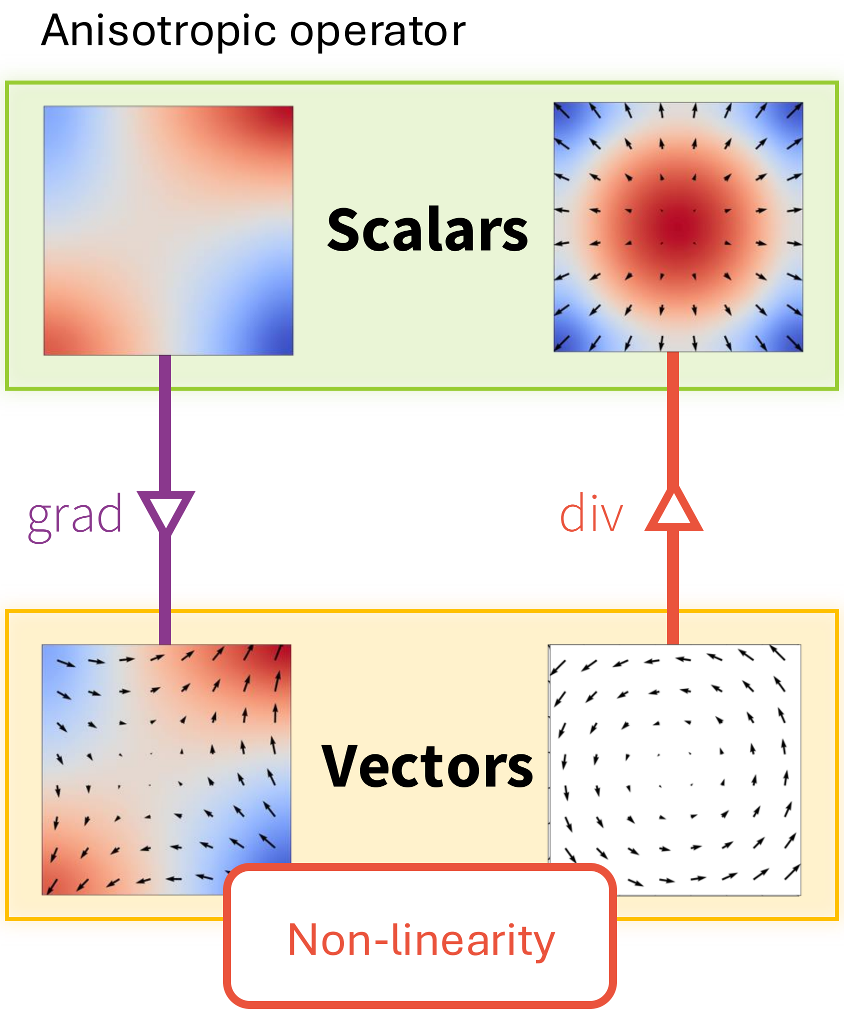
CNNs for 3D

- Graph- and point based
  - GCN, PointNet++, EdgeConv
- 3D kernel (extrinsic)
  - KPConv, MinkowskiNet, SSCN
- 2D kernels *on* surfaces (intrinsic)
  - GCNN, ACNN, MoNet, MDGCNN, HSN



CNNs for 3D

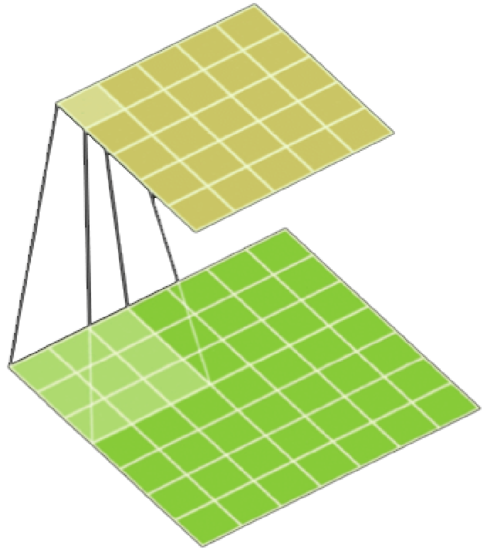
- Graph- and point based
  - GCN, PointNet++, EdgeConv
- 3D kernel (extrinsic)
  - KPConv, MinkowskiNet, SSCN
- 2D kernels *on* surfaces (intrinsic)
  - GCNN, ACNN, MoNet, MDGCNN, HSN
- Operator-based (e.g., Laplacian)
  - DeltaConv

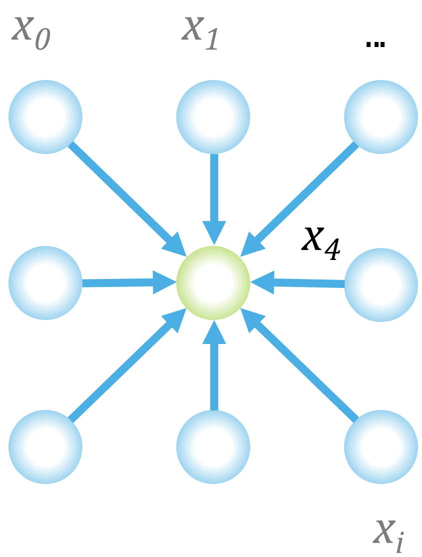


Transformers

Attention on a Graph

Single CNN layer with 3x3 filter

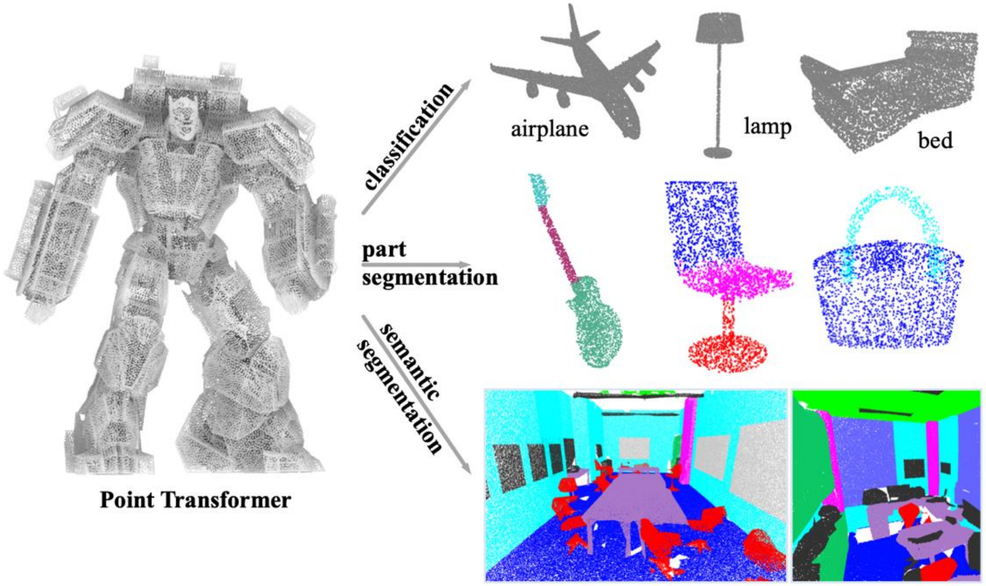




$$x_i' = \sum_{j=0}^{N} \mathrm{softmax}\left(\left(\mathrm{W}_q x_j\right)^T \mathrm{W}_k x_i\right) \mathrm{W}_v x_j$$

Transformers for 3D

- PointTransformer - Zhao et al. 2021
- Challenge: how to reduce cost?
  - Apply on local neighborhoods
  - Apply on serialized point clouds (space-filling curves, v3 2024)
- Challenge: positional encoding
  - PointNet++-like positional encoding, $\mathrm{MLP}(p_j - p_i)$
  - Rotation invariance?

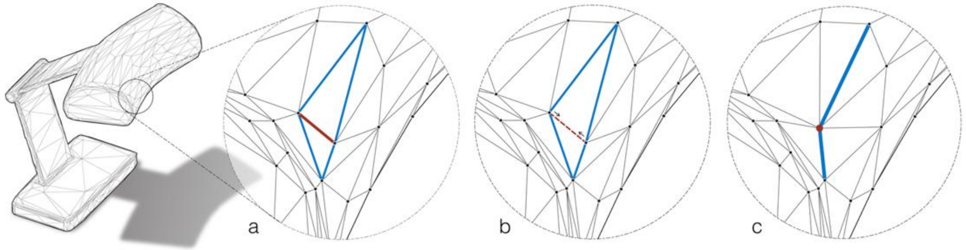


Current state-of-the-art

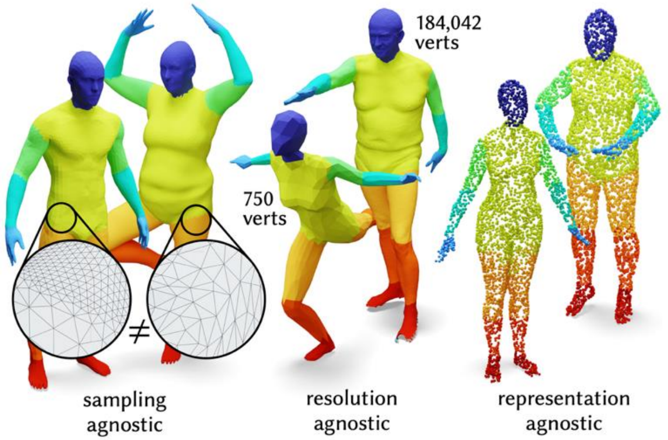
- Many approaches, *but* PointNet++ still seems to work quite well
  - Due to tasks/benchmarks?
- Other options often seen
  - DGCNN (EdgeConv), PointTransformer v3, Sparse Voxel Convolutions
- Which one to use depends on your task
  - Do you expect the network to understand curvature? Maybe not PointNet++/DGCNN?
  - Do you want efficiency, simplicity? – Try the simple networks first.

Methods you should know, but we didn't cover

- MeshCNN (Hanocka et al. 2019)
  - Operates on mesh edges
  - Geometric features
    - dihedral angle, two inner angles and two edge-length ratios for each face

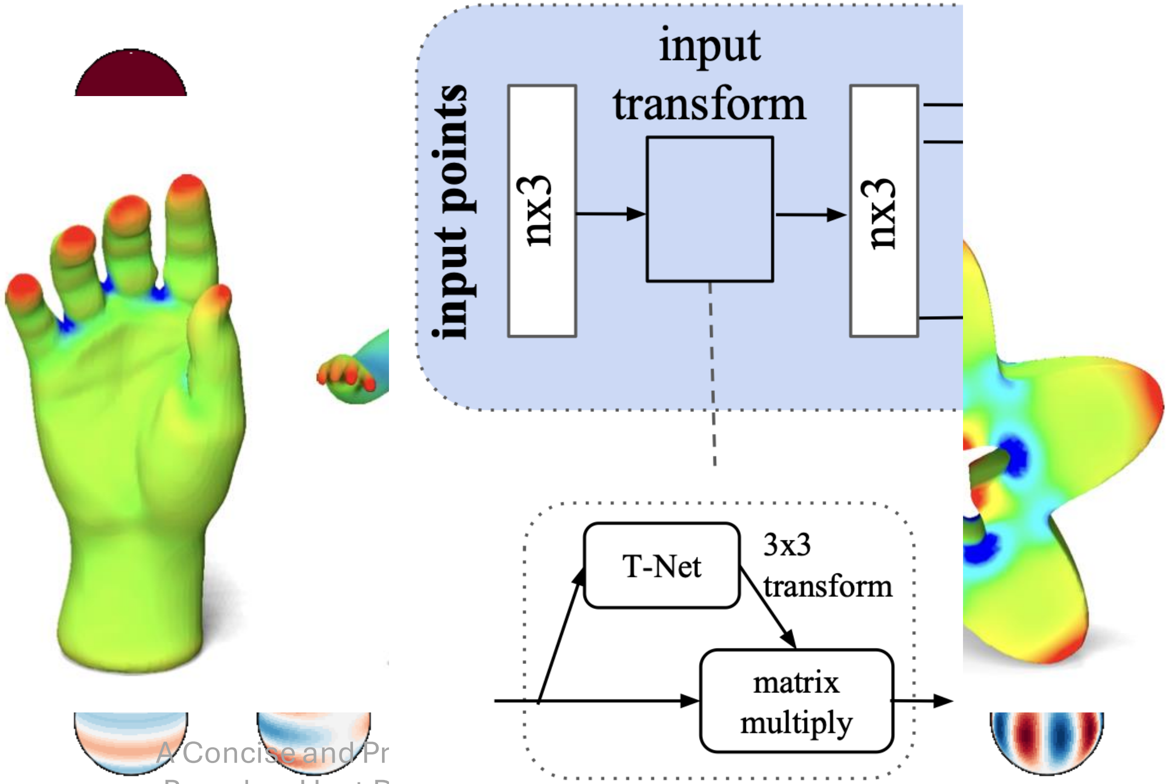


- DiffusionNet (Sharp et al. 2022)
  - Learning to diffuse
  - Accelerated in frequency domain (Eigenvectors of Laplacian)
  - Robust to discretizations (e.g., mesh → point cloud)



Rotation invariance, translation invariance?

- Data augmentation
  - Random rotation/translation
- ‘Learn’ the transformation
  - Spatial transformer, e.g., in PointNet
- Invariant input features
  - Heat-Kernel Signatures (DiffusionNet)
  - MeshCNN – Edges, intrinsic features
- Encode features in local frames
  - Point difference, normals (translation-invariant)
  - Distances (translation-rotation-invariant)
- Rotation-equivariance
  - E.g., Tensor Field Networks (Thomas et al. 2018), Spherical Harmonics (Poulenard et al. 2019)



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Based on Heat D
PointNet: Deep Learning on Point Sets for 3D Classification
and Segmentation, Qi, Su, Mo, Guibas (2016)
github.com/nvidia/torch-harmonics

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Swiss Federal Institute of Technology Zurich

Chapter 4: Hands-on with PyTorch Geometric

PyTorch Geometric

https://pytorch-geometric.readthedocs.io/en/latest/

- Implementation of message-passing paradigm
- Supports many convolution types
- Many helpful utilities

- Alternatives
  - Kaolin, PyTorch3D, GraphGym

Setting up the environment

Dataset, loading and visualization

Model

Training

Deep Learning on Meshes and Point Clouds

Ruben Wiersma

Graduate School - SGP 2025, Bilbao