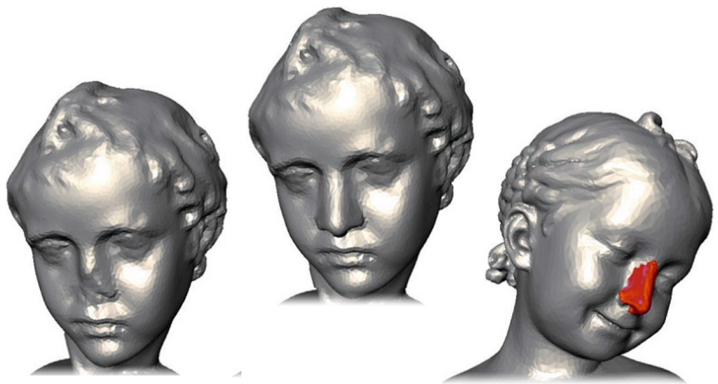
imati

Geometric Techniques for Digital Fabrication

Marco Attene and **Marco Livesu**



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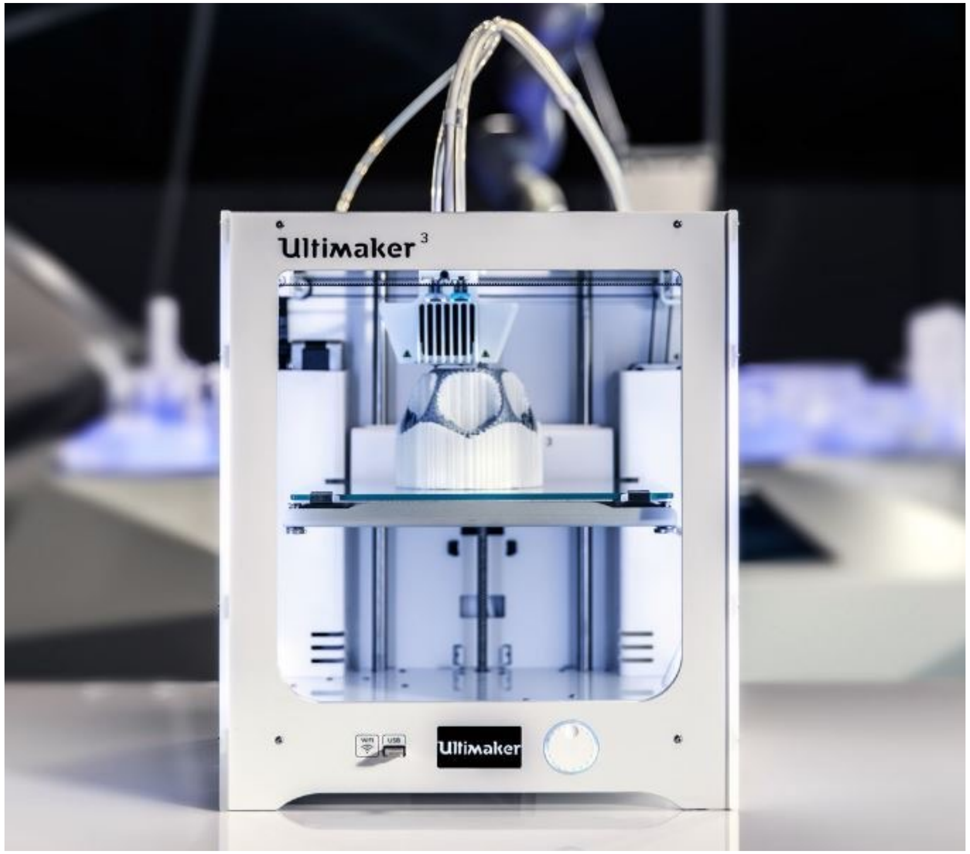
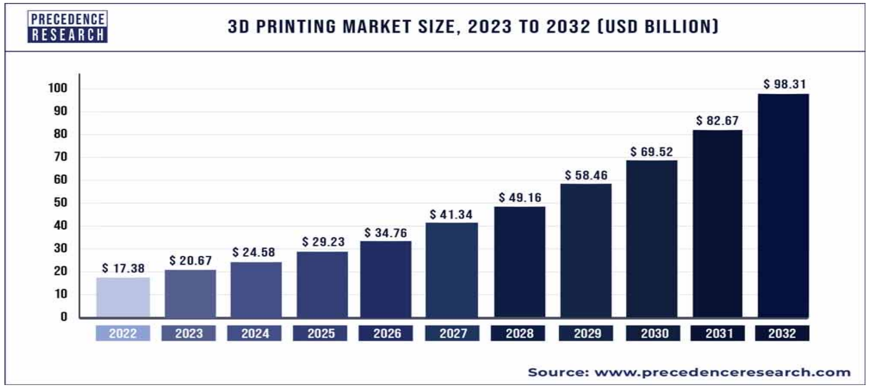
RAISE
Robotics and AI for socio-economic empowerement
Ecosistema dell'Innovazione della Liguria
PNRR – M4C2 – I1.5

Digital fabrication

- First “digital” milling machine
  - (MIT, 1952)
- Cutting tools (laser, waterjets, wires, …)
- Now: additive manufacturing
  - Powderbed 3D printers
  - FDM 3D printers
    - Flexible
    - Cheap !



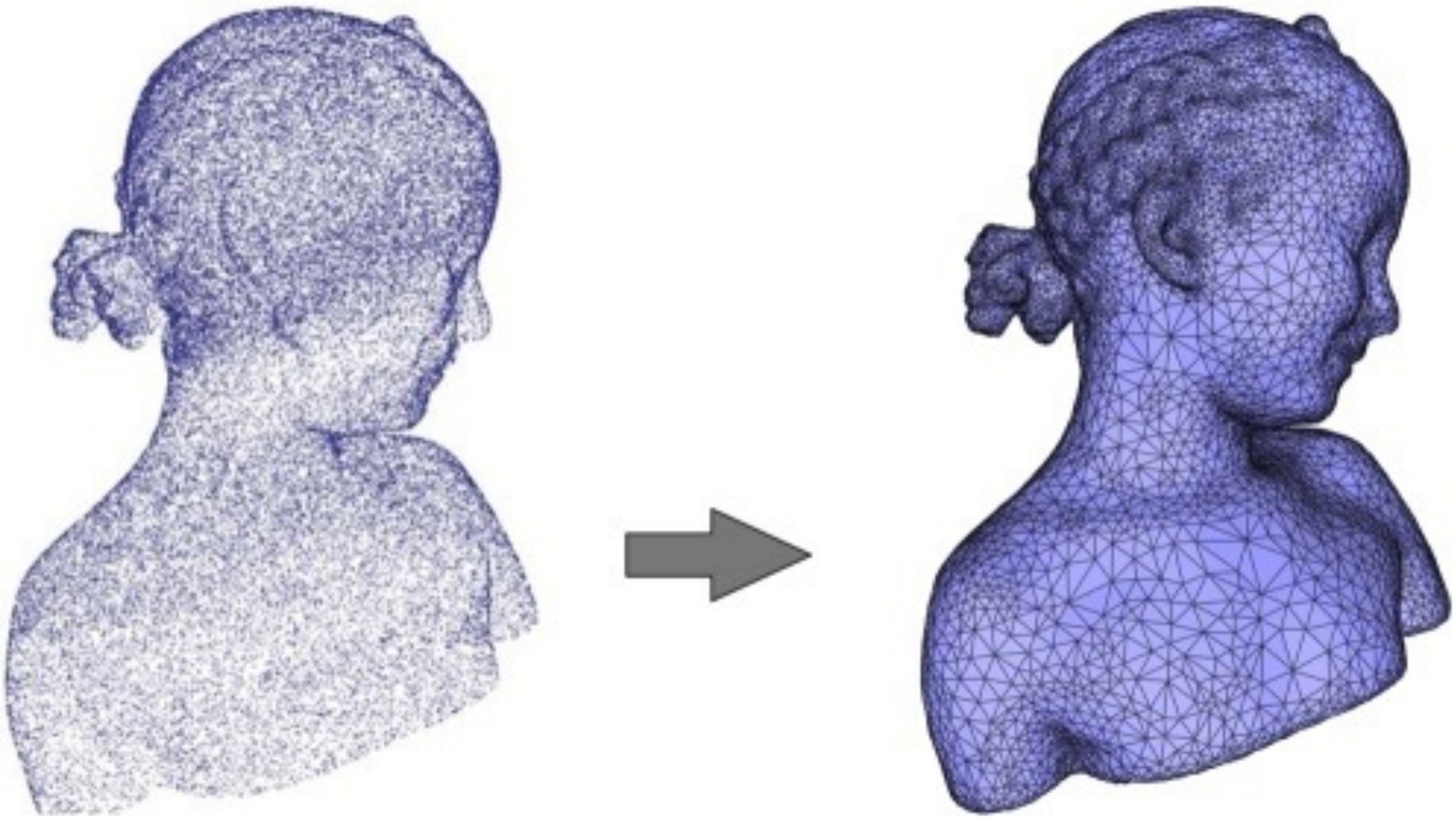
Potential Impact



- $98 billion exp. in 2032
  - Precedence Research, 2023
- Turning a 3D digital model into a physical prototype will be as easy as printing a text document
  - EU Digital Agenda for 2020

How to create 3D models ?

Model creation



Convert to mesh

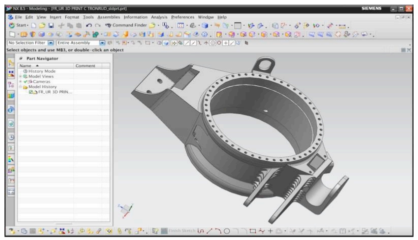
Computational Fabrication pipeline

Design
- Digitization
- CAD

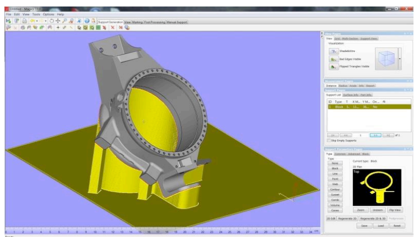
Planning
- Processing
- CAM

Fabrication
- Machine setup
- Actual construction

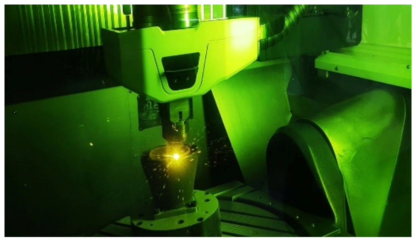
Surfaces



Triangles

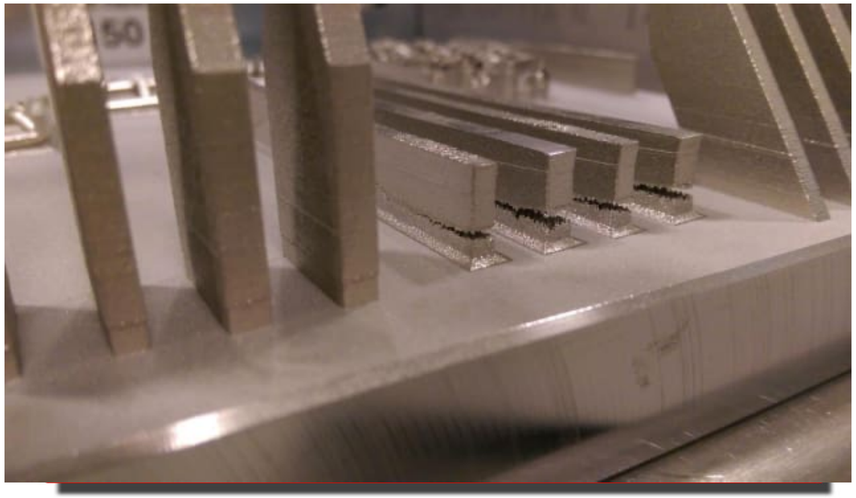


Toolpath



Process Planning for AM

Process Planning is somehow easy for AM (few constraints) but it is not trivial at all...



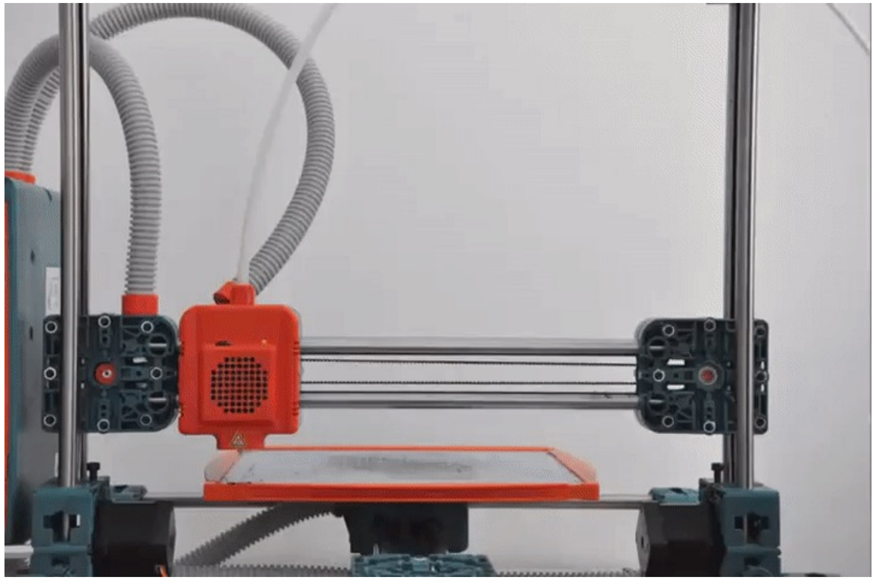
Geometry processing for AM

- 3D printing vs traditional fabrication
  - More flexibility
  - Ideally, automatic/algorithmic planning
- Geometry processing key for
  - Fitting the printing chamber (reorient, decompose, ...)
  - Converting surfaces to solid models (disambiguate)
  - Thickening thin parts
  - Support simulation (volume meshing)
  - Analyze printability / repair / adapt
  - ...

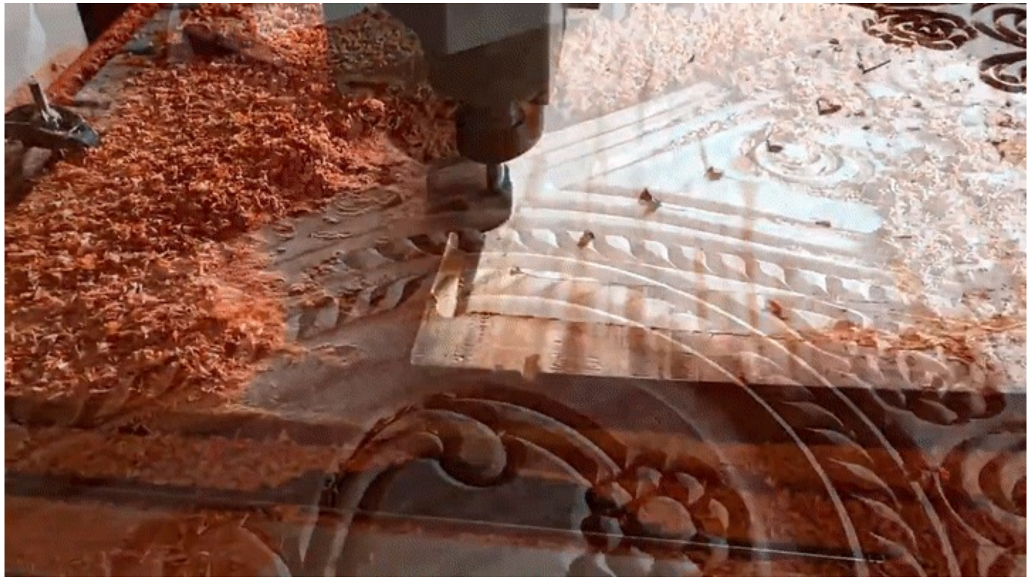
Outline

- Fabrication Technologies
- Modeling for fabrication
- Model orientation
- Slicing
- Internal Supports
- External supports
- Decomposition
- Toolpath generation
- Conclusions and outlook

Shape Synthesis



ADDITIVE



SUBTRACTIVE

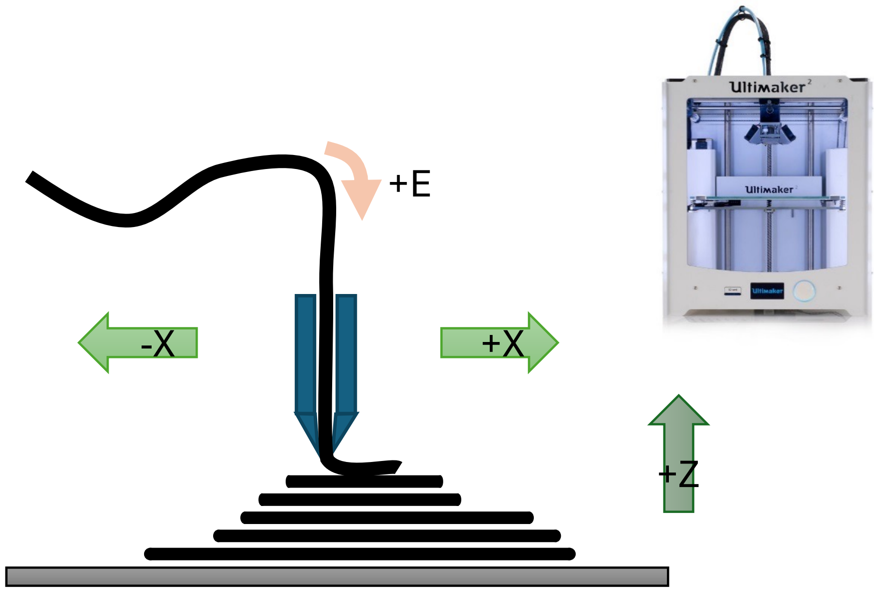
Additive Synthesis

There are several alternatives...

Additive Synthesis

There are several alternatives...

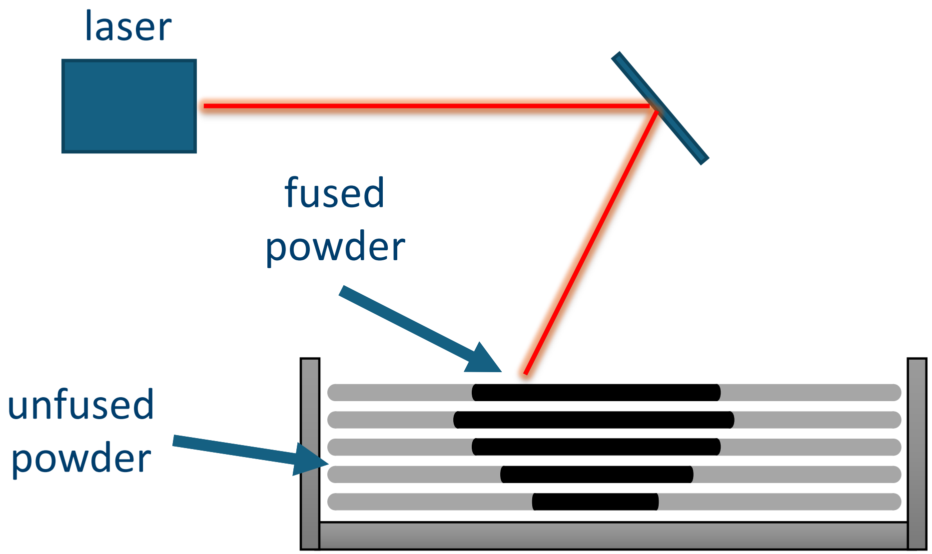
- **Material Deposition (FDM)**



Additive Synthesis

There are several alternatives...

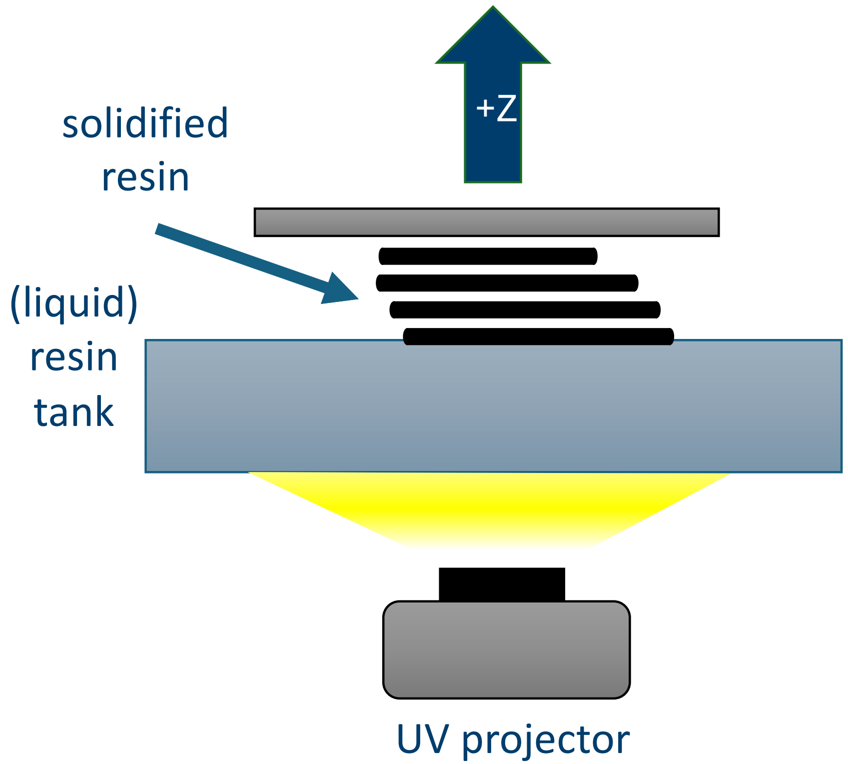
- Material Deposition (FDM)
- **Laser on Powder (SLS/SLM)**



Additive Synthesis

There are several alternatives...

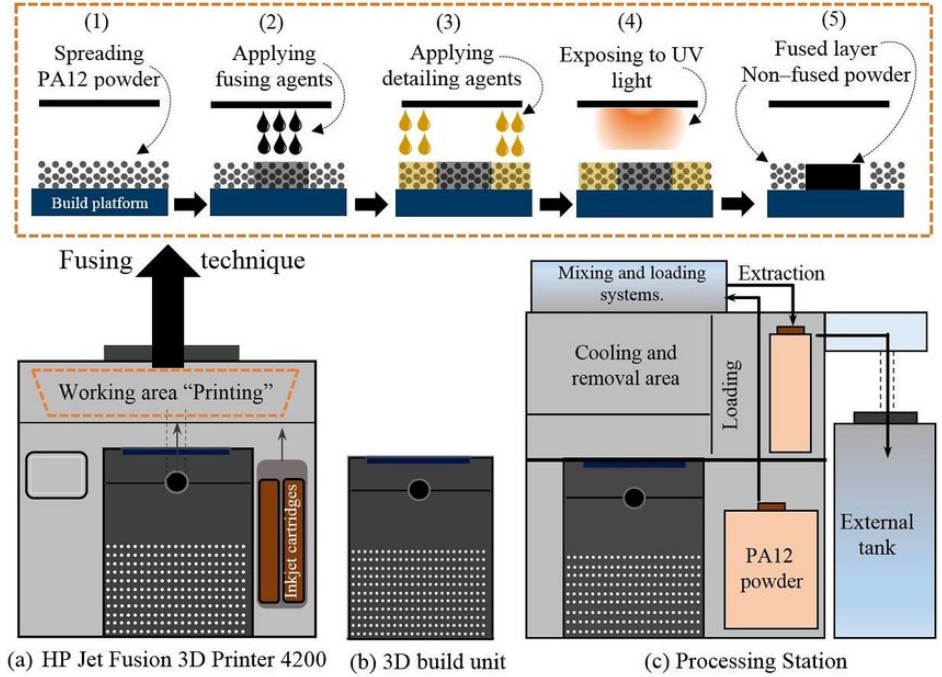
- Material Deposition (FDM)
- Laser on Powder (SLS/SLM)
- **Image on Resin (SLA/DLP)**



Additive Synthesis

There are several alternatives…

- Material Deposition (FDM)
- Laser on Powder (SLS/SLM)
- Image on Resin (SLA/DLP)
- **Droplets on Powder (Z-Corp, HP Jet Fusion)**



Additive Synthesis

There are several alternatives...

- Material Deposition (FDM)

Local deposition

- Laser on Powder (SLS/SLM)
- Image on Resin (SLA/DLP)
- Droplets on Powder (Z-Corp, HP Jet Fusion)

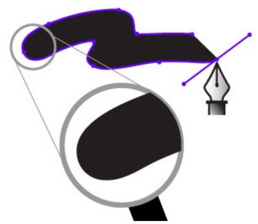
Layer solidification

Additive Synthesis

There are several alternatives…

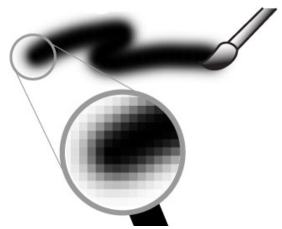
- Material Deposition (FDM)
- Laser on Powder (SLS/SLM)

Vector



- Image on Resin (SLA/DLP)
- Droplets on Powder (Z-Corp, HP Jet Fusion)

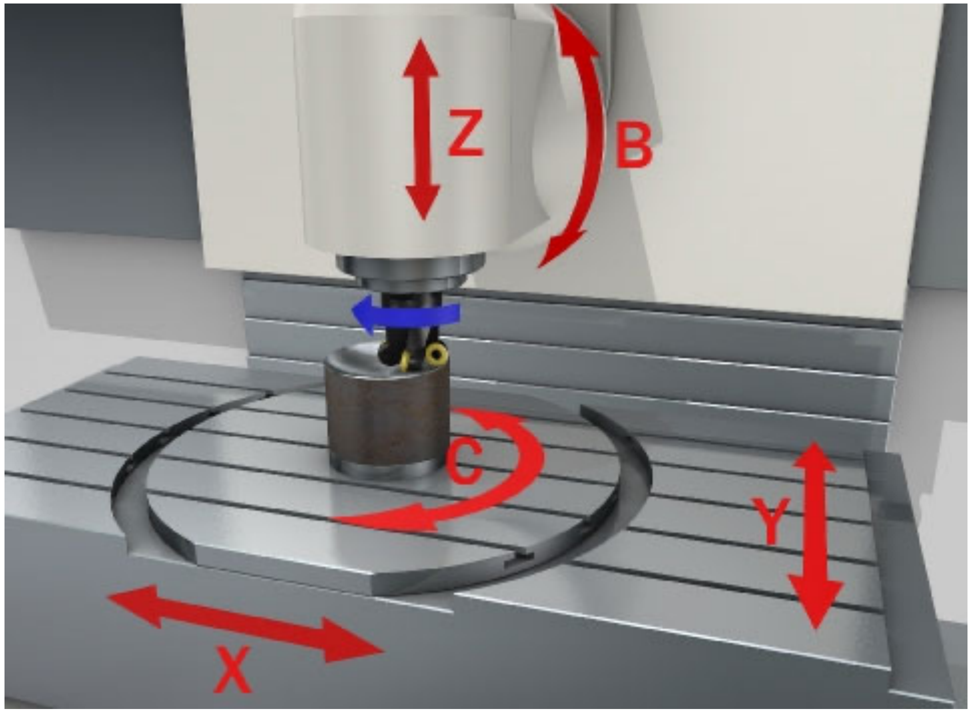
Raster



Subtractive Synthesis

Subtractive Synthesis

- Machines mainly differ on the number of degrees of freedom to control the drill bit



3 axis
- *X,Y,Z*

Easy to program

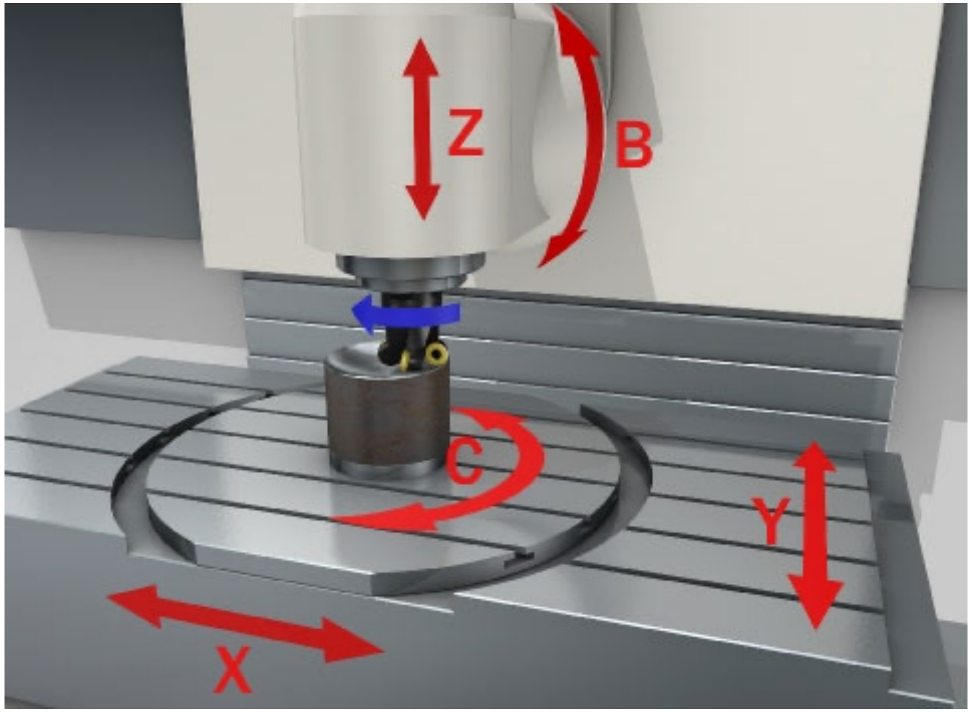
4 axis
- *X,Y,Z,C*

5 axis
- *X,Y,Z,C,B*

Hard to program

Subtractive Synthesis

- Machines mainly differ on the number of degrees of freedom to control the drill bit



3 axis
- *X,Y,Z*

Simple shapes (HF)

4 axis
- *X,Y,Z,C*

5 axis
- *X,Y,Z,C,B*

Complex shapes

Outline

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- **Modeling for fabrication**
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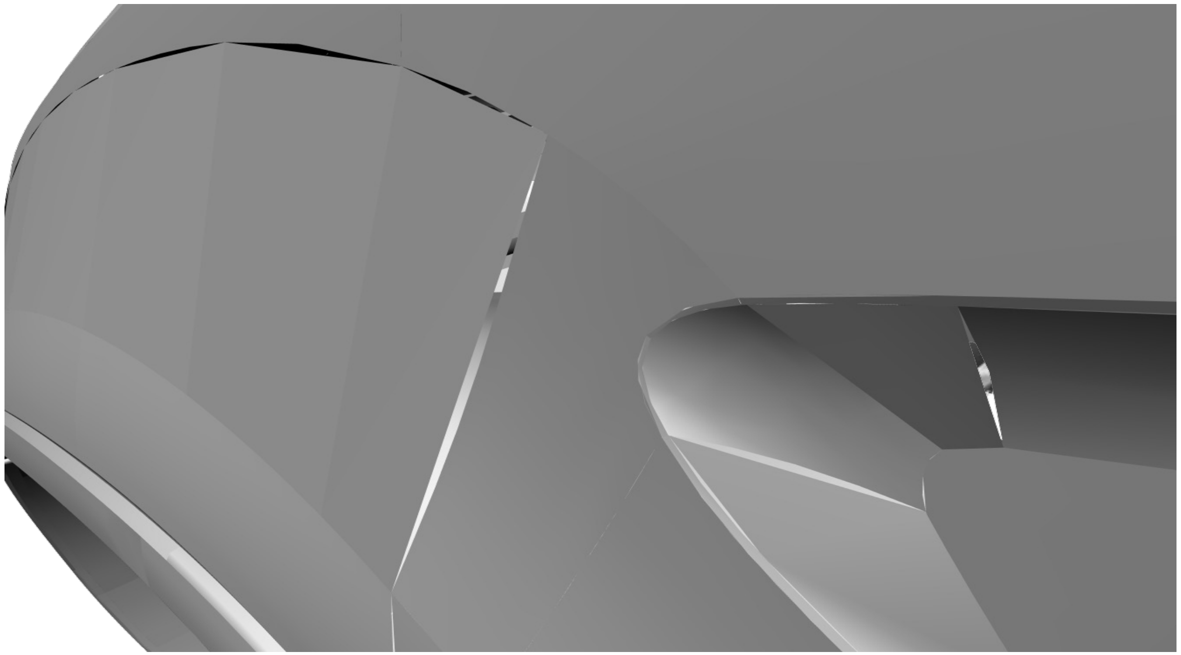
Printability

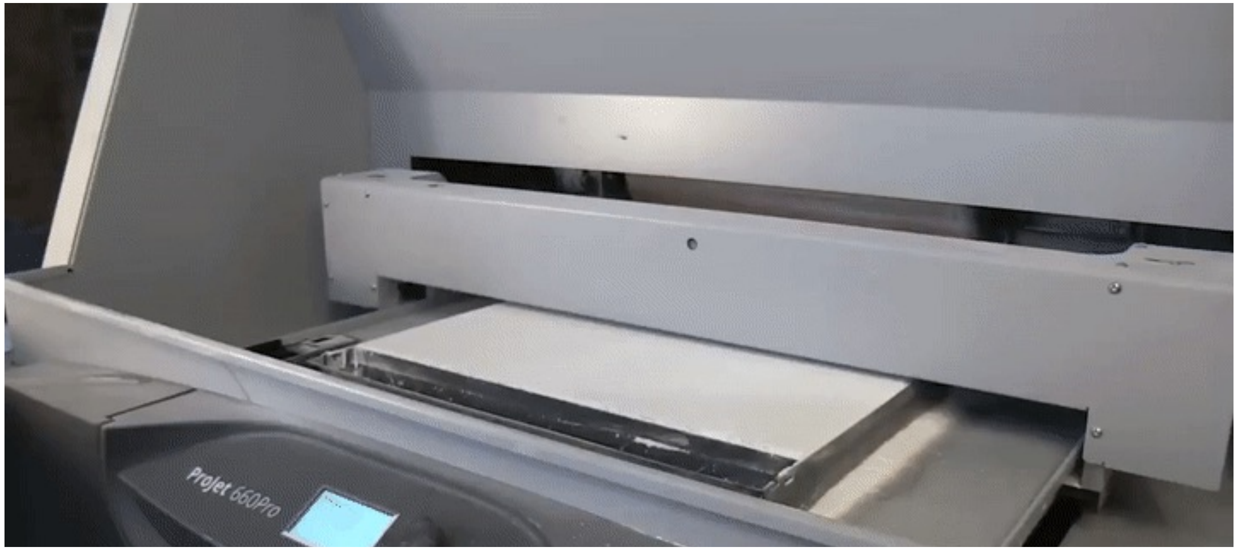
Fabrication is demanding when modeling 3D shapes

- Common requirement – 3D model must be a solid
- Requirements that depend on fabrication technology

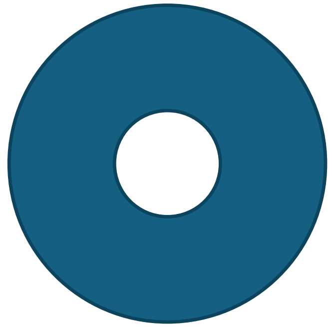


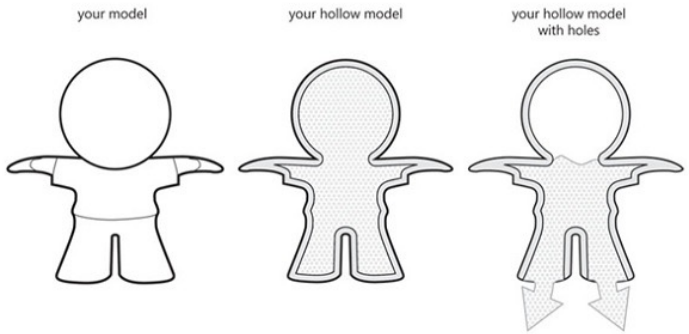
Printability

Fabrication is demanding when modeling 3D shapes



Closed voids

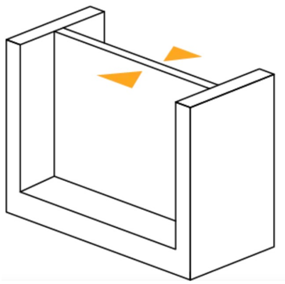


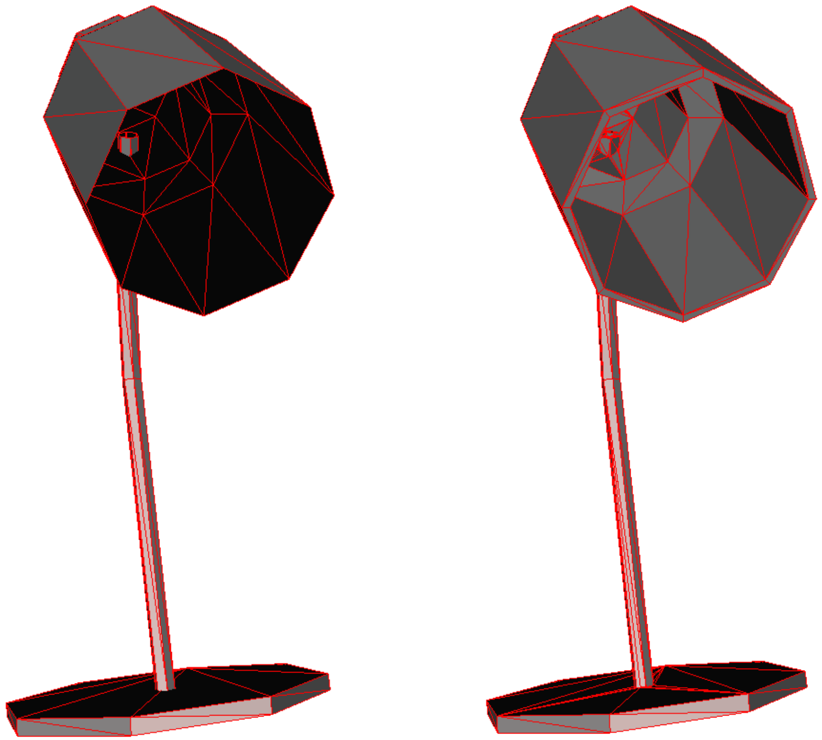


Evacuation channels

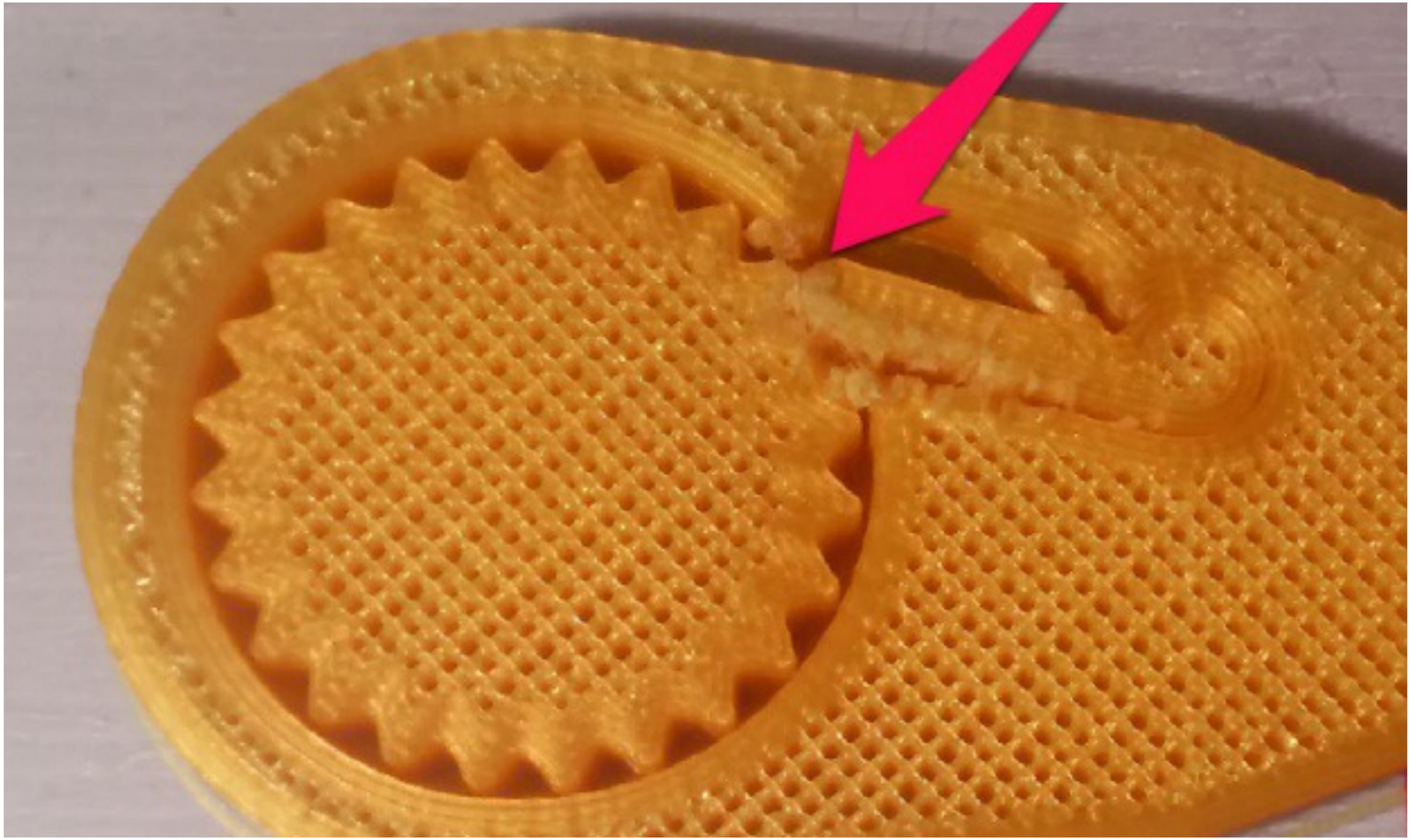
Printability

Fabrication is demanding when modeling 3D shapes





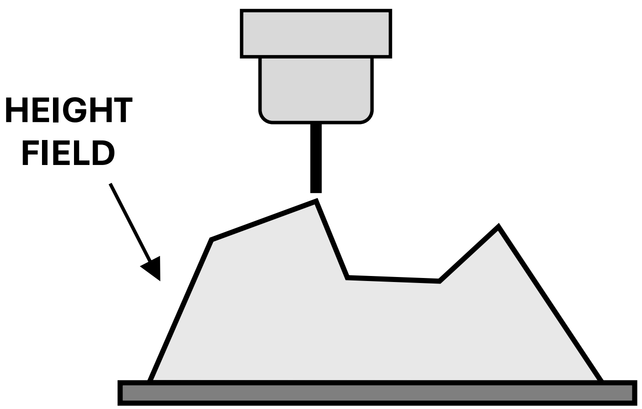
Printability

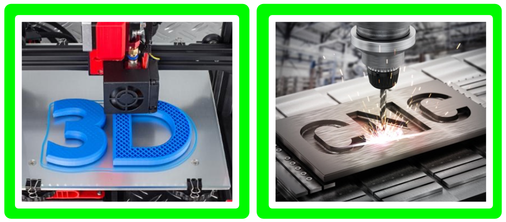


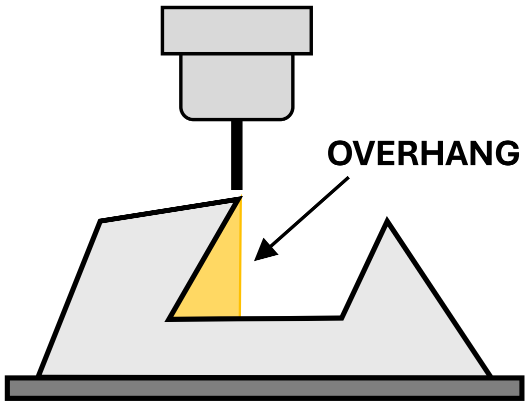
Overhangs

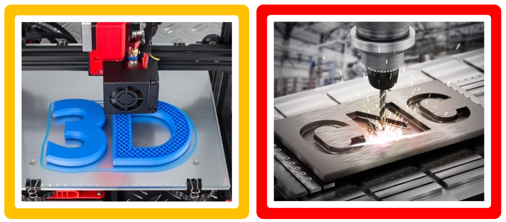
Each manufaturing hardware imposes a number of ***constraints*** on the class of shapes that can be fabricated with it

– *one of the major issues has to do with **surface orientation***





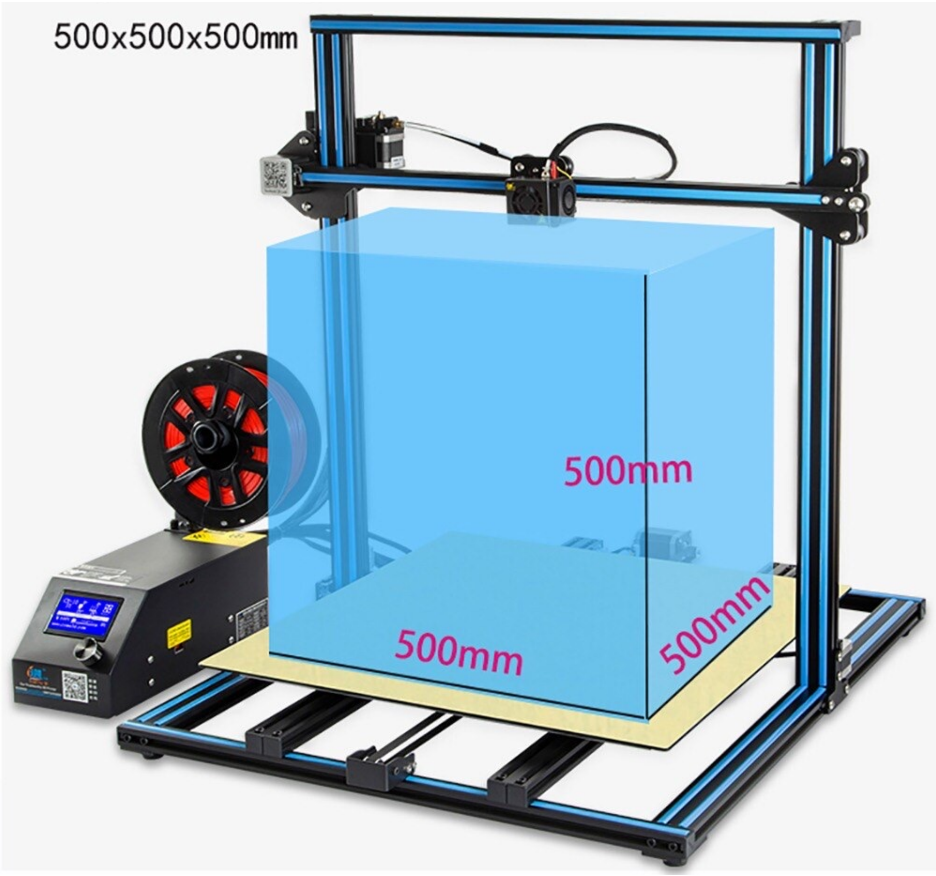


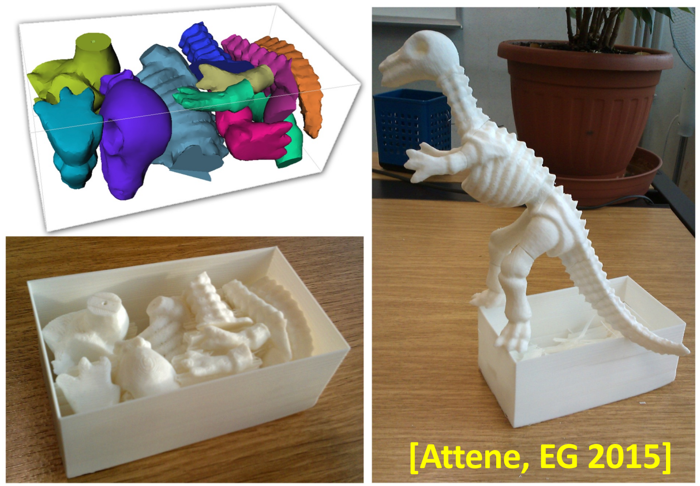


supports needed

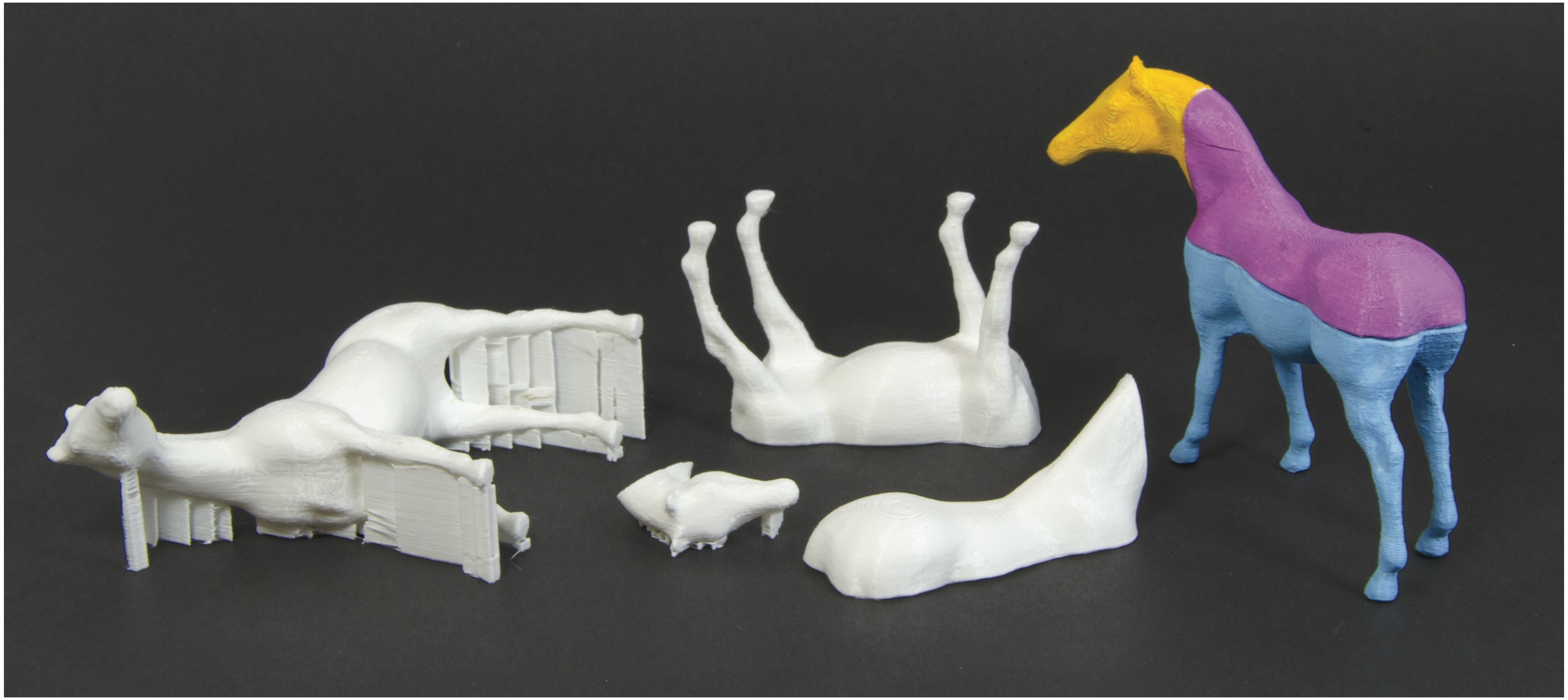
not fabricable

Size limitations



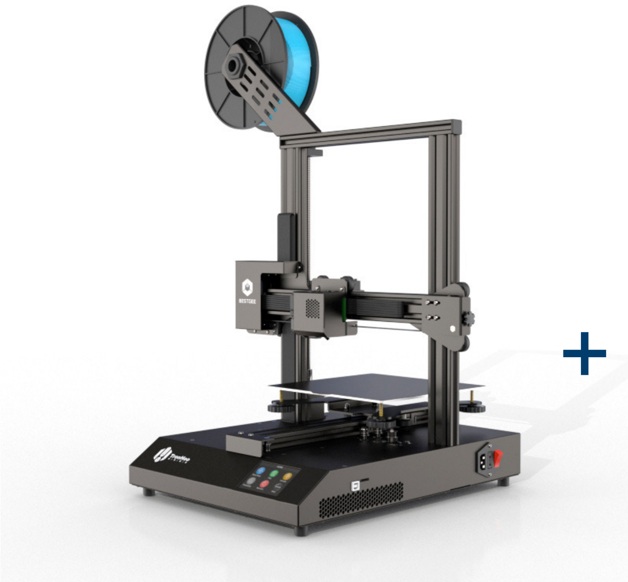


Material use and printing time



[Hu et al., SIGGRAPH Asia 2014]

Multiple colors/materials



3D printer with
single filament

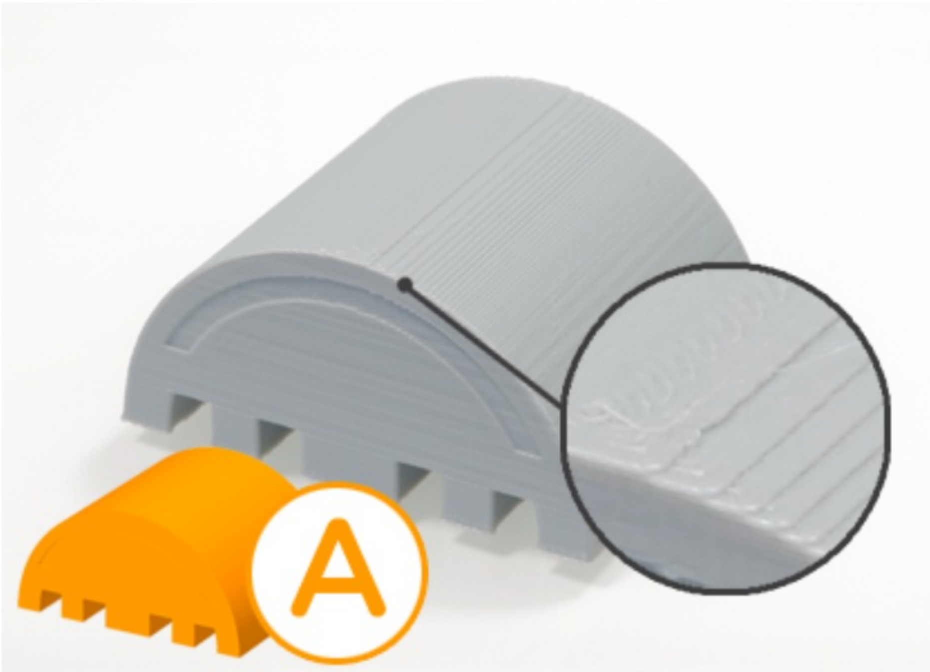
+

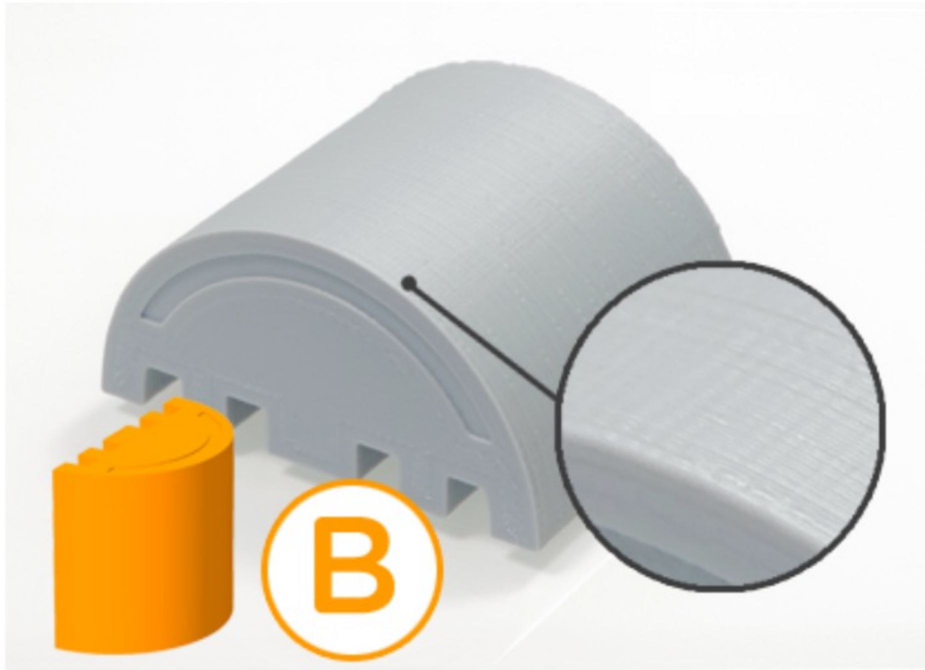
composite object
(multiple colors/materials)

=

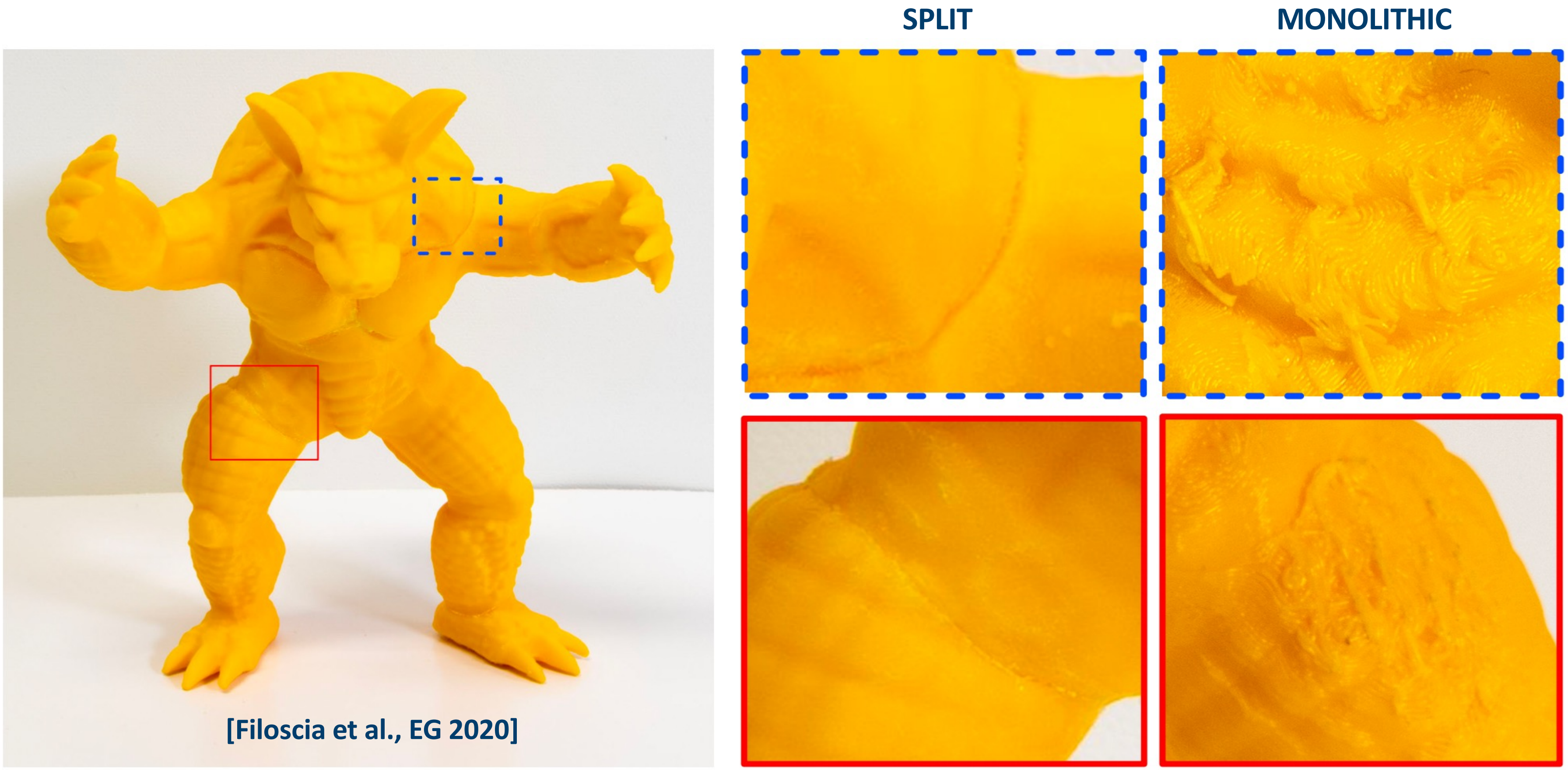
Araujo et al 2019

Surface quality





Surface quality

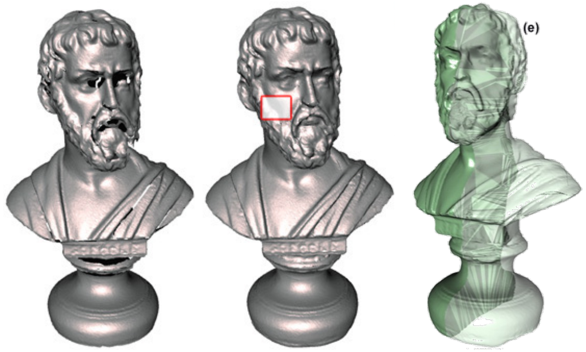


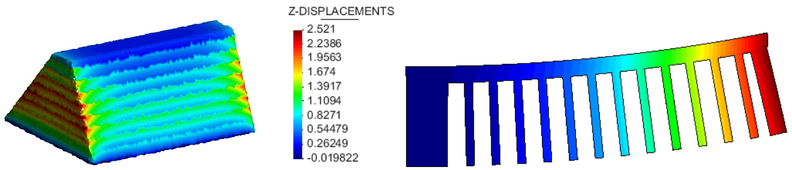
Equilibrium

[Prevost et al., SIGGRAPH 2013]

We will not talk about

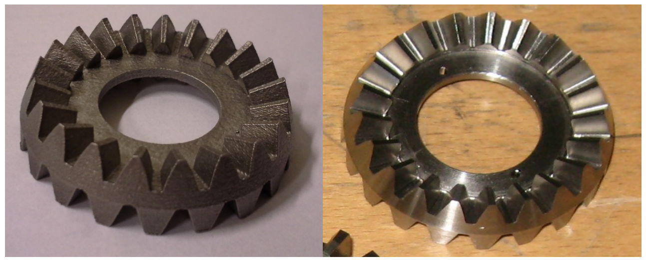
- Model repairing
  - Would require a full course (SGP 2013, 2019)





- Distortions and stresses
  - Out of scope

- Combined additive-subtractive fabrication
  - Mostly engineering issues, though interesting!



Outline

- Fabrication Technologies
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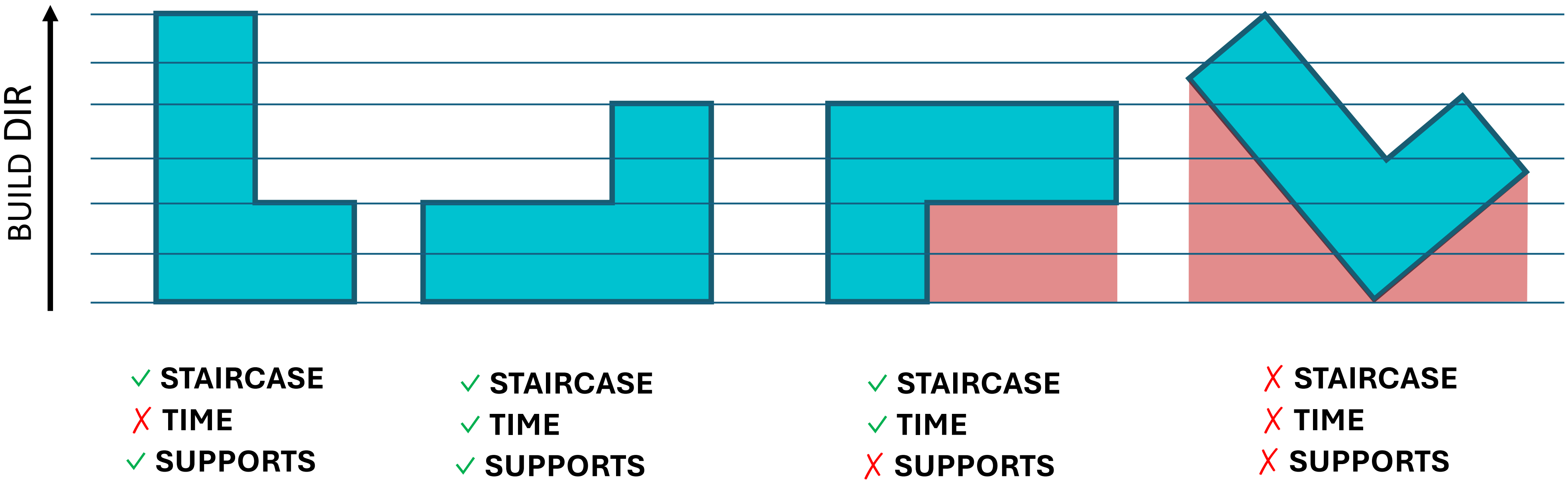
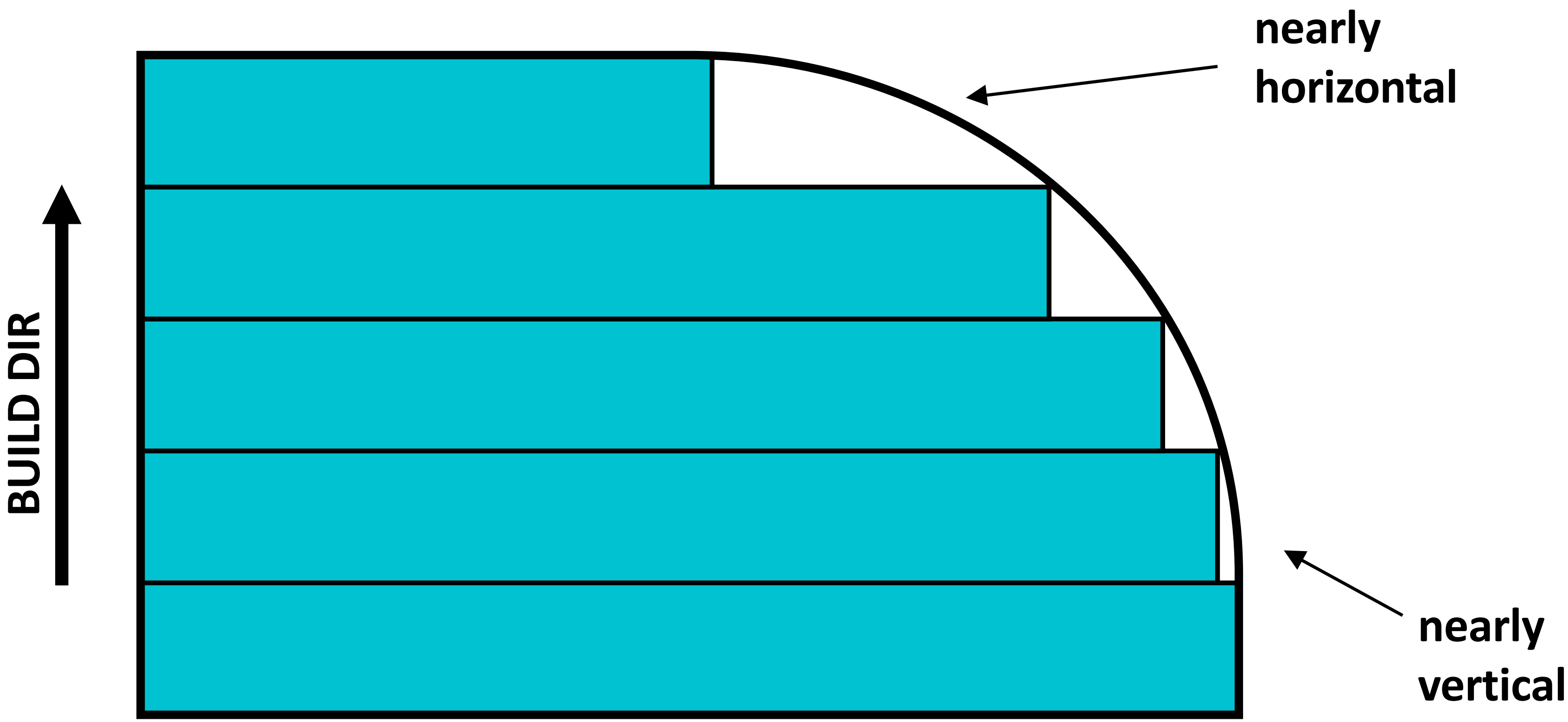
Effects of the Build Direction

- **Input:** the object, in the reference system in which it was designed (object space)
- **Output:** the same object, in the orientation in which it will be fabricated

- **The choice of the build direction *heavily impacts* many fabrication aspects**

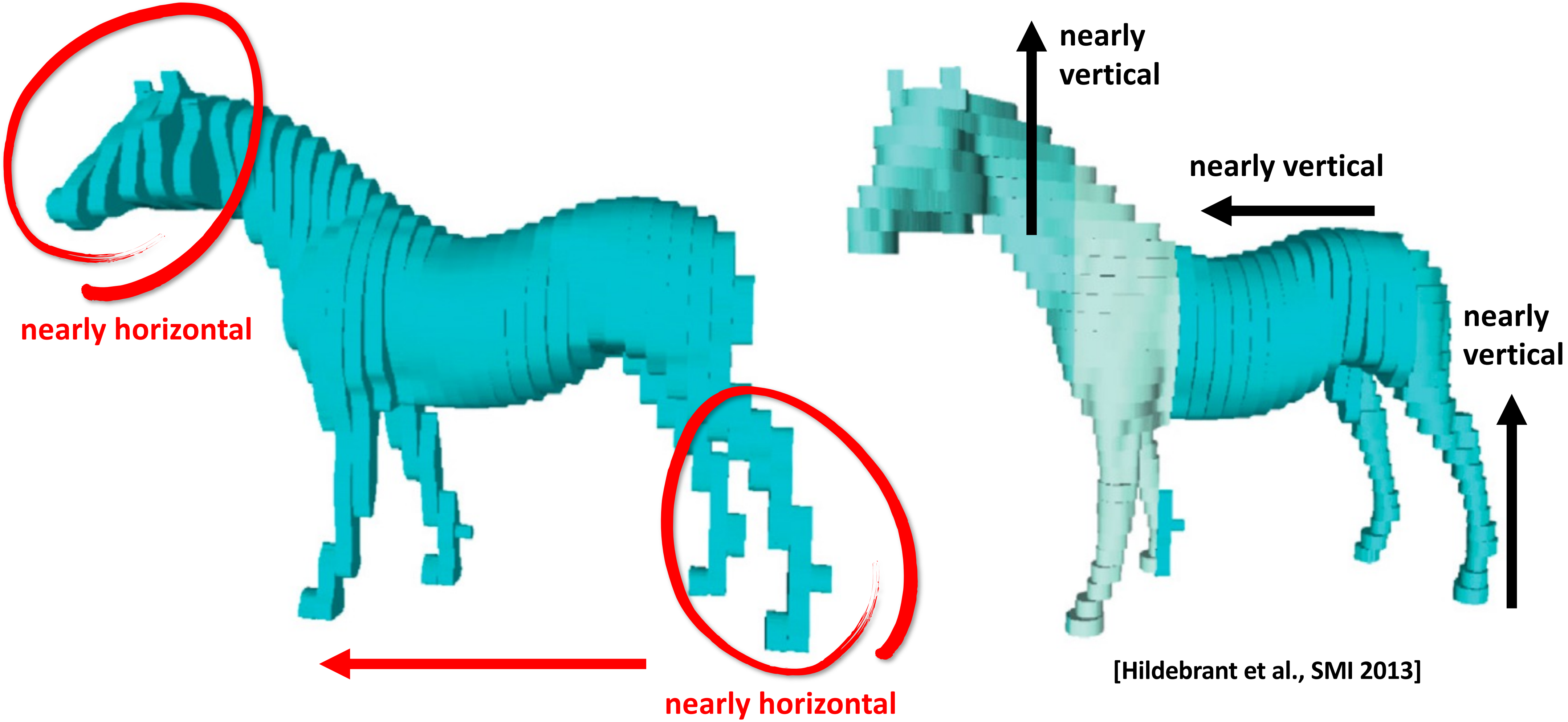


Staircase Effect



E(nearly horizontal) >> E(nearly vertical)

Staircase Effect

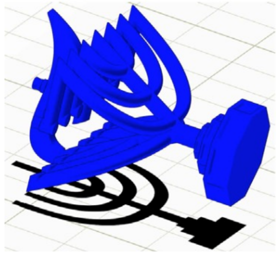


[Hildebrant et al., SMI 2013]

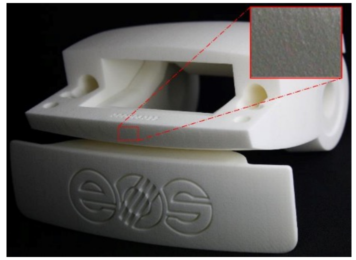
Common Metrics

- ***Cost***
  - Min support structures
  - Min build time
- ***Fidelity***
  - Min Staircase Error
  - Min post processing (e.g., artifacts due to support removal)
- ***Functionality***
  - Min critical stress areas
- ***Mixed Factors***
  - Min weighted sums of the aforementioned criteria

[Morgan et al., 2016]



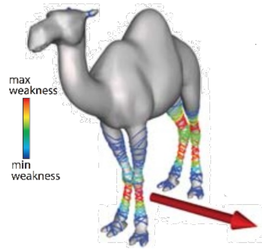
[Ezair et al., 2015]



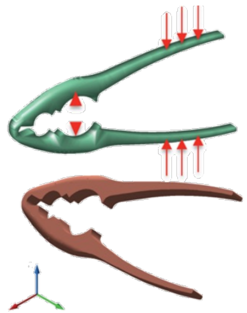
[Delfs et al., 2016]

[Zhang et al., 2015]

[Whang et al., 2016]

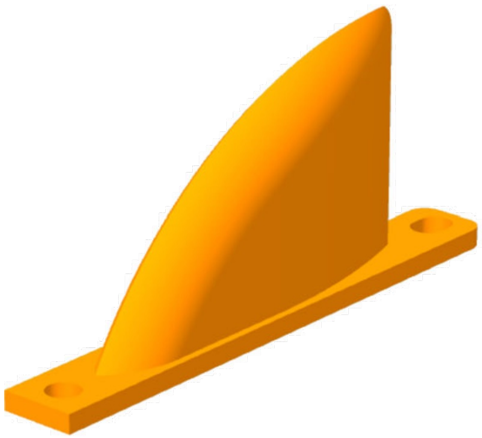


[Umetani et al., 2013]

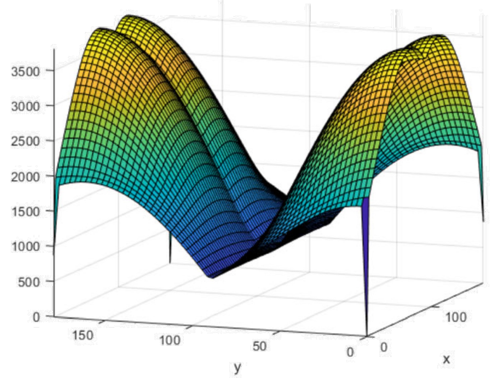


[Ulu et al., 2015]

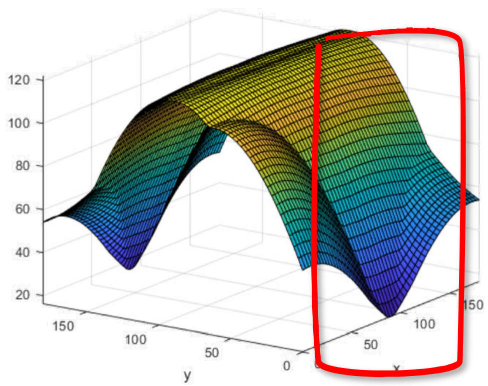
Optimization Landscape



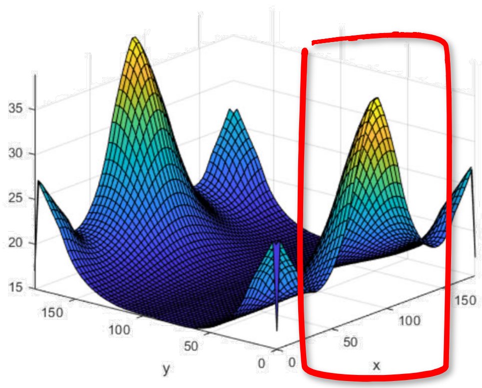
- Non convex functionals
- Multiple local minima, many not so good
- Contraddicting objectives!



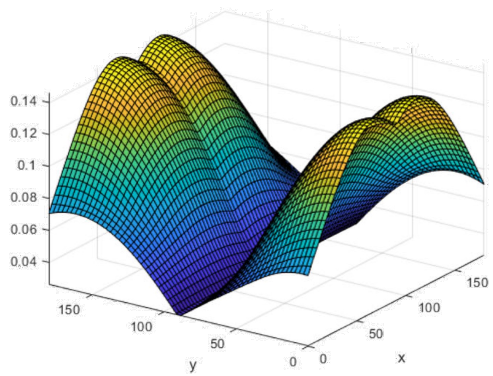
SUPPORT AREA



BUILD TIME



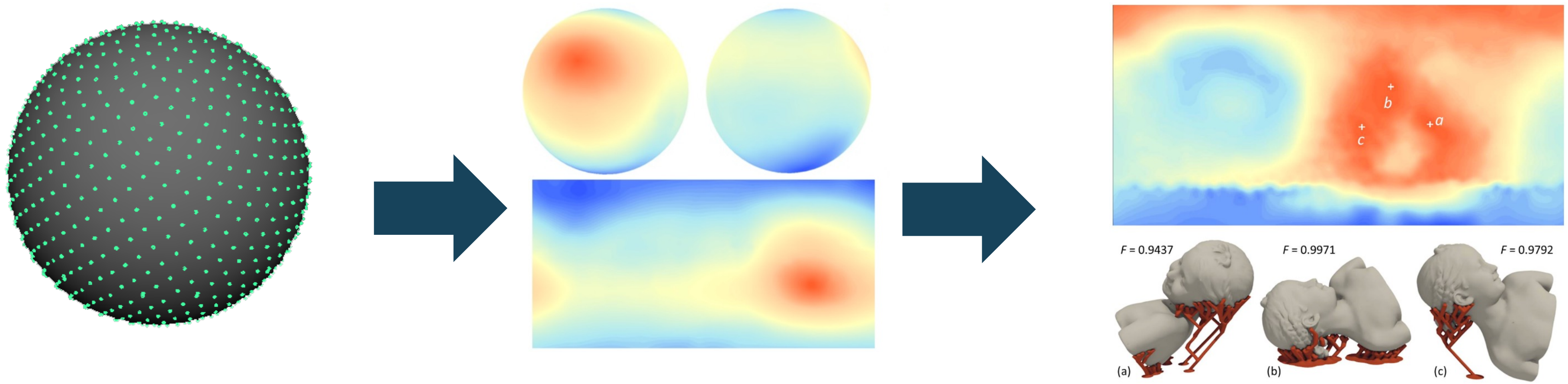
SURFACE ROUGHNESS



SURFACE QUALITY

Recurrent approaches

- Many recent methods are based on similar ***heuristics***



SAMPLE GAUSS SPHERE (UNIFORM/SALIENCY)

EVALUATE SAMPLES

SELECT LOCAL MAXIMA/MINIMA

- ***Genetic Algorithms*** *and* ***Simulated Annealing*** *are also used!*

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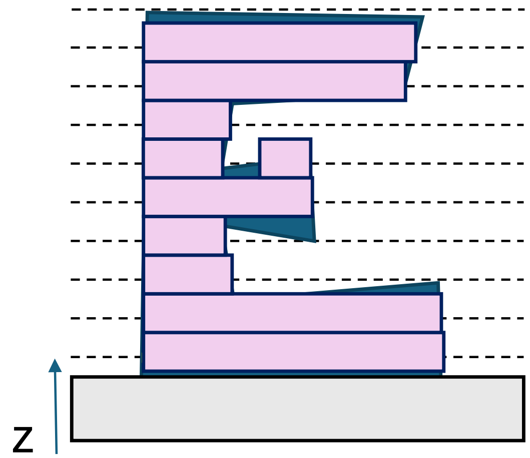
Slicing

- ***Input:*** model in its final orientation
- ***Output:*** contours to become slabs (layers) of material

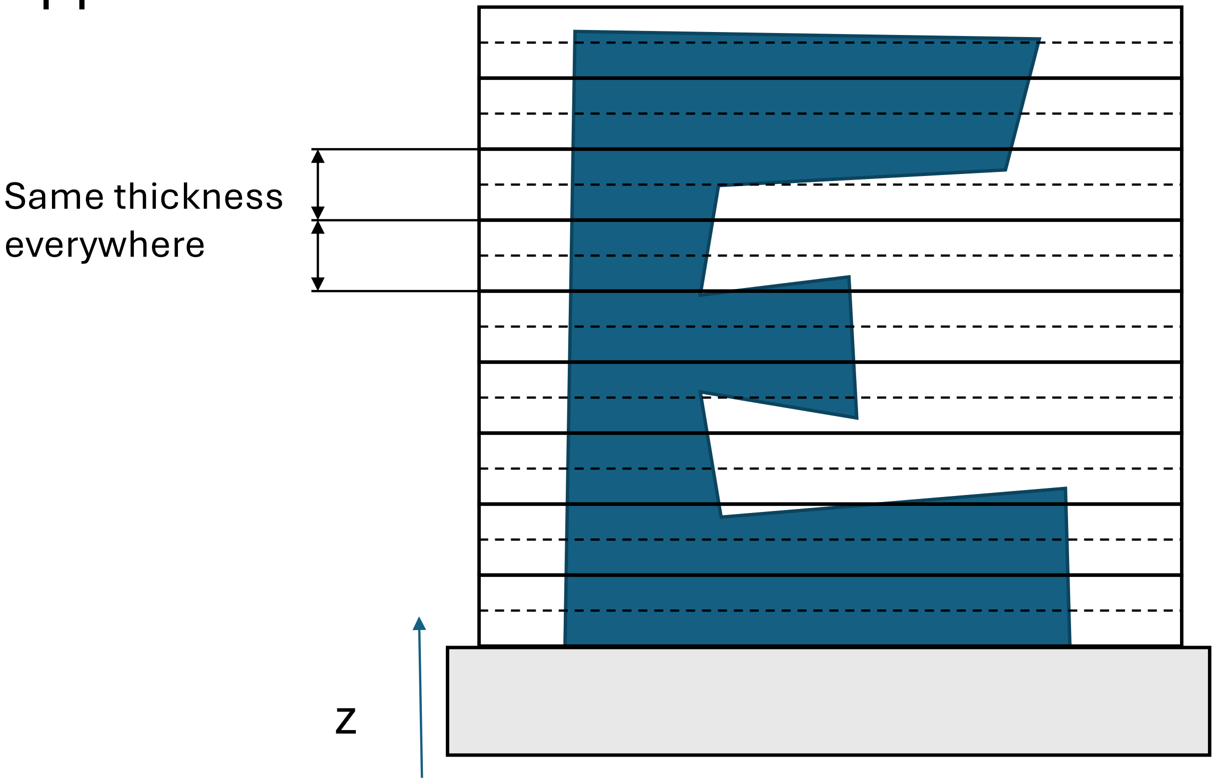


- Two main questions to answer:
  - ***Where*** to position the slicing planes
  - ***How*** to efficiently compute the contours

- Objective: ***accuracy!***

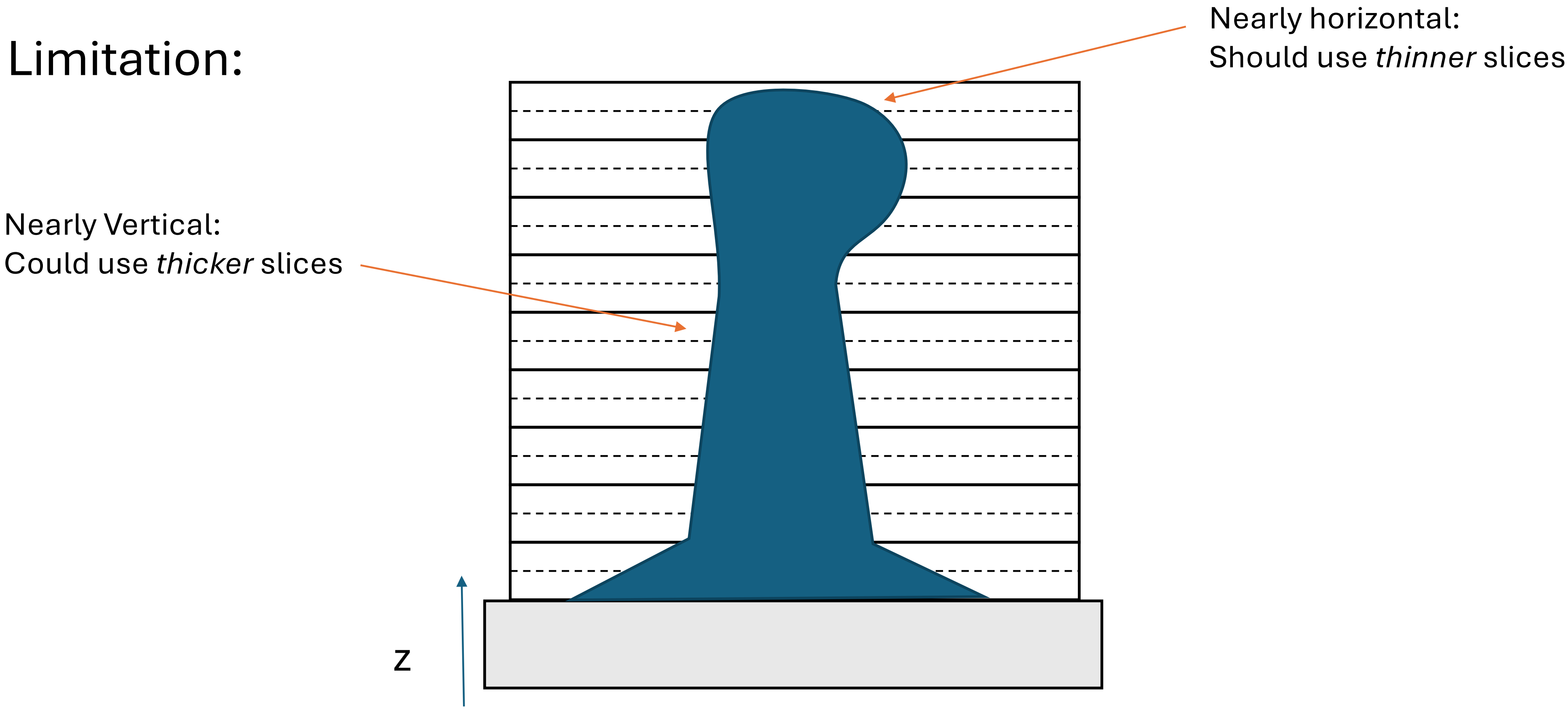
Uniform Slicing

- Main approach:



Uniform Slicing

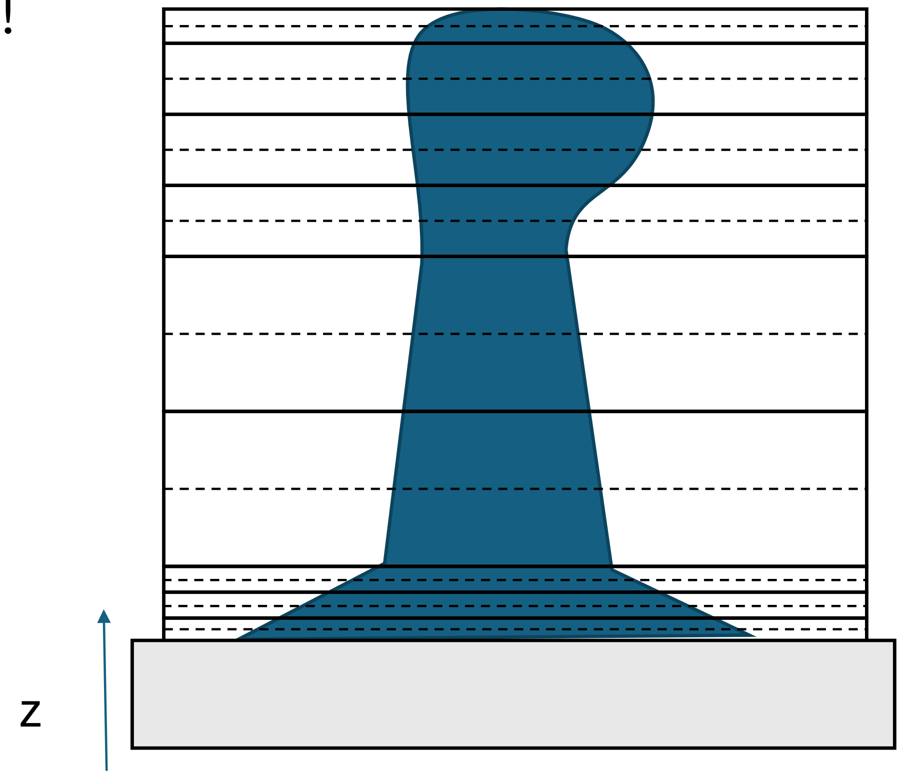
- Limitation:



Adaptive Slicing

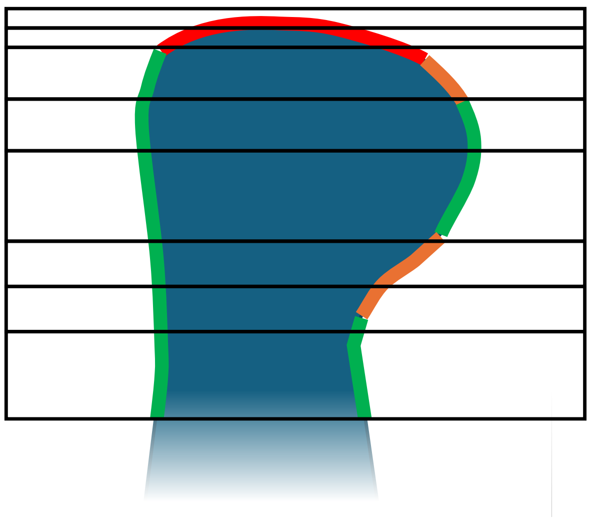
- Many technologies can vary layer height
  - Adapt the slices!
  - Same # slices

    ➔ less *error*



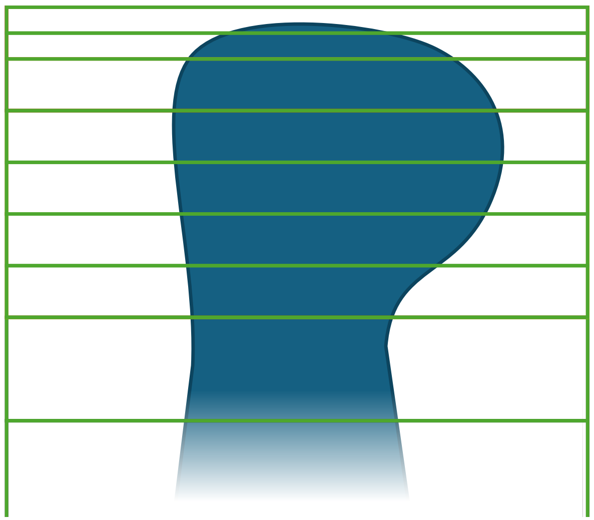
Adaptive Slicing

- How to determine the slice thicknesses?
  - From local error



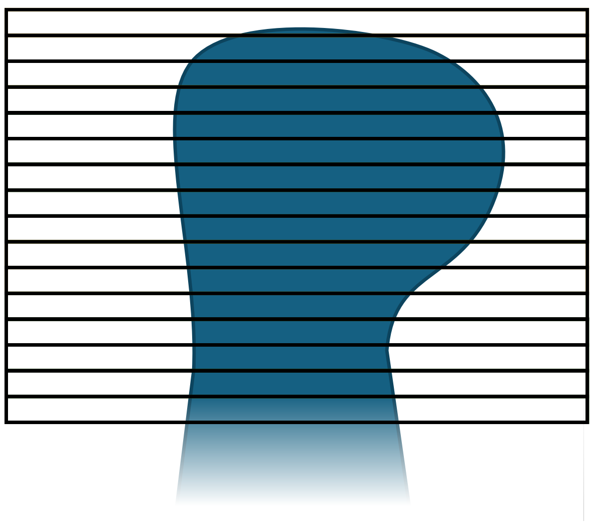
Adaptive Slicing

- How to determine the slice thicknesses?
  - From local error
  - Subdivide from coarsest uniform



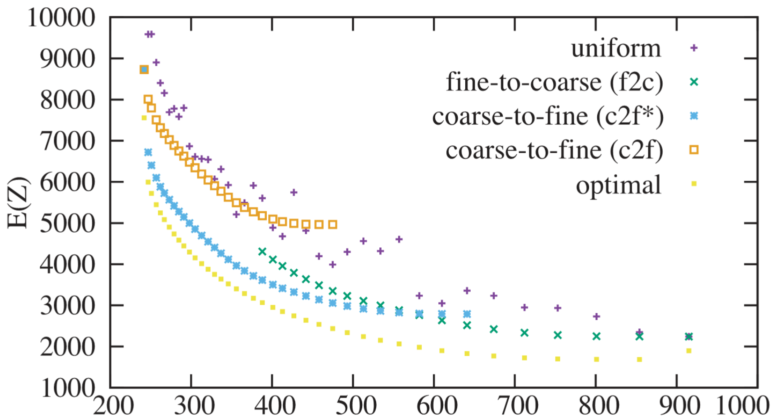
Adaptive Slicing

- How to determine the slice thicknesses?
  - From local error
  - Subdivide from coarsest uniform
  - Merge from thinnest uniform



Adaptive Slicing

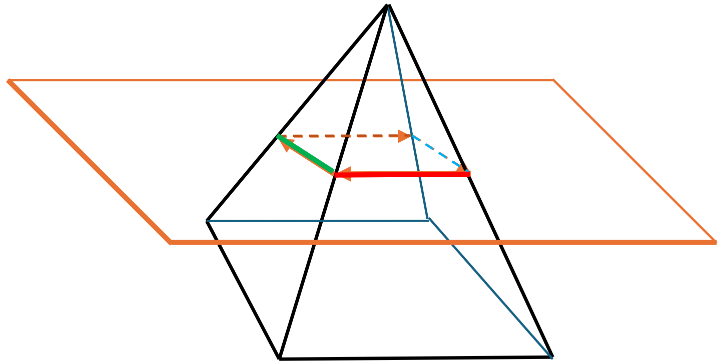
- How to determine the slice thicknesses?
  - From local error
  - Subdivide from coarsest uniform
  - Merge from thinnest uniform
  - Global optimization



[AHL17] Optimal Discrete Slicing, M. Alexa, K. Hildebrand, S. Lefebvre, ACM TOG, 2(

Indirect Contouring of Triangle Meshes

- **Input:** 3D model + slice planes
- **Output:** 2D contours for each slice (raster or vector)



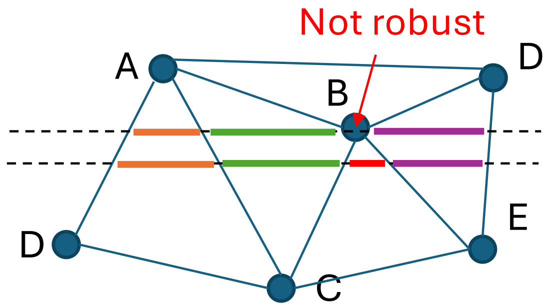
1 intersect each slice plane with triangles

➔ produces edges

2 connect the edges into closed loops

➔ guaranteed to form a closed, simple loop

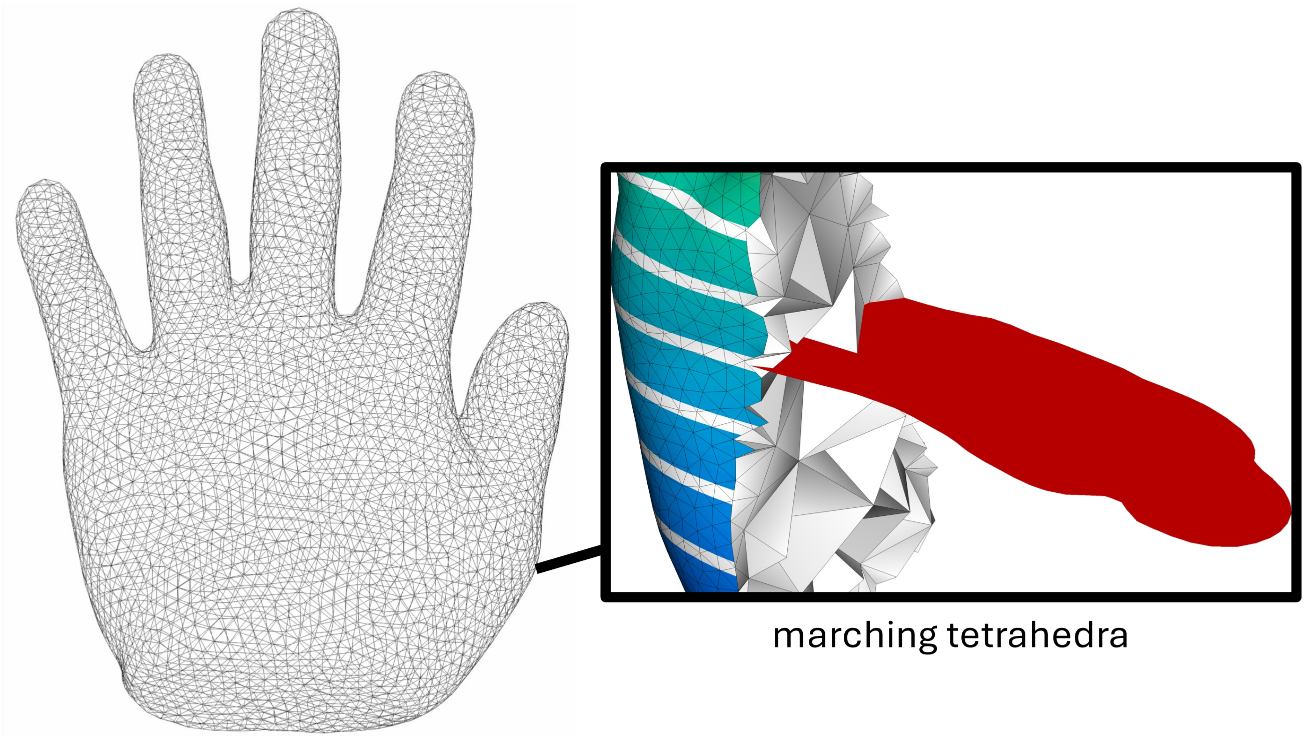
Beware of numerical issues!!



Implicit Contouring of Triangle Meshes

- Define slices as level sets of implicit functions

Surface Mesh
(contours as 1D polylines)

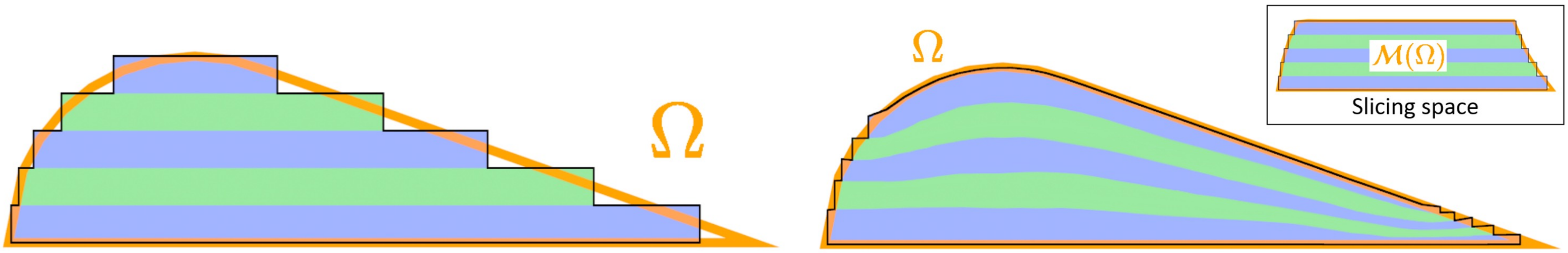


Volume Mesh
(contours as 2D polygons)

marching tetrahedra

Non Planar Slicing

- Many FDM printers can deposit material along *slightly* curved paths
- Exploit this feature to ***reduce the staircase effect!***
- ***Key idea***: map to a *slicing space*



[CurvySlicer, 2019]

Non Planar Slicing

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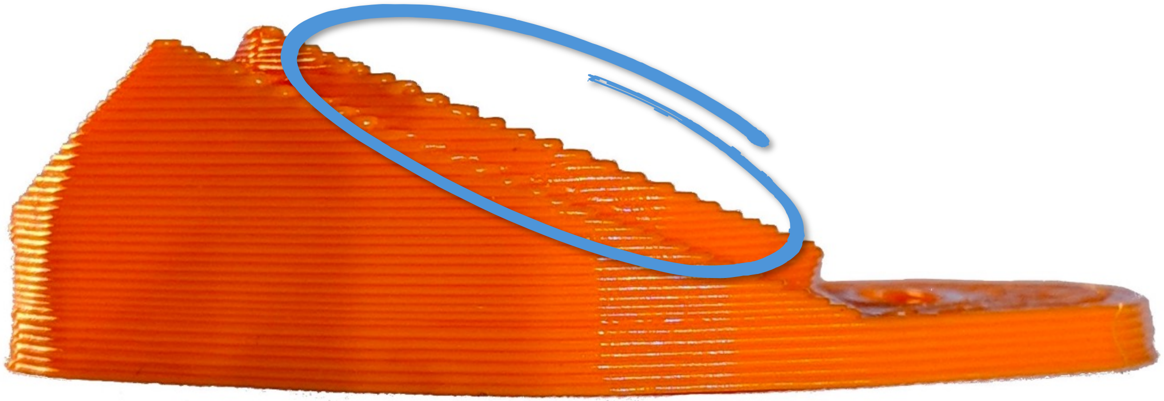


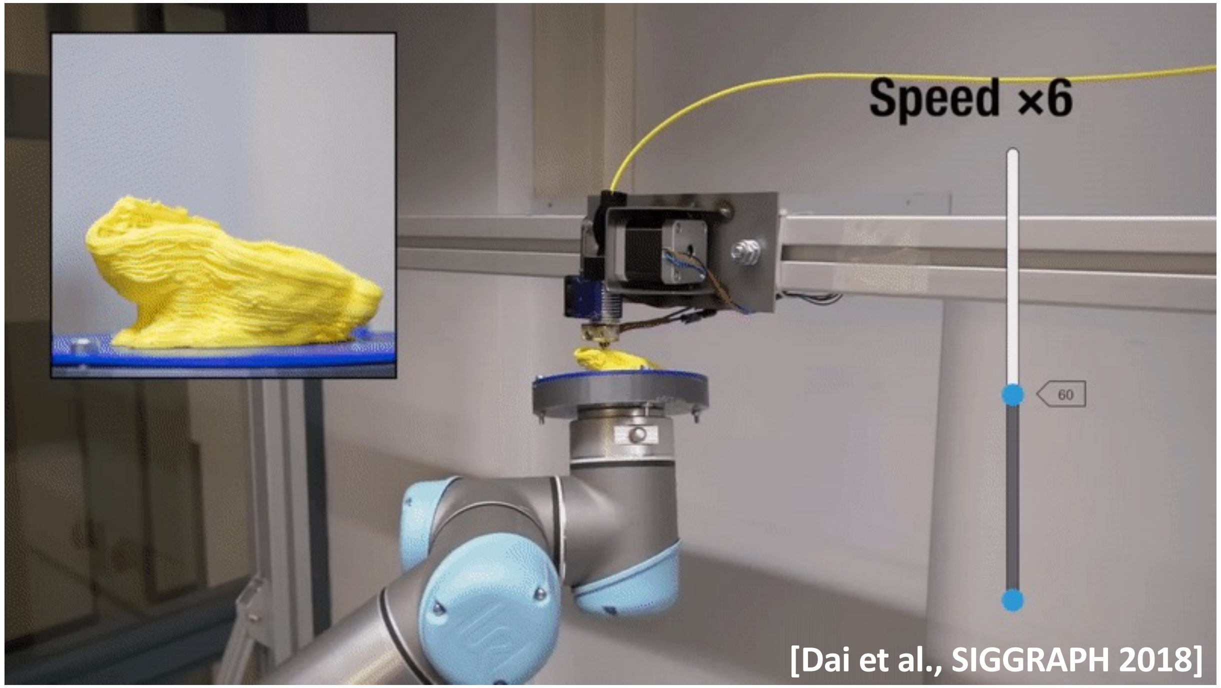
[CurvySlicer, 2019]

Non Planar Slicing

- Many FDM printers can deposit material along *slightly* curved paths
- Exploit this feature to ***reduce the staircase effect!***
- ***Key idea***: map to a *slicing space*

MAJOR ISSUES:

- *Meet fabricability constraints*
- *Avoid collisions with the tool*

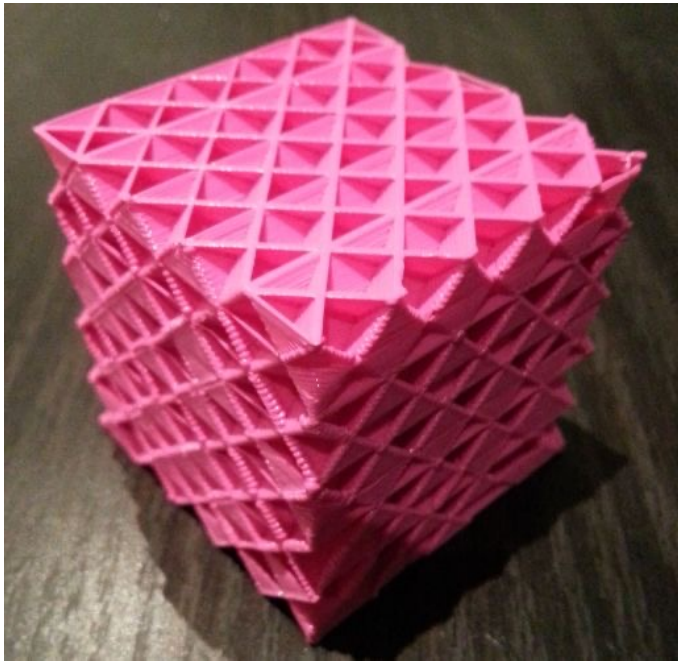


Outline

- Fabrication Technologies
- Modeling for fabrication
- Model orientation
- Slicing
- **Internal Supports**
- External supports
- Decomposition
- Toolpath generation
- Conclusions and outlook

Support structures

Internal structure

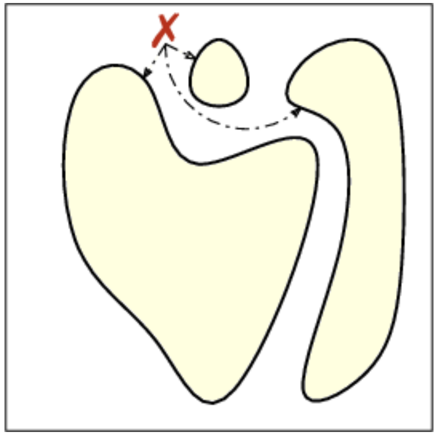


External structures

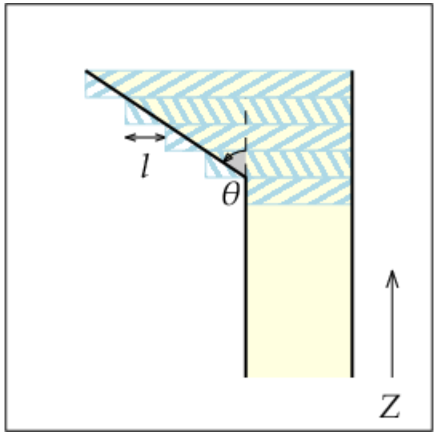
Constraints and Technologies

	Material Deposition	Layer Solidification
Connectivity	✗	(✗) SLA
Overhangs	✗	
Islands	✗	(✗) SLA
Cavities		✗
Heat dissipation	(✗)	(✗)

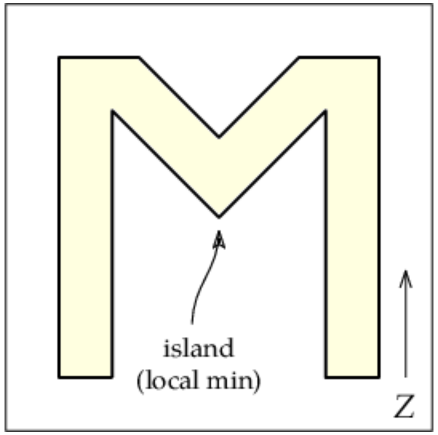
Connectivity



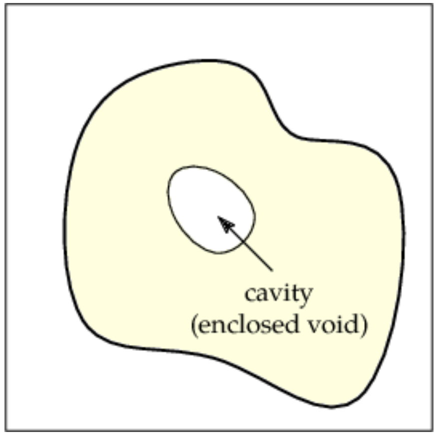
Overhang



Islands

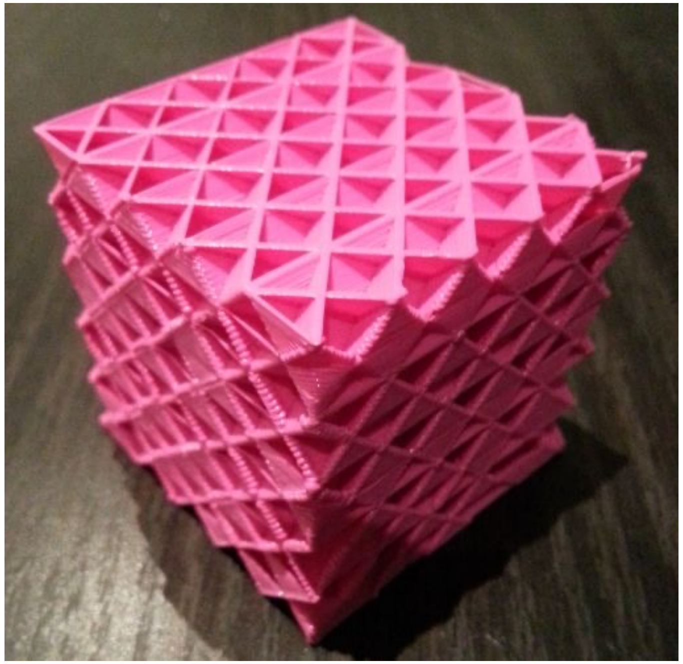


Cavities



Support structures

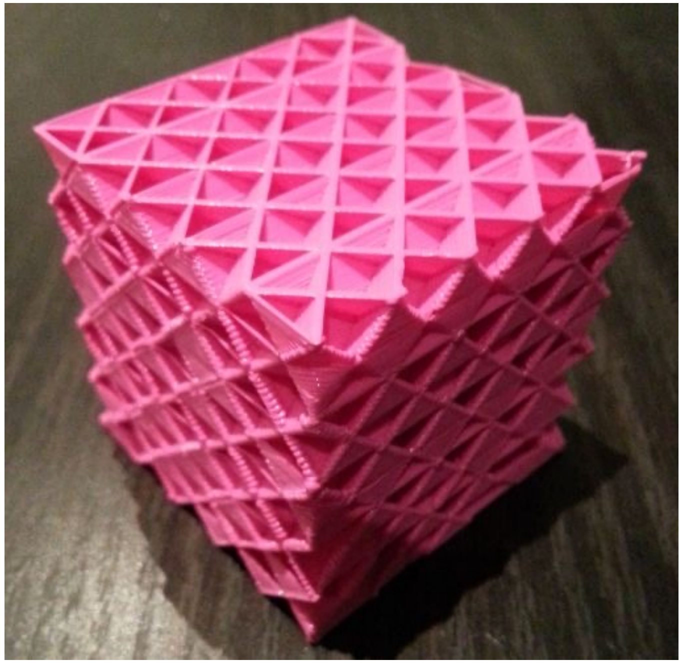
Internal structure



External structures

Internal Structures: Motivation

- Save material consumption
- Reduce printing time
- Change physical properties
  - ➔ try to preserve rigidity
  - ➔ or introduce flexibility



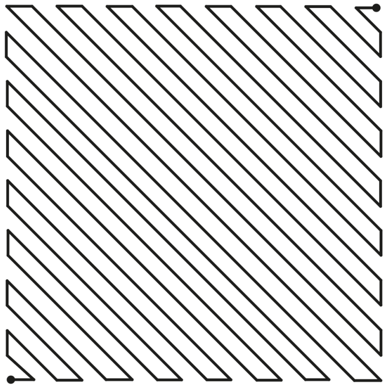
Courtesy of [Lef15]

Internal Structures: Overview

Hollowing

Courtesy of [MHCL15]

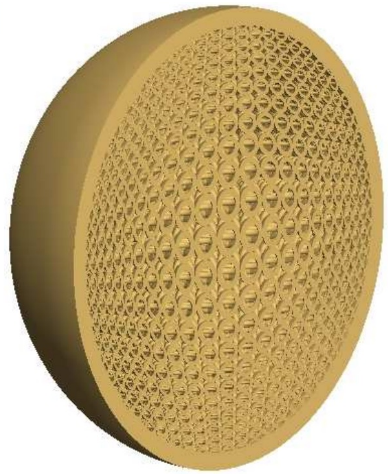
Dense and sparse infills



Internal frame structures

Courtesy of [LSZ*14]

Microstructures



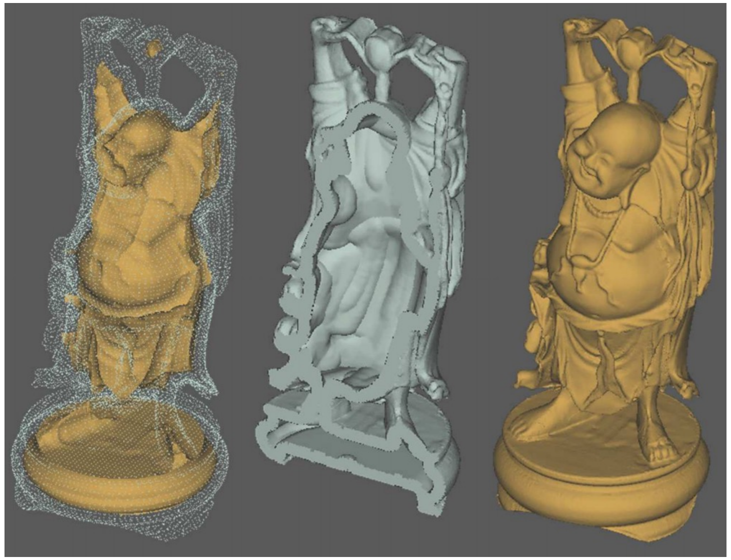
Courtesy of [FVP13]

Internal Structures: Hollowing

- Morphological erosion:

Preprocessing

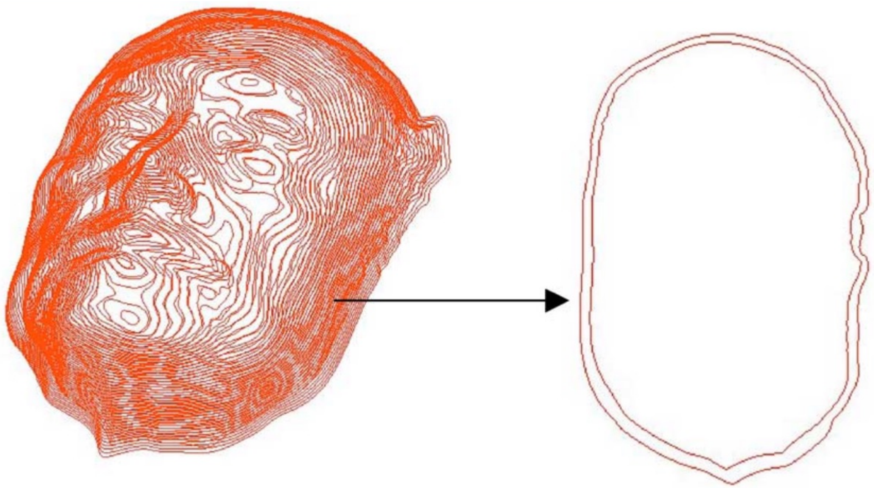
[CT98,CW11, WM13b,LM14,CB14,MHCL15]



Courtesy of [WM13b]

At slice level

[MSWS00, Par05]

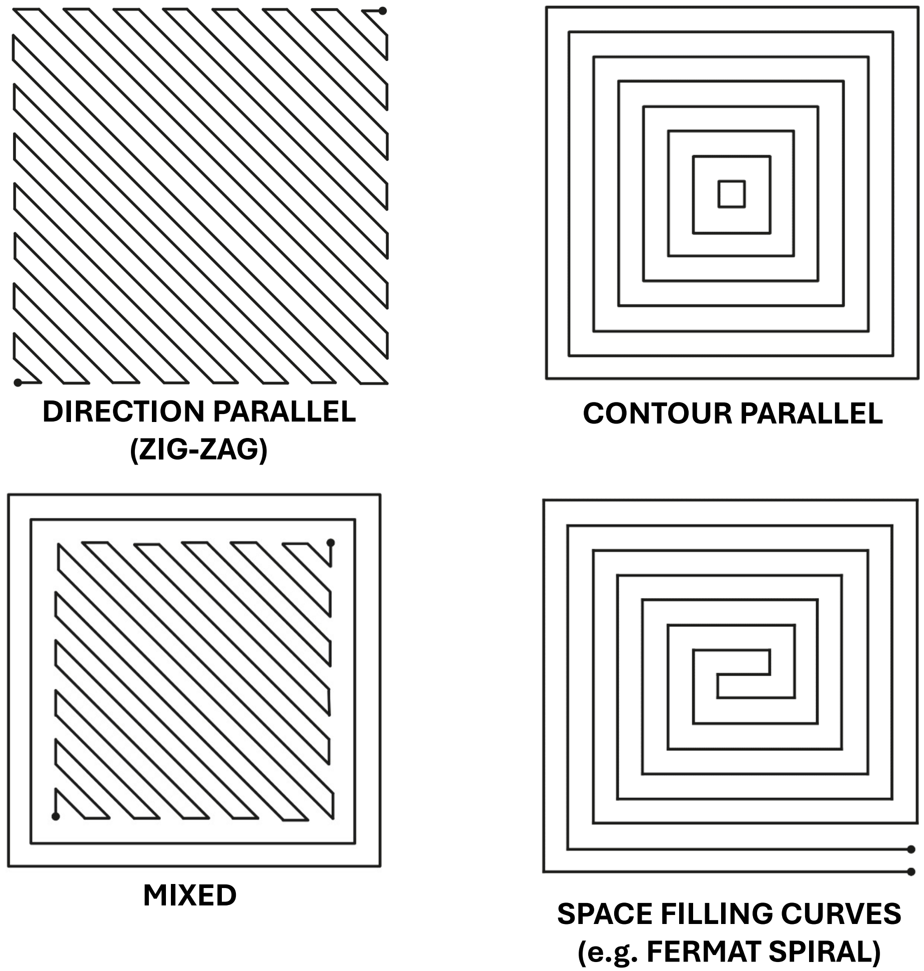


Courtesy of [Par05]

Dense Infills

- Space filling curve
- Challenges:
  - Print time, quality, robustness
  - Object strength

Courtesy of [CZH23]

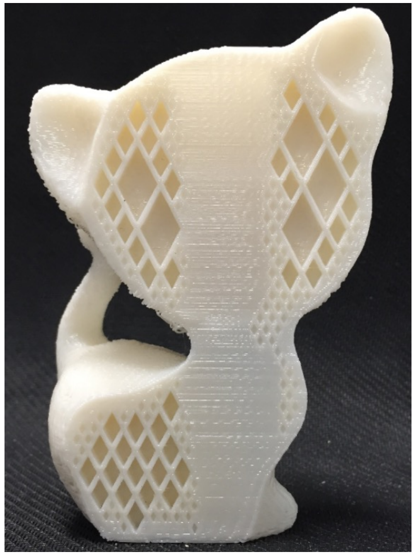


Sparse Infills

Spatial tessellations

Courtesy of [LSZ*14]

Self-supporting with load direction

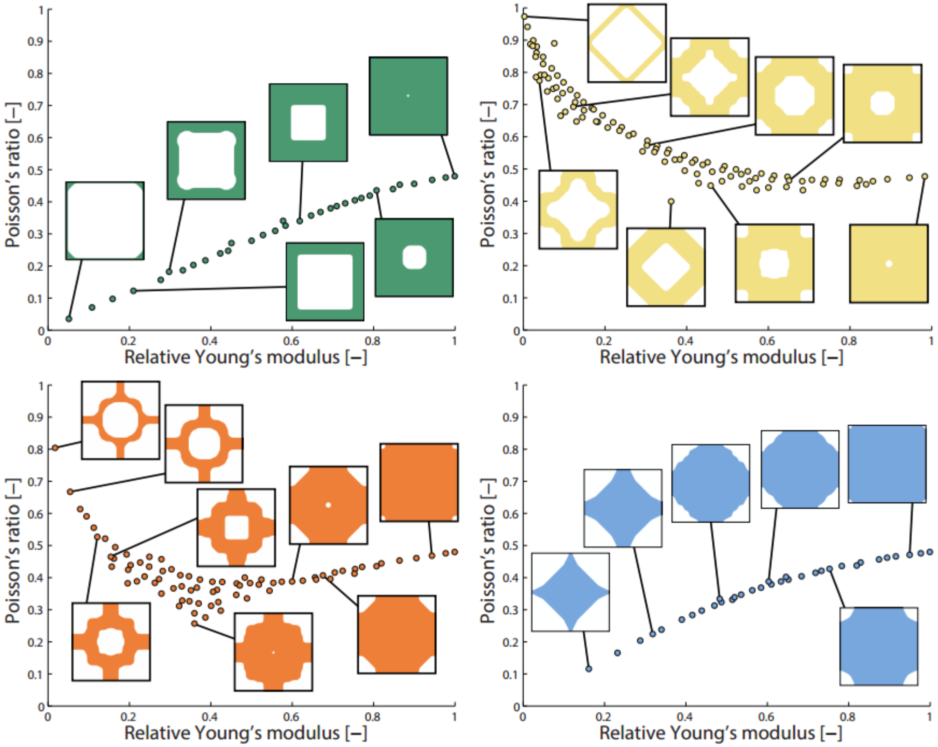


Courtesy of [WWZW16]

Self-supporting (no pref. direction)

Courtesy of [WLD*22]

Microgeometry -> Physics



Courtesy of [PZM*15]

Microstructures to Control Elasticity in 3D printing (Schumacher et al. 2015)

Outline

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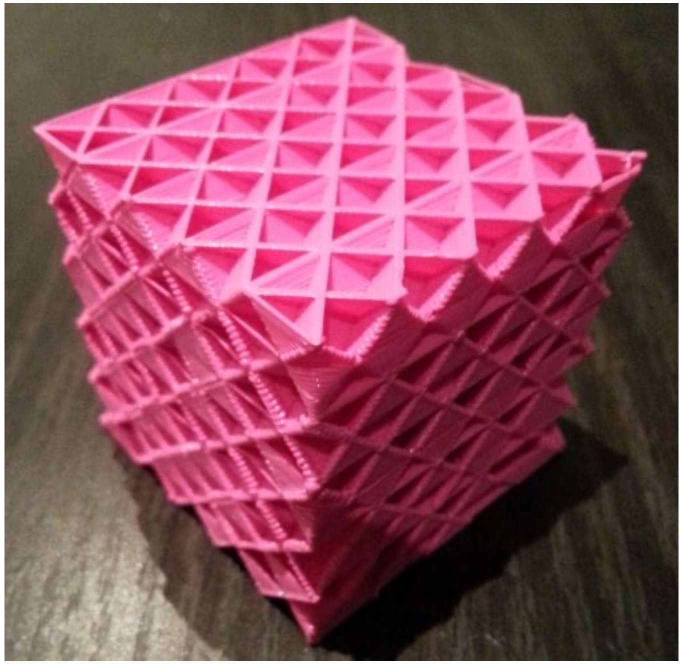
- **External supports**
- Decomposition
- Toolpath generation
- Conclusions and outlook

Support structures

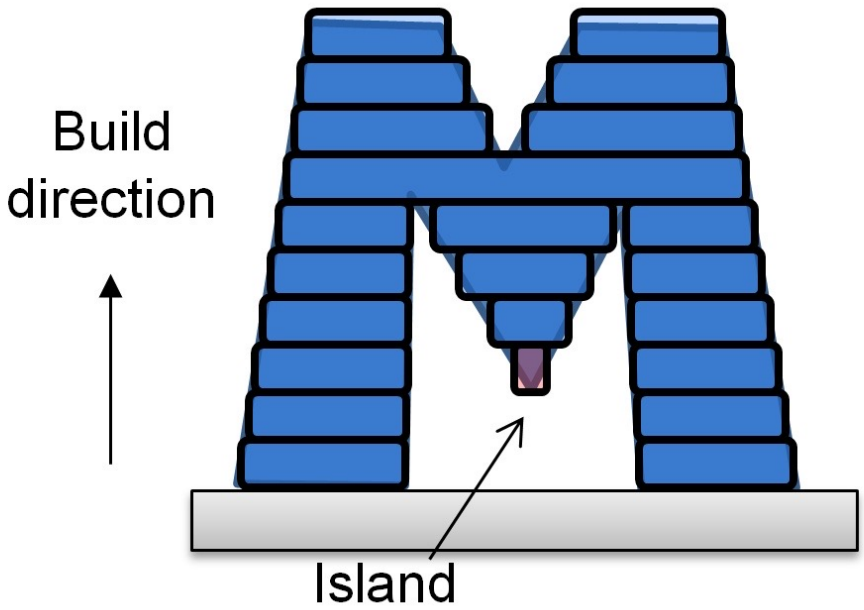
- Internal structures.



- External structures.

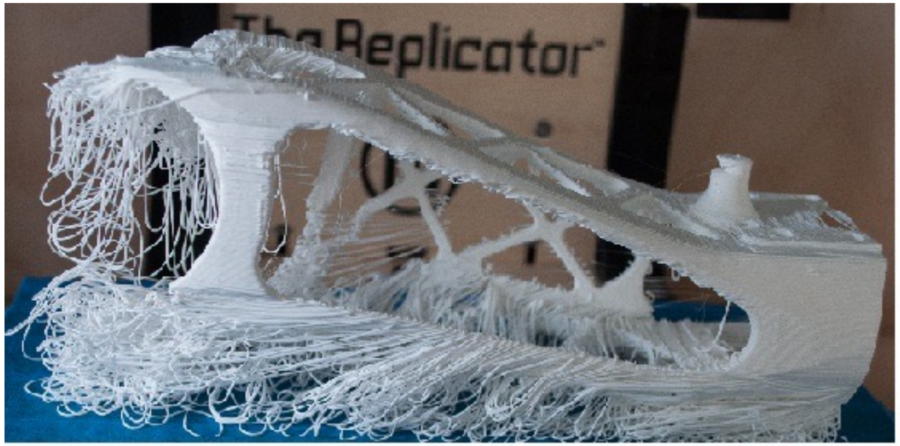
External Structures: Motivation

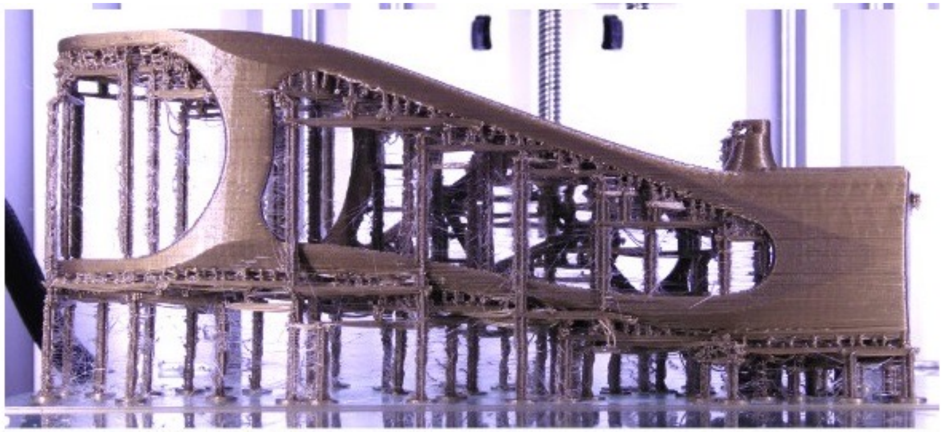
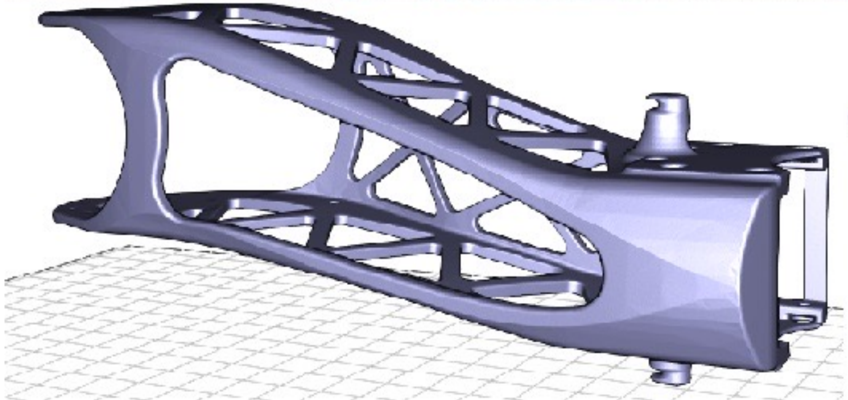
- Overhangs and islands:



- Other factors: shape deformation, heat diffusion, etc.

Overhangs and Islands




Standard Process

1. **Detect** surfaces requiring supports

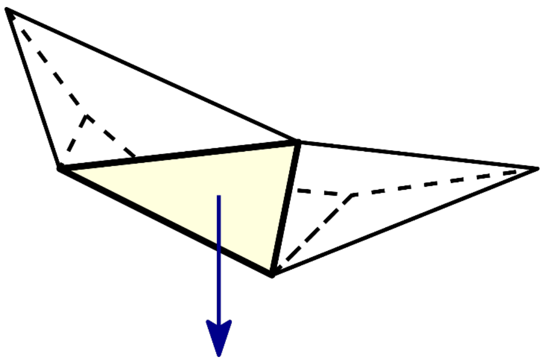
2. **Generate** support structures

3. **Remove** of supports after printing

Detect need for supports

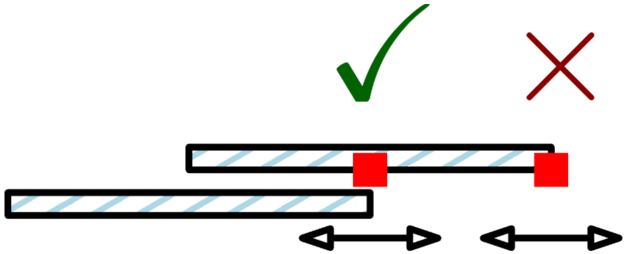
Face orientation

[KJAB*91,AD95]



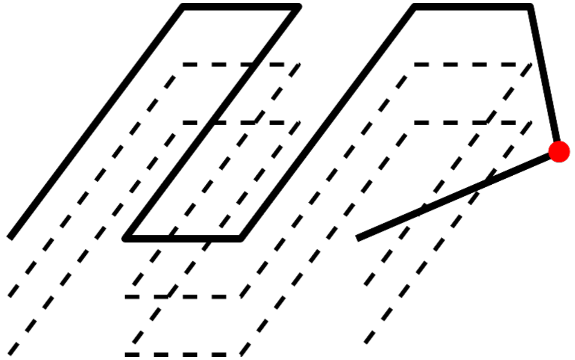
At slice level

[ACC*88,CJR95,HYML09,CLQ13,HWC14]



At toolpath level

[DHL14]



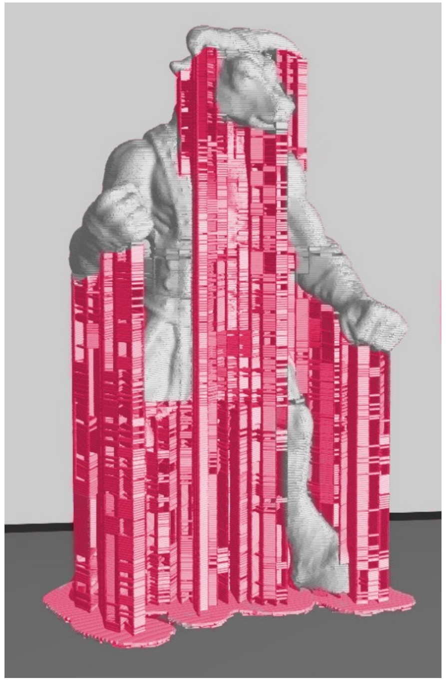
- Select subset of support points [ER07, CLQ13, DHL14, HWC14]

Generate support structures

- **Trade-off:**
  - Print time
  - Material use
  - Reliability

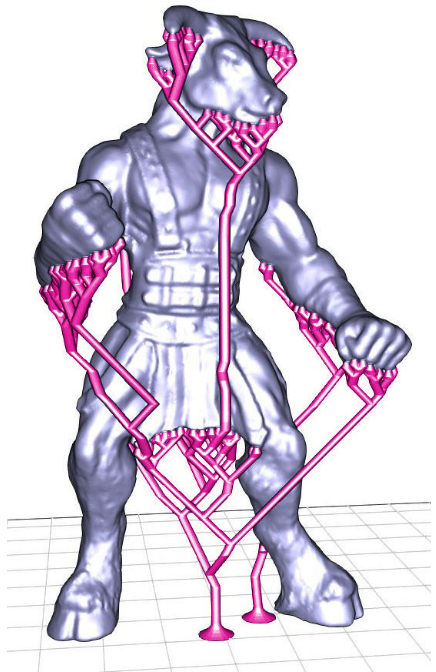
Dense

[HYW*09,Hei10]



Sparse

[VGB14a,HWC14,DHL14,SU14]



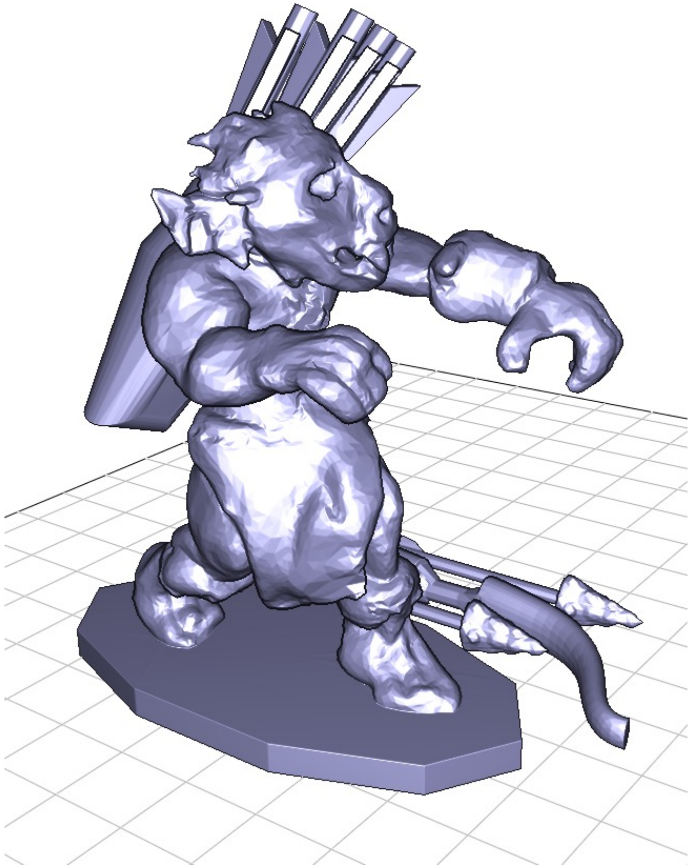
Meshmixer [SU14]

Steady scaffolds

1. Overhang detection.

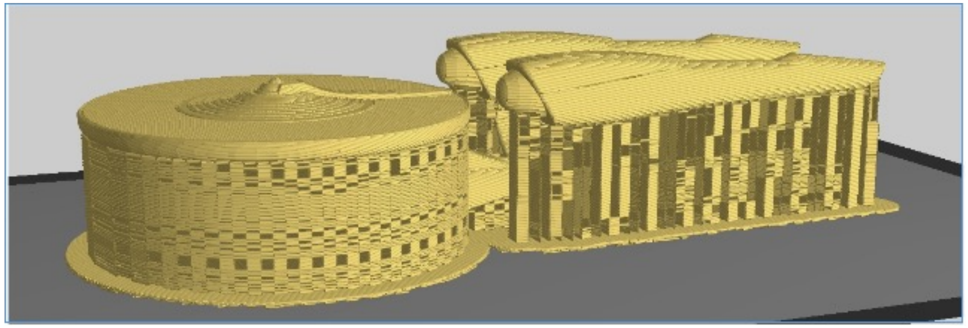
2. Bridge synthesis.

Bridging the gap: automated steady scaffoldings for 3D printing (Dumas et al. 2014)

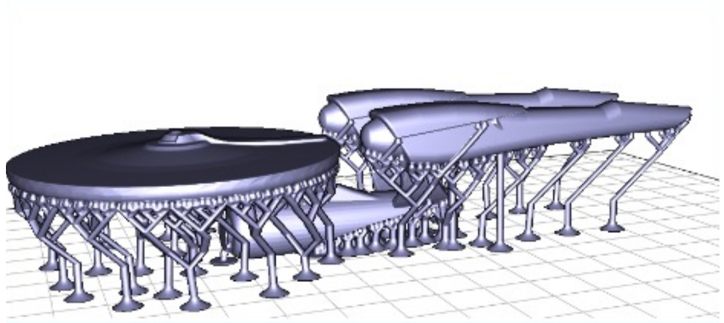


thing:347046

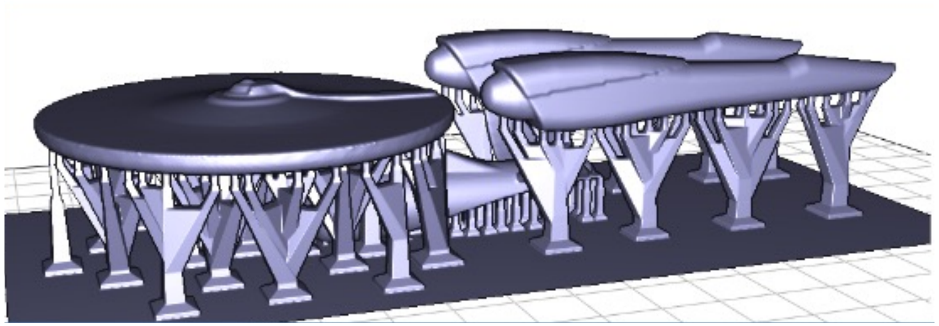
External Structures: Comparison



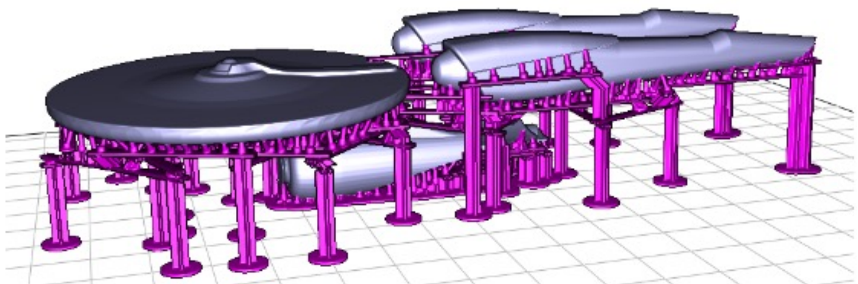
Makerware



Meshmixer [SU14]



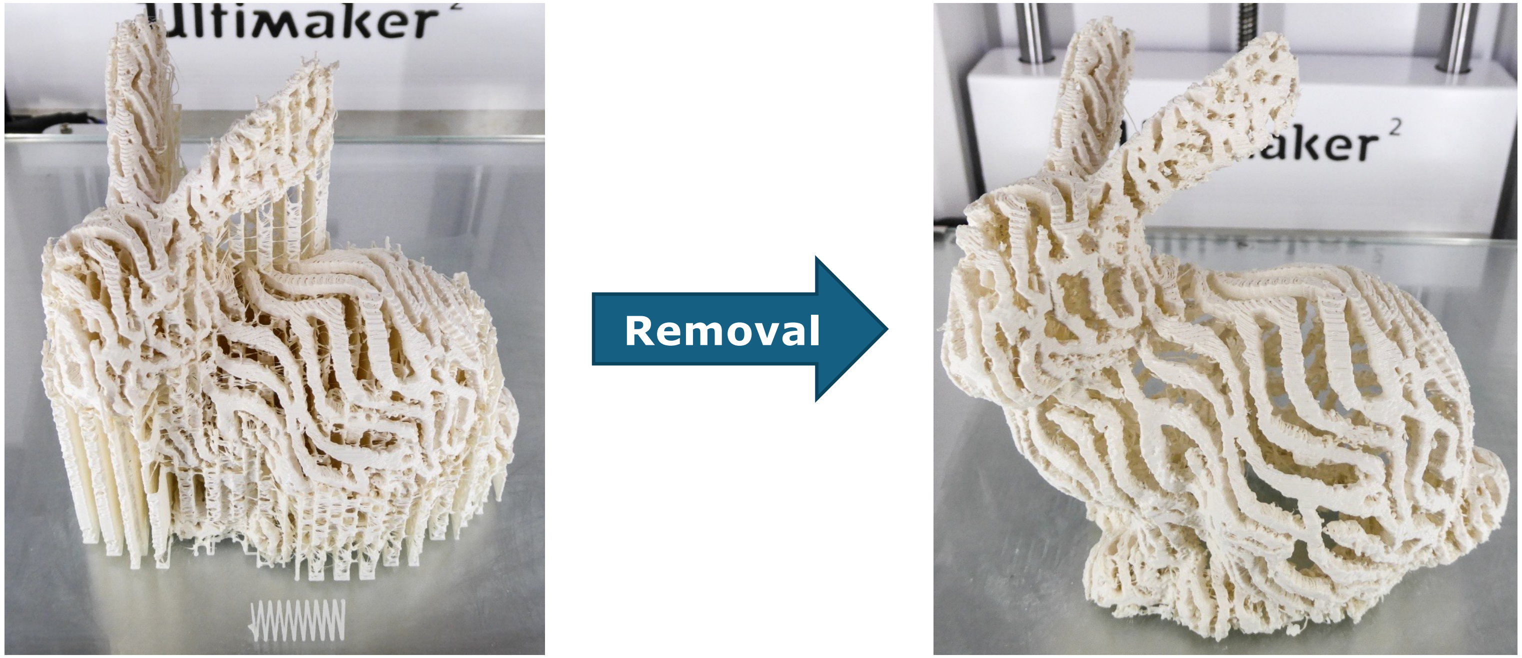
PhotshopCC



Dumas et al. 2014 [DHL14]

Support removal after printing

- Can be challenging:



- Dissolvable supports [PJB04, HNCS16].

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Shape Decomposition

- Often used to ***overcome the limits*** of a fabrication technique or hardware

  - Size
  - Geometry
  - Colors/Materials
  - Staircase effect
  - Support artifacts
  - Packing

  …and many others!

Shape Decomposition

IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. IT-29, NO. 2, MARCH 1983

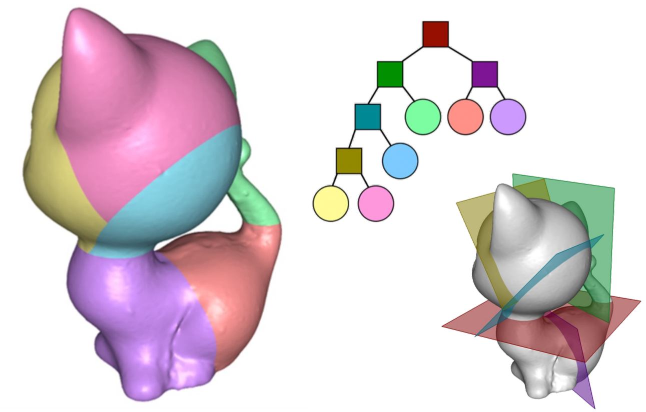
Some NP-Hard Polygon Decomposition Problems

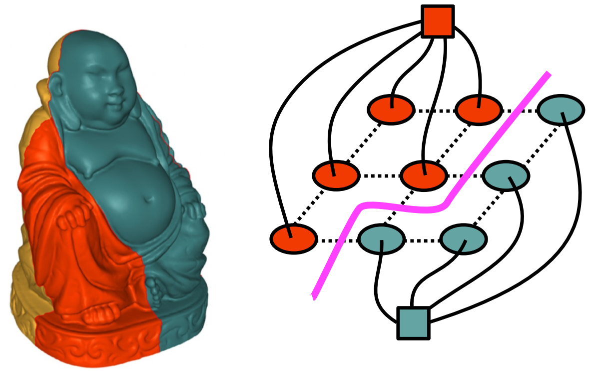
JOSEPH O'ROURKE, MEMBER, IEEE, AND KENNETH J. SUPOWIT

Abstract—The inherent computational complexity of polygon decomposition problems is of theoretical interest to researchers in the field of computational geometry and of practical interest to those working in syntactic pattern recognition. Three polygon decomposition problems are shown to be NP-hard and thus unlikely to admit efficient algorithms. The problems are to find minimum decompositions of a polygonal region into (perhaps overlapping) convex, star-shaped, or spiral subsets. We permit the polygonal region to contain holes. The proofs are by transformation from Boolean three-satisfiability, a known NP-complete problem. Several open problems are discussed.

Approaches

- Despite the variety of goals, manufacturing paradigms and fabrication hardware, all methods:
- Aim to control the same two aspects
  - Part size (either static or in motion)
  - Local surface orientation
- Mostly exploit similar techniques
  - Binary Space Partitions
  - Graph Labeling
  - Mesh booleans



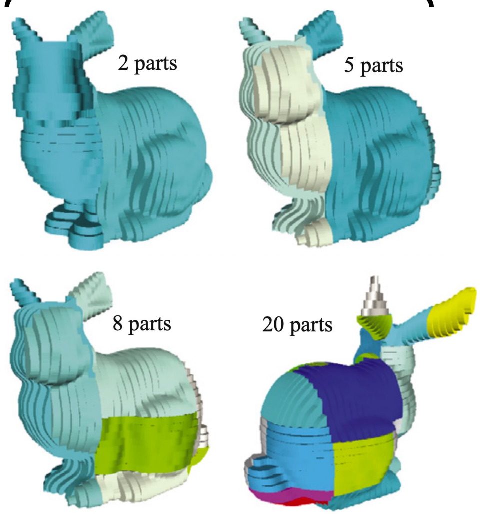


Binary Space Partitions

Size

[Luo et al., SIGGRAPH 2012]

Staircase Effect



[Hildebrand et al., SMI 2013]

Packing

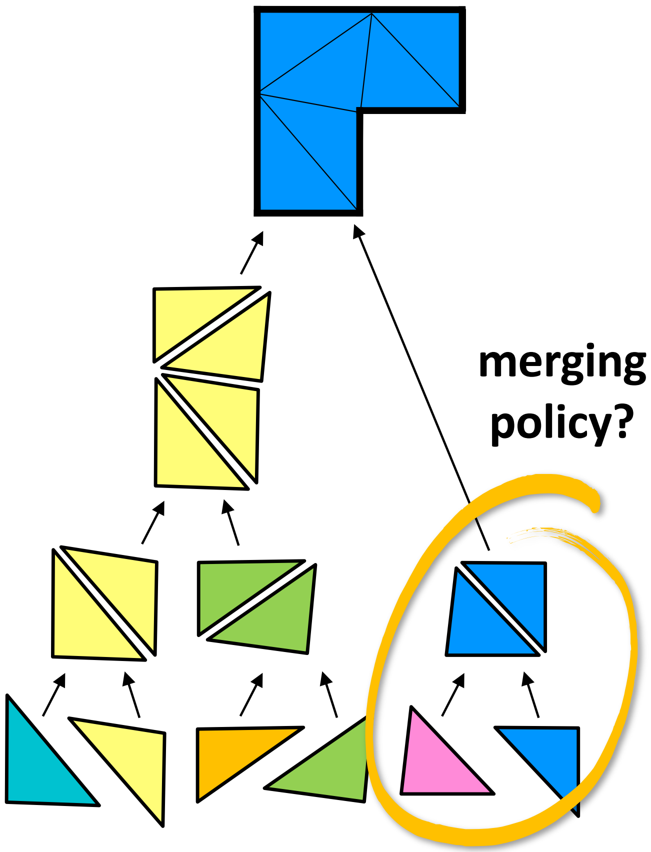
[Chen et al., SIGGRAPH Asia 2015]

[Attene, EG 2015]

Top Down
(with beam search)

Bottom Up
(greedy)

Bottom Up vs Top Down

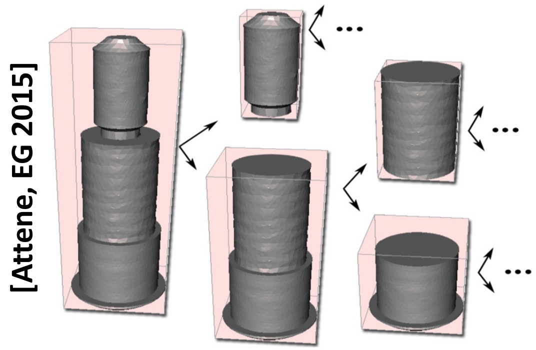


Shapes in a Box:

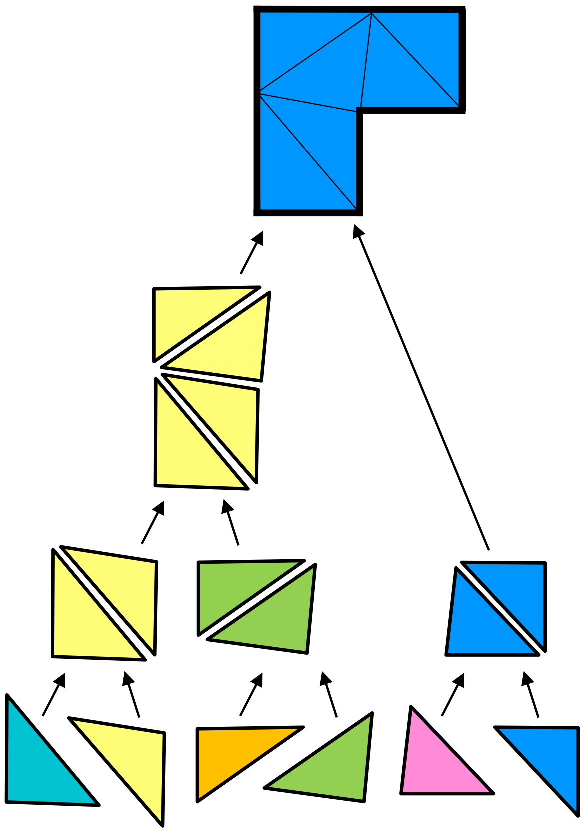
- ***discretize*** domain (tetrahedralization)
- minimize absolute aboxiness

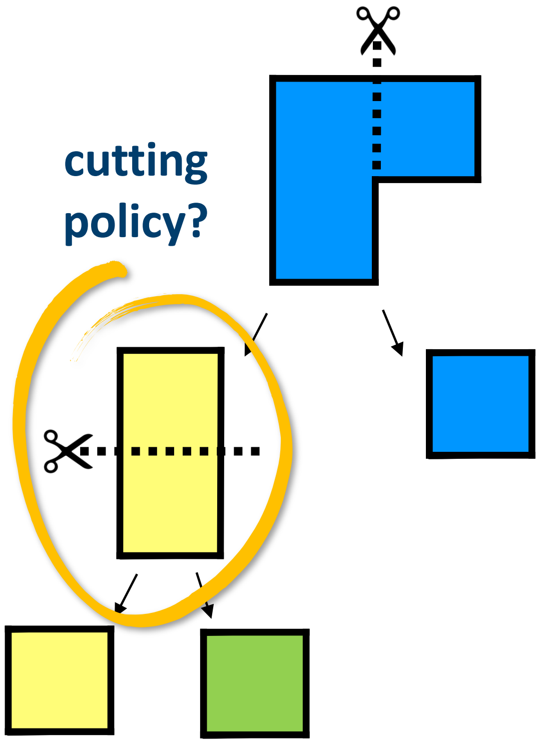
$$A(\mathcal{O}) = VOL(OBB(\mathcal{O})) - VOL(\mathcal{O})$$

cluster of tetrahedra — minimal oriented bbox



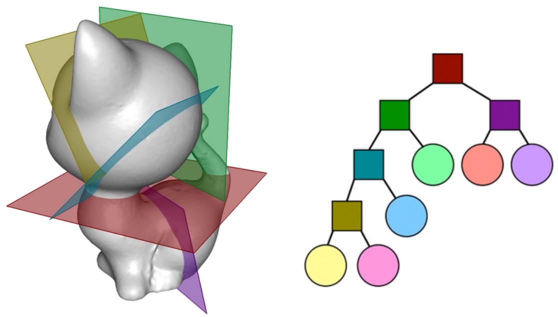
Bottom Up vs Top Down





Chopper:

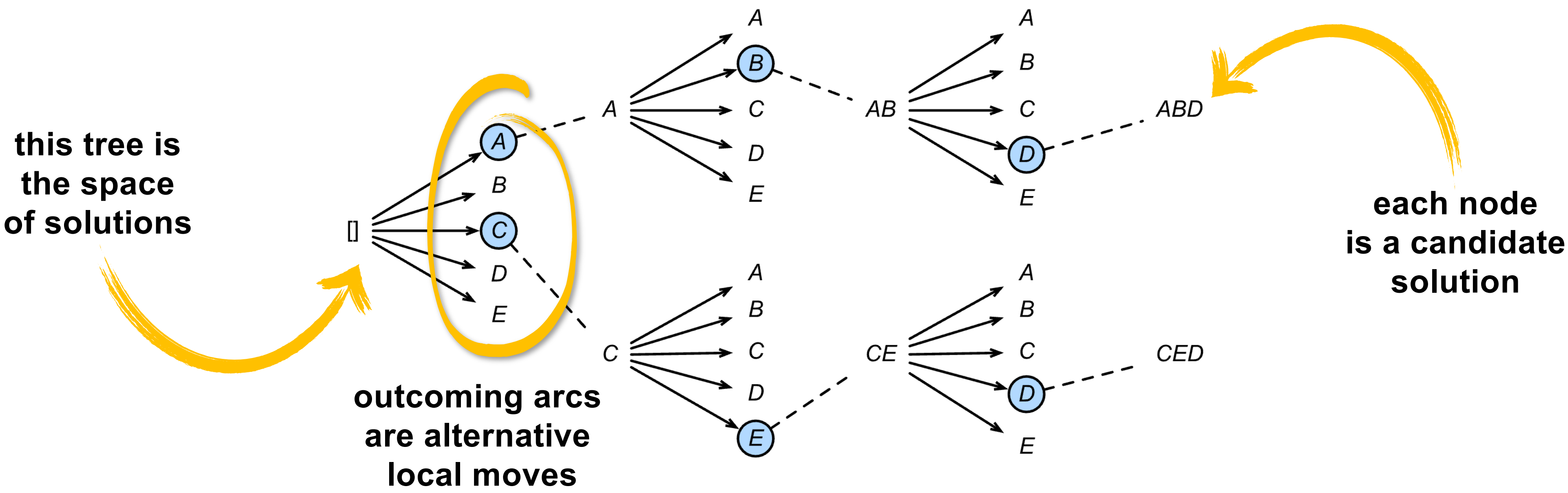
- ***discretize*** set of planar cuts
- minimize split metric based on
    - # of parts
    - connectors
    - structure/fragility
    - aesthetics (hide seams)
    - symmetry



[Luo et al., SIGGRAPH 2012]

Exploring the Space of Solutions

- **Greedy:** at each step pick the best move
- **Beam Search:** explore a wider portion of the feasible space
  - **Assumption:** partial solutions can be ranked
  - **Algorithm:** at each stage, continue exploring only the *N* best solutions (**beam width**)



Decomposition by Labeling

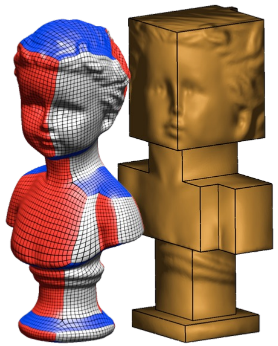
- Solves a multi-labeling problem on a generic graph *G(N,A)* by minimizing

$$L = \arg\min \sum_{i \in N} C_i(l) + \sum_{ij \in A} C_{ij}(l_i, l_j)$$

DATA TERM
cost of assigning label l to node i

SMOOTH TERM
cost of assigning labels l_i, l_j to adjacent nodes i, j

- The problem is NP-Complete
  - finds a local minimum
  - depends on initialization and processing order
  - heavily used in Vision/Graphics
  - it works remarkably well in practice!

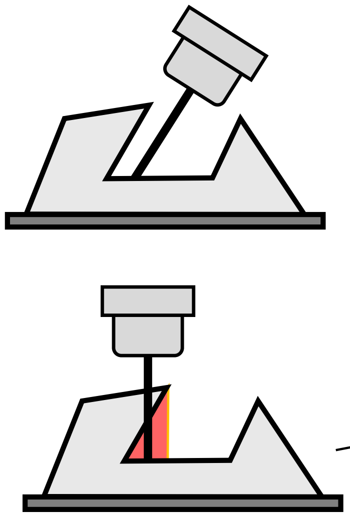


[PolyCut, SIG Asia 2013]

[GrabCut, SIGGRAPH 2004]

Labeling formulation

- The **graph** is the dual mesh
  - one node per triangle / tetrahedron / voxel
- The **labels** are candidate machining / extraction directions



$$L = \arg\min \sum_{i \in N} C_i(l) + \sum_{ij \in A} C_{ij}(l_i, l_j)$$

DATA TERM

$$\begin{cases} f_i(l) & \text{if feasible} \\ \infty & \text{otherwise} \end{cases}$$

SMOOTH TERM

$$\begin{cases} 0 & \text{if } l_i = l_j \\ f_{ij}(l_i, l_j) & \text{otherwise} \end{cases}$$

Labeling

- The **graph** is the dual mesh
  - one node per triangle / tetrahedron / voxel
- The **labels** are candidate machining / extraction directions

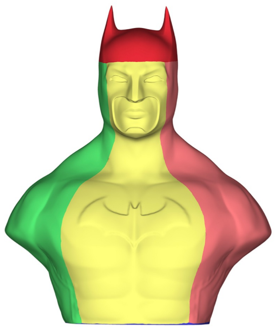
Surface2Volume
G: dual tetmesh
L: extraction directions

[Araùjo et al., SIGGRAPH 2019]

HF Decomp
G: dual trimesh
L: HF directions

[Herholz et al., EG 2015]

4 Axis Milling
G: dual trimesh
L: milling directions

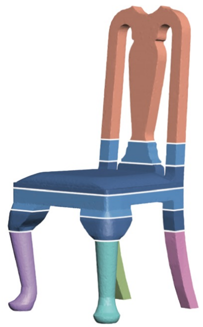


[Nuvoli et al., EG 2021]

Rigid Molding
G: dual tetmesh
L: molding directions

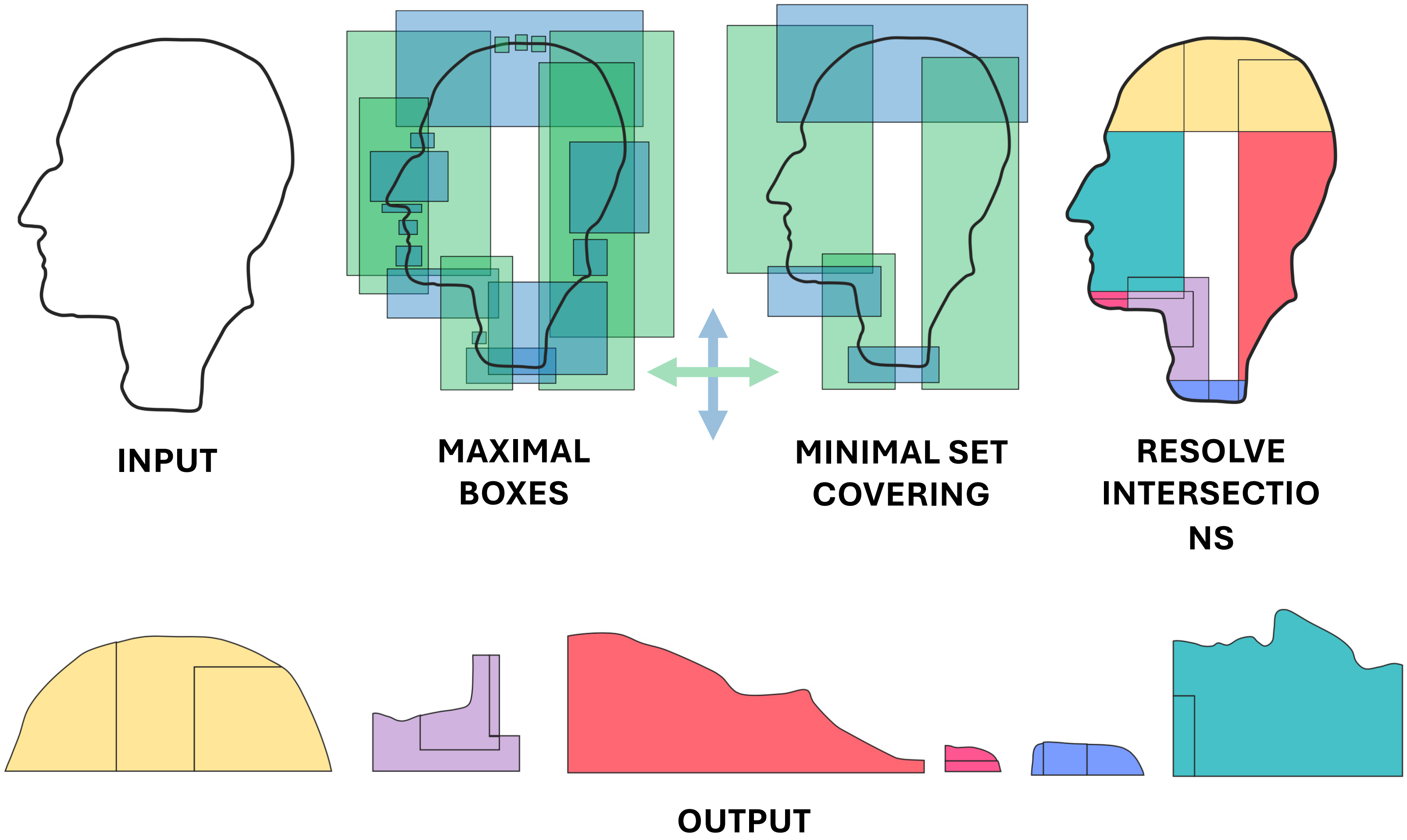
[Alderighi et al., SIG Asia 2021]

DHF Slicer
G: dual trimesh
L: DHF directions



[Yang et al., SIG Asia 2020]

Booleans [Muntoni et al., TOG 2018]

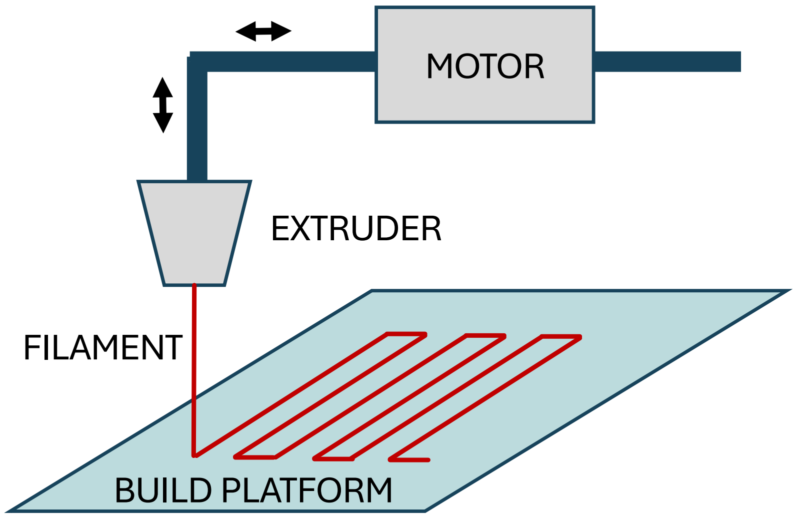


Outline

- Fabrication Technologies
- Modeling for fabrication
- Model orientation
- Slicing
- Internal Supports
- External supports
- Decomposition
- Toolpath generation
- Conclusions and outlook

Machine Toolpaths

VECTOR (e.g. FDM)



Operate on each slice like a ***plotter***
(i.e., connect points with lines/arcs)

RASTER (e.g. DLP)

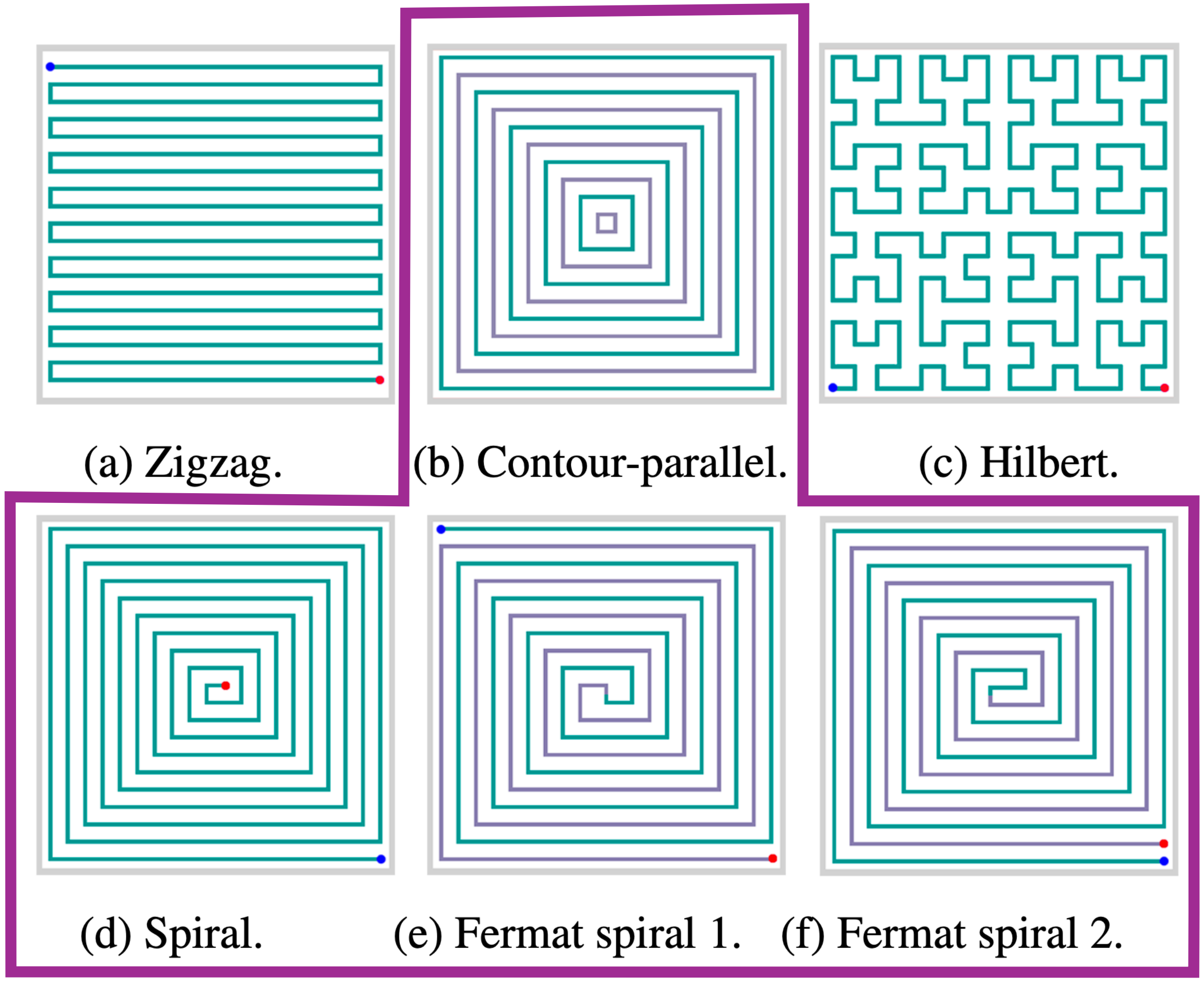
Operate on each slice like a ***printer***
(i.e., slices as a 2D images)

Path types

- Path ***continuity***
  - avoid starts/stops
  - ***link paths*** if possible

- Path ***geometry***
  - avoid abrupt direction changes (**homogeneous deposition**)
  - few, low curvature paths

- Can be derived from a ***distance field***



(a) Zigzag. (b) Contour-parallel. (c) Hilbert.
(d) Spiral. (e) Fermat spiral 1. (f) Fermat spiral 2.

What to Optimize For

- Usually multiple paths per slice are needed
- Optimize ***START***/***END***/***LINK*** placement
  - ***Efficiency*** => minimize ***airtime*** => relates to the ***Traveling Salesman Problem***

(NP-Complete!)

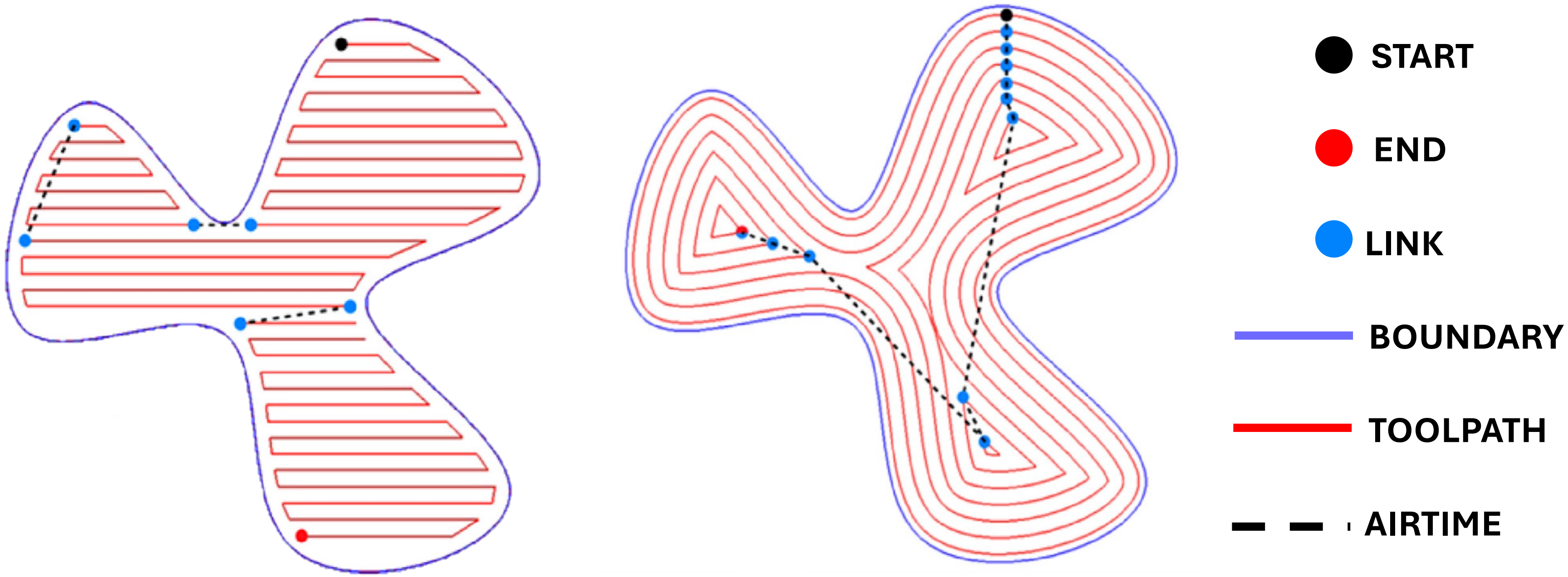
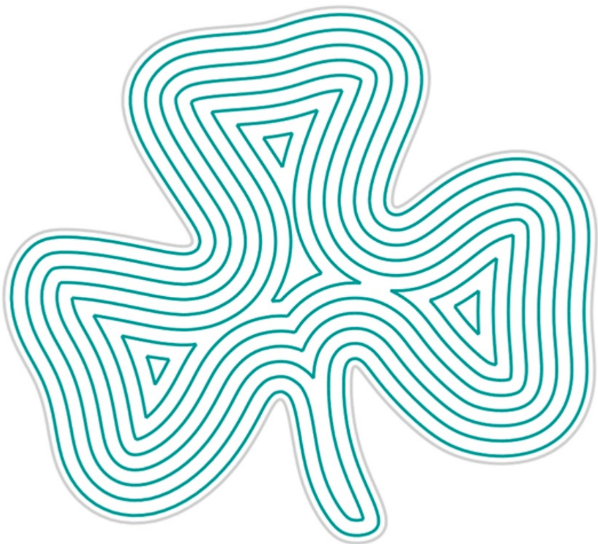


IMAGE COURTESY OF: (Optimization of toolpath generation for material extrusion-based additive manufacturing technology, AM, 2014)

Fermat Spiral [Zhao et al., SIGGRAPH 2016]

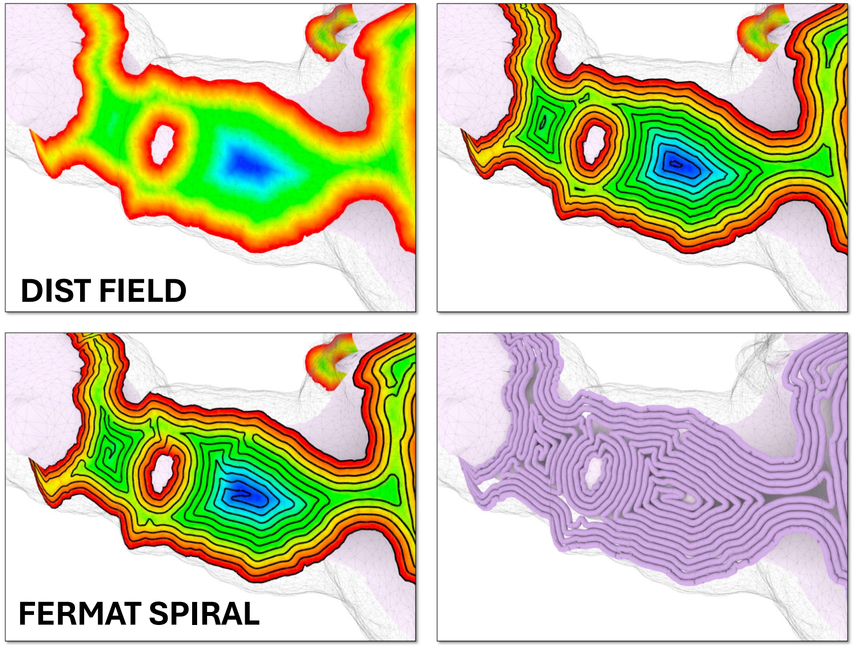
- Two nice properties:
  - Allow to control endpoint positioning **(useful for linking disjoint paths)**
  - Promote long and low-curvature paths ***(useful for homogeneous deposition)***



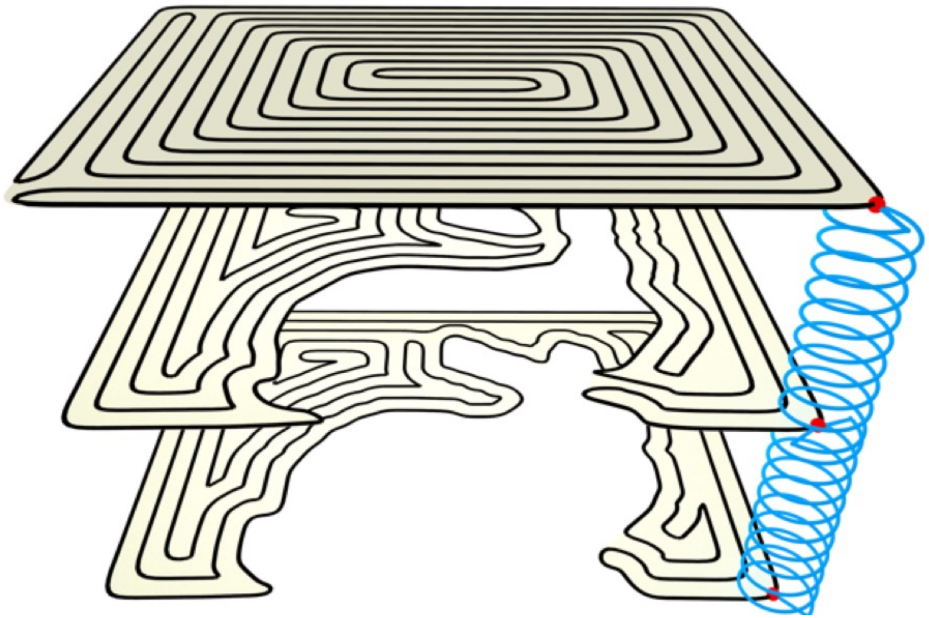
DISTANCE FIELD

Fermat Spiral [Zhao et al., SIGGRAPH 2016]

- Two nice properties:
  - Allow to control endpoint positioning **(useful for linking disjoint paths)**
  - Promote long and low-curvature paths ***(useful for homogeneous deposition)***



LINKING PATHS ON THE SAME SLICE



LINKING PATHS ON ADJACENT SLICES

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Ongoing research

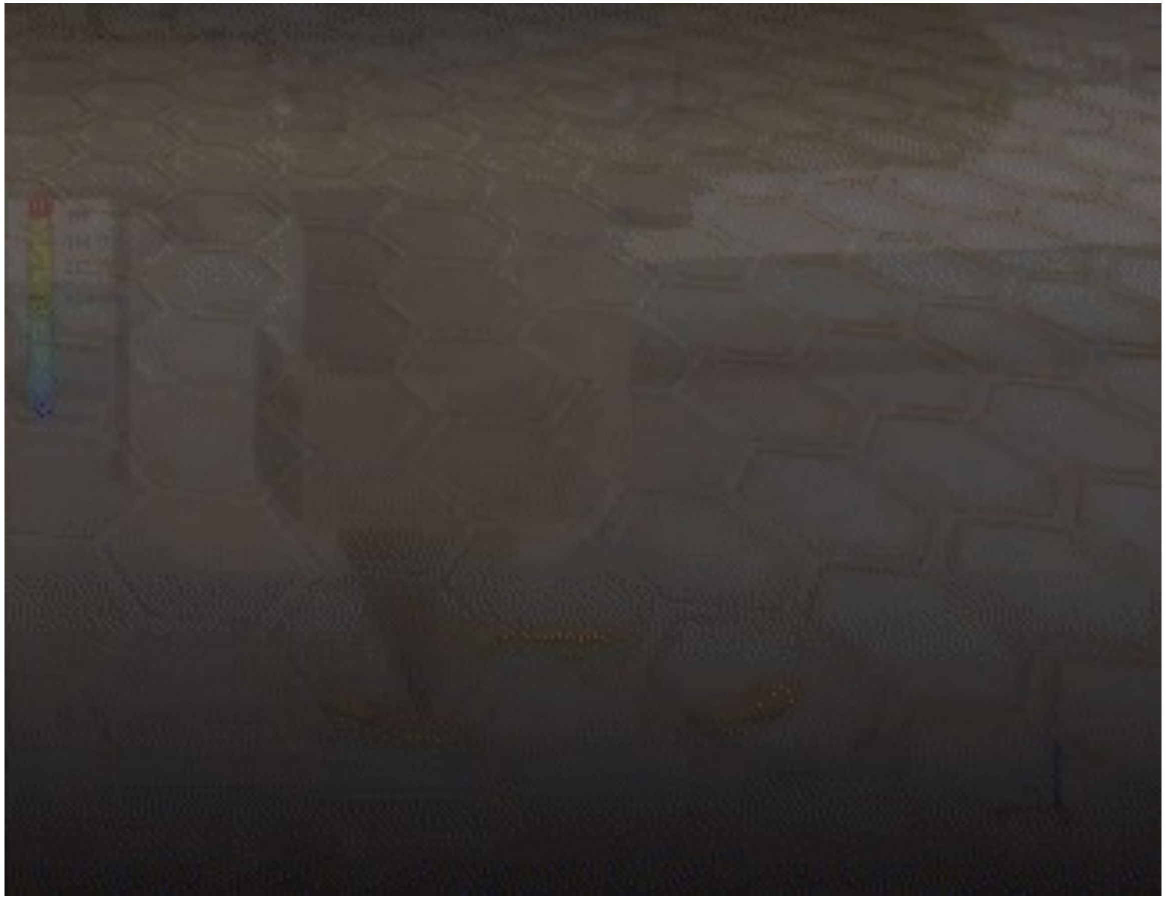
5 axis 3D printing

- Issues
  - Occlusions
  - Path Planning
  - Costs

Ongoing research

Process simulation

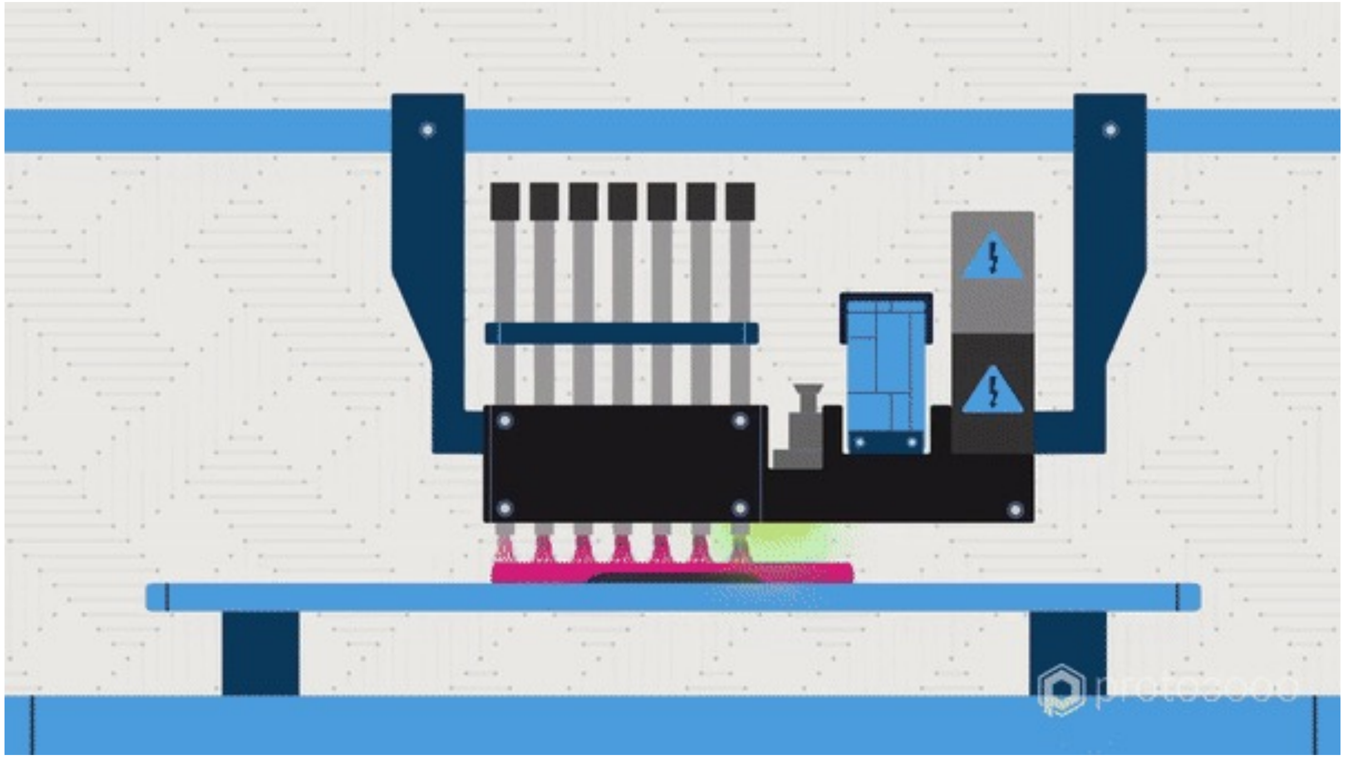
- Predict
  - Temperature gradient
  - Distortions
  - Residual stress
  - Breaks



Ongoing research

Multi-material Modeling

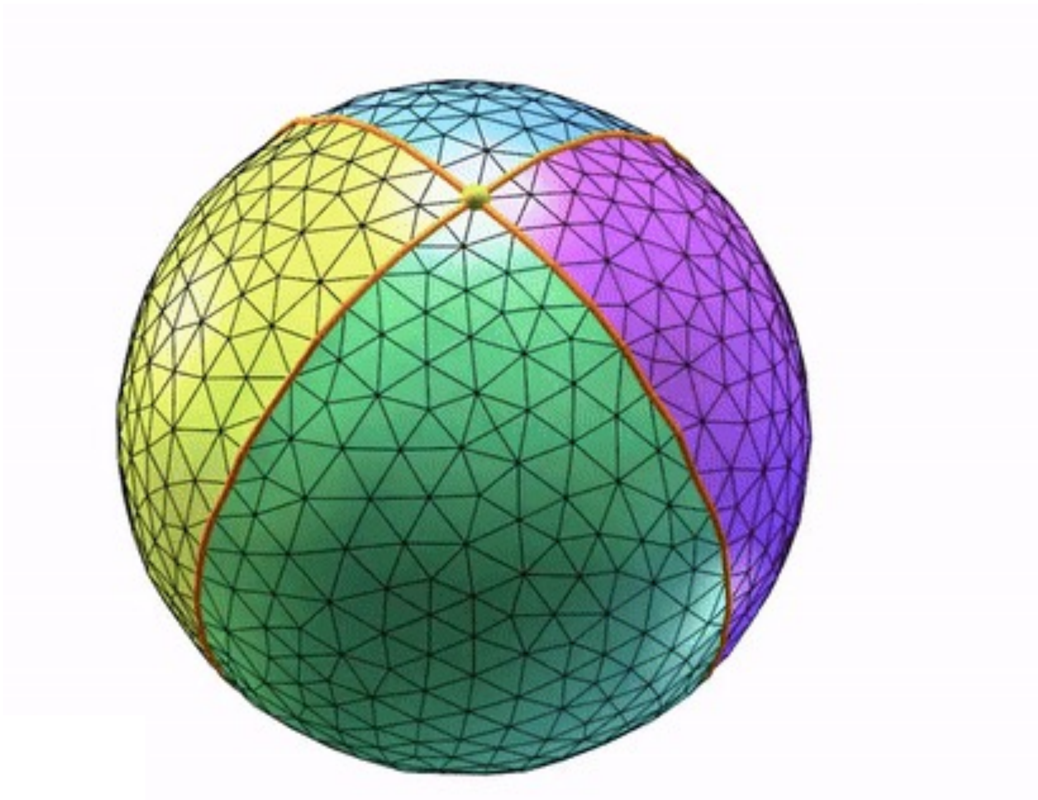
- Issues
  - Lack of intuitive tools
  - «body»-based modeling
  - Graded materials
  - Material vs microstructure
  - ...



Ongoing research

Multi-material Meshing

- Issues
  - Meshing is hard!
  - Interfaces must be properly handled
  - Graded materials?
  - Adaptivity?



Concluding remarks

Digital fabrication is cool! But…

- 3D modeling is not as accessible as 2D drawing
- Fab technology to be considered
- Physics to be considered
- From model to physical prototype -> Process planning
- Still a lot of room for research

Question time

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Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA

RAISE
Robotics and AI for socio-economic empowerement
Ecosistema dell'Innovazione della Liguria
PNRR – M4C2 – I1.5

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