Topology Optimization for Computational Fabrication

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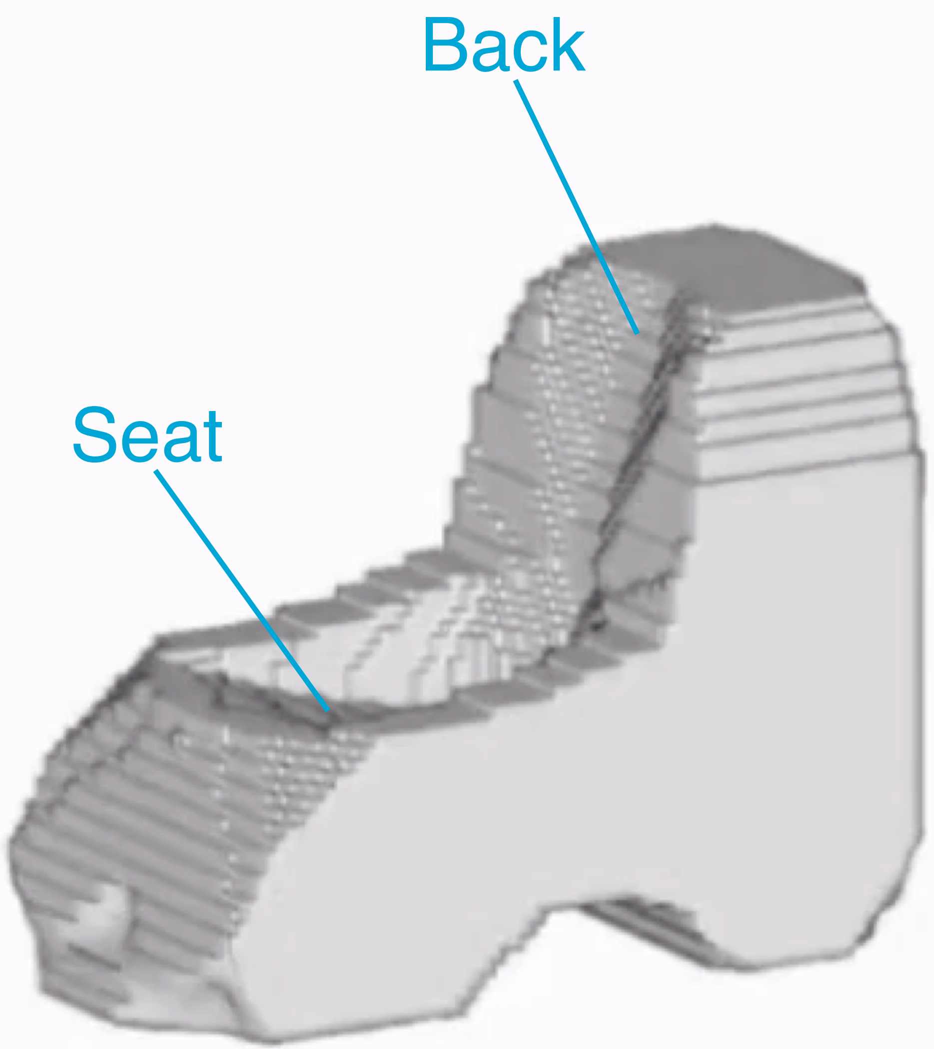
Bone Chair by Joris Laarman



Optimization of Bone Chair
by Lothar Harzheim & Opel GmbH

www.jorislaarman.com

More Examples

- Lightweight design -> material cost, operational cost, environmental impact
- Byproduct: Organic shapes!

Frustum Inc.

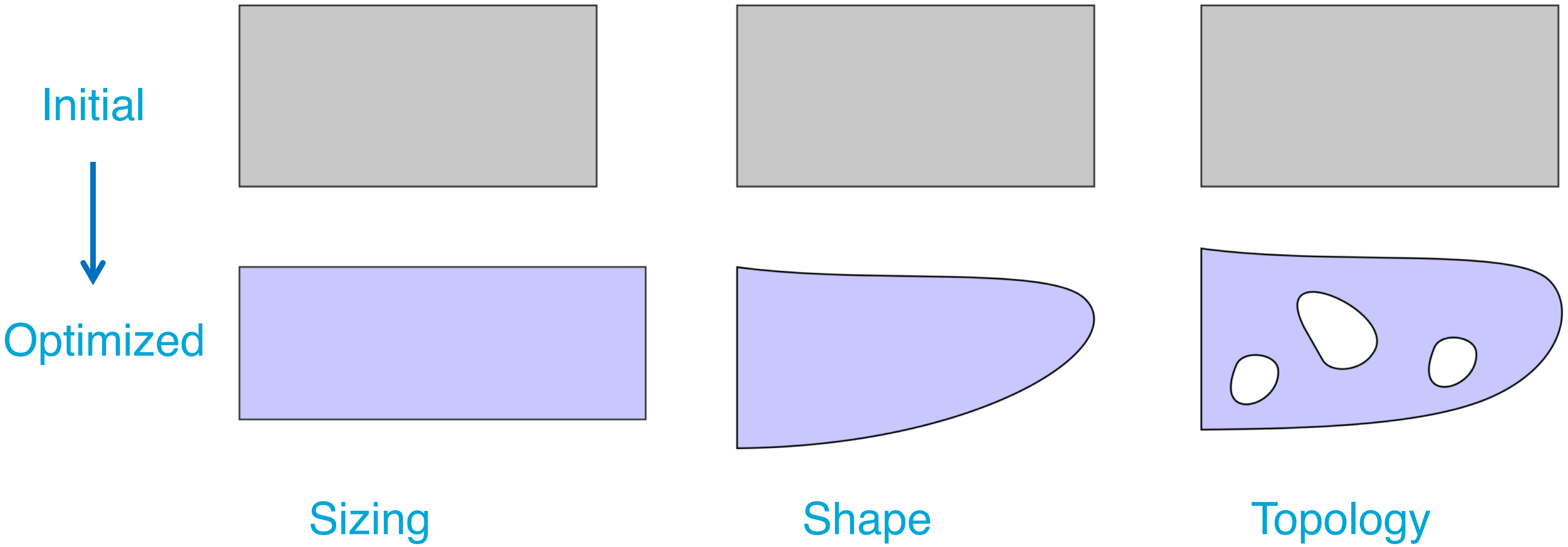
Airbus APWorks, 2016

Qatar national convention

Schedule

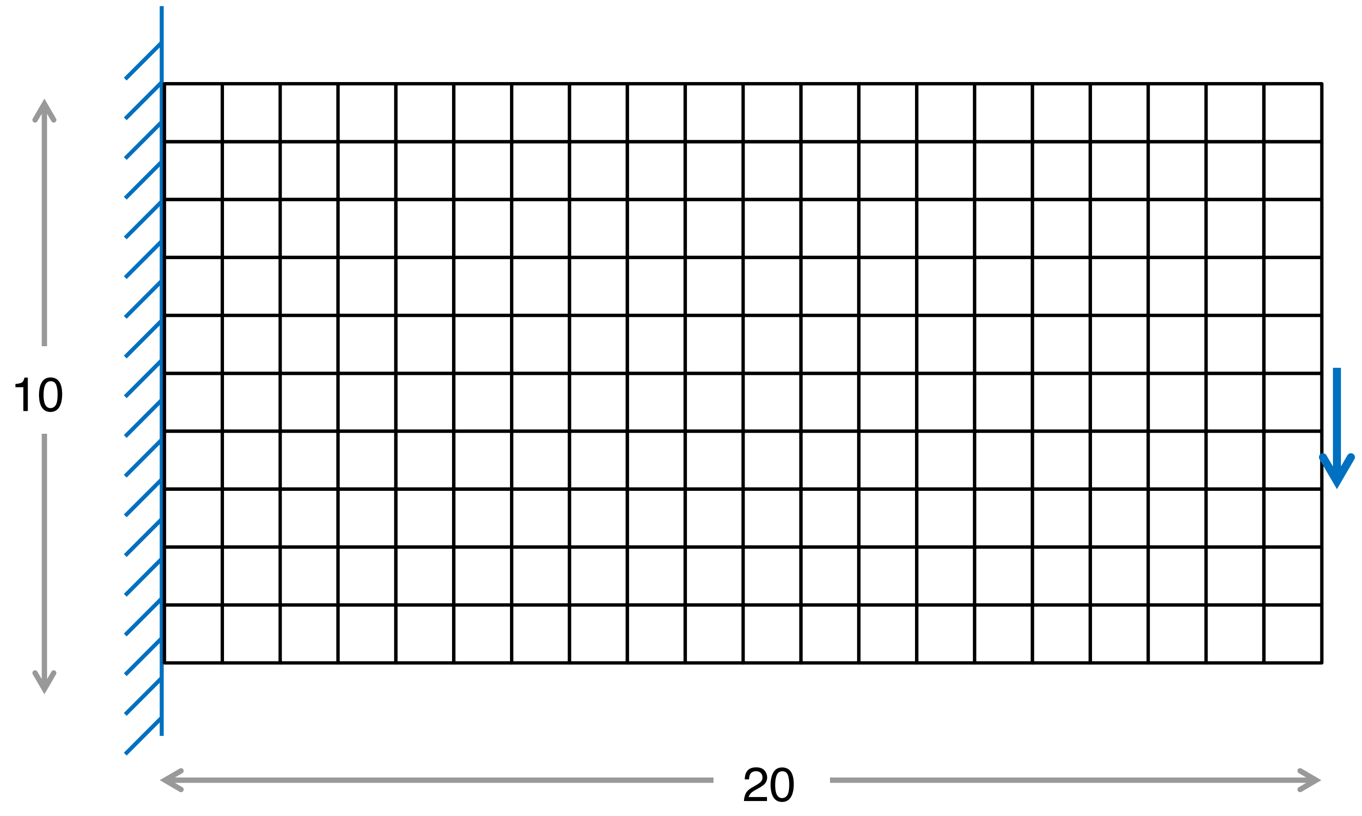
- Basics of Topology Optimization
- Topology Optimization for Additive Manufacturing

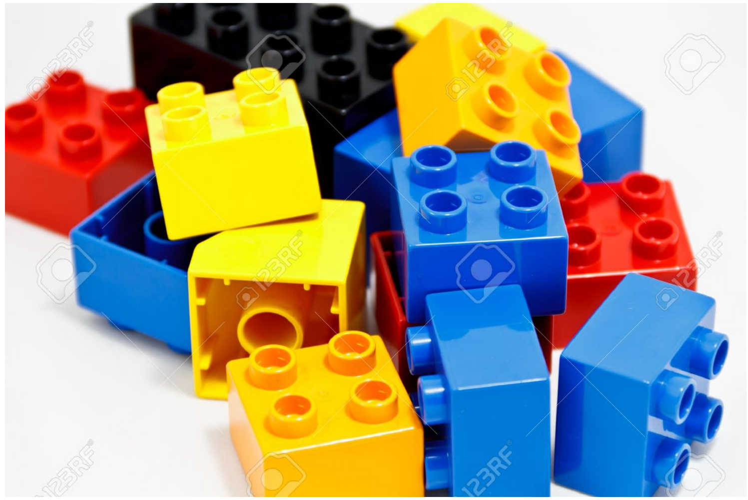
Classes of Structural Optimization: Sizing, Shape, Topology



A Toy Problem

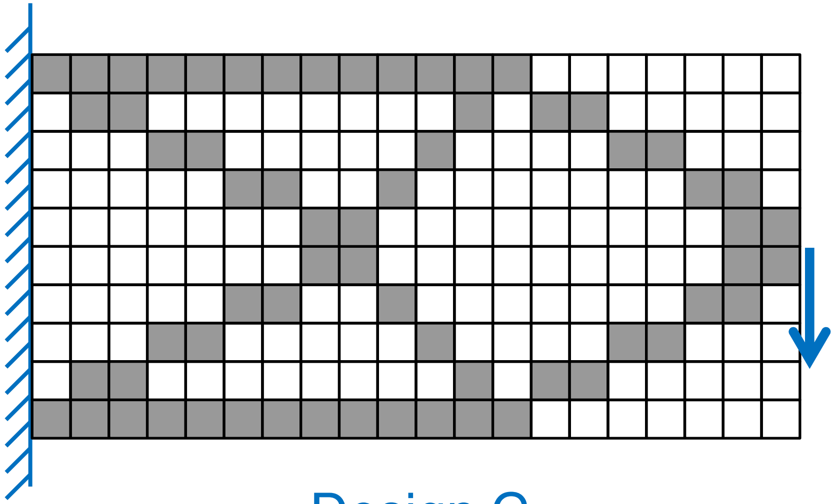
- Design the **stiffest** shape, by placing $\mathbf{60}$ Lego blocks into a grid of $\mathbf{20 \times 10}$



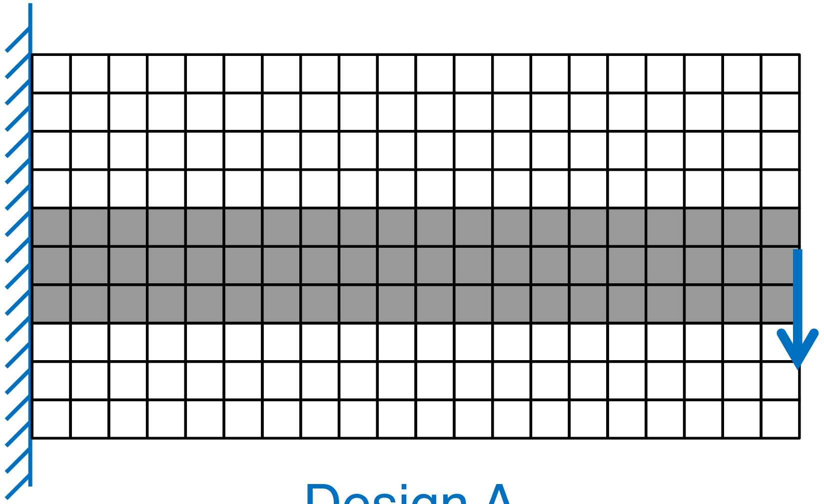


A Toy Problem: Possible Solutions

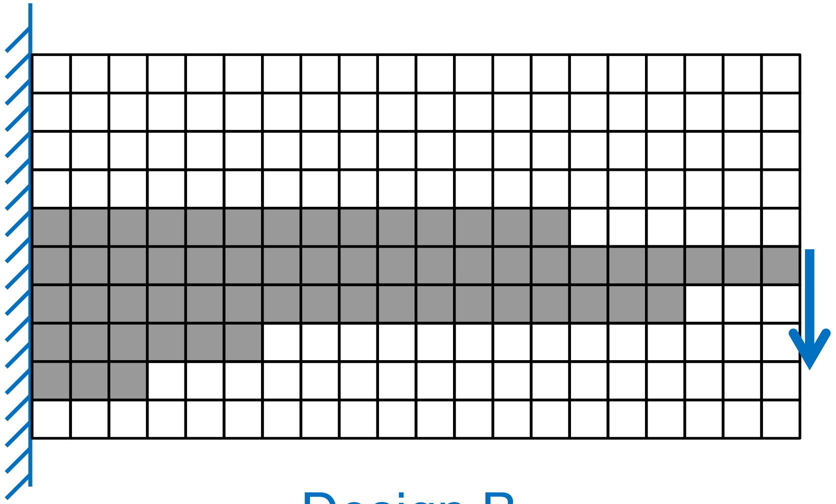
- Number of possible designs
  - $C(200{,}60) = \frac{200!}{60!(200-60)!} = 7.04\times10^{51}$
- Which one is the stiffest?



Design C



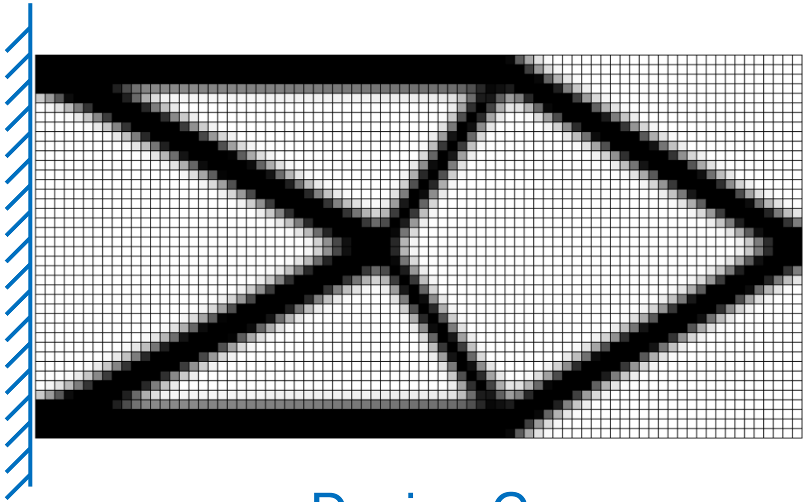
Design A



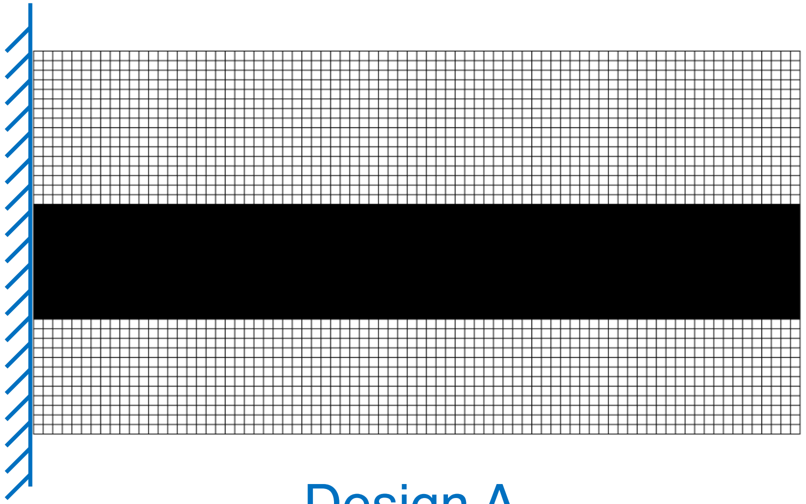
Design B

A Toy Problem: Possible Solutions

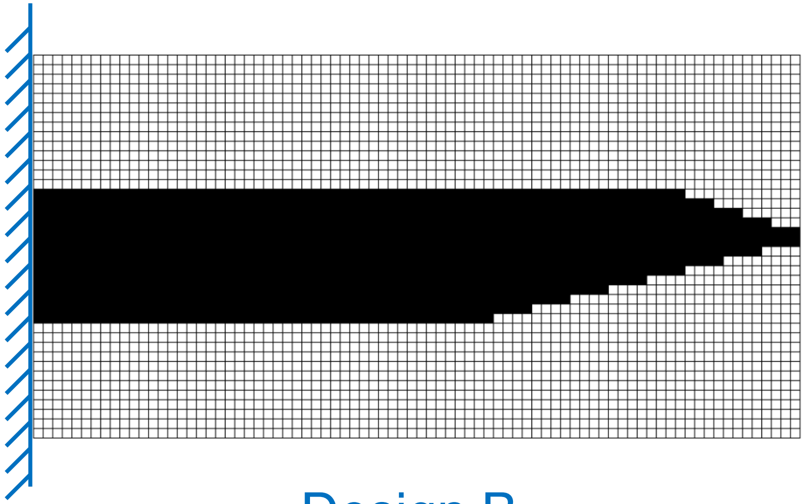
- Which one is the stiffest?



Design C



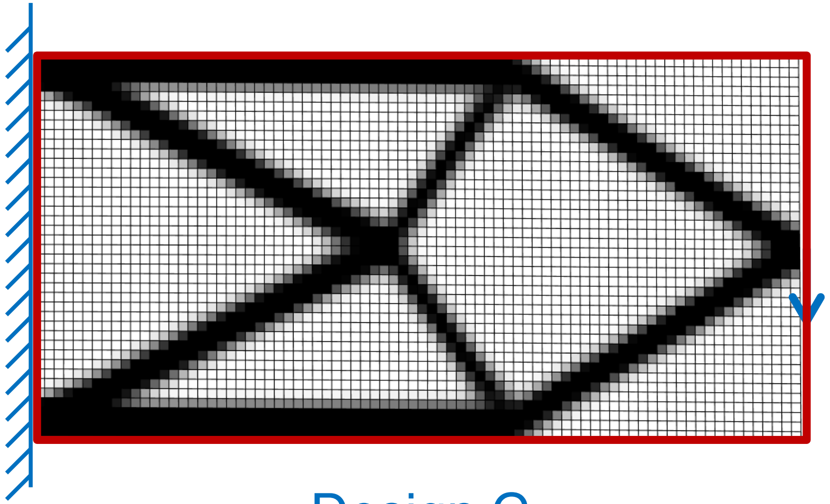
Design A



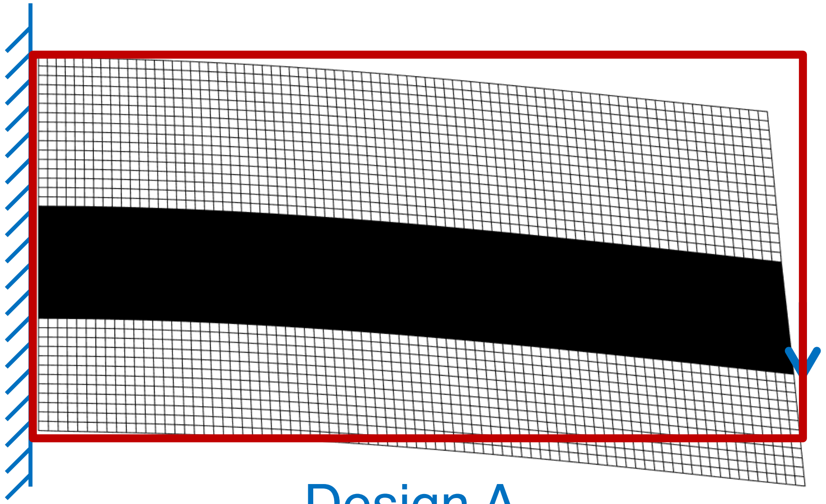
Design B

A Toy Problem: Possible Solutions

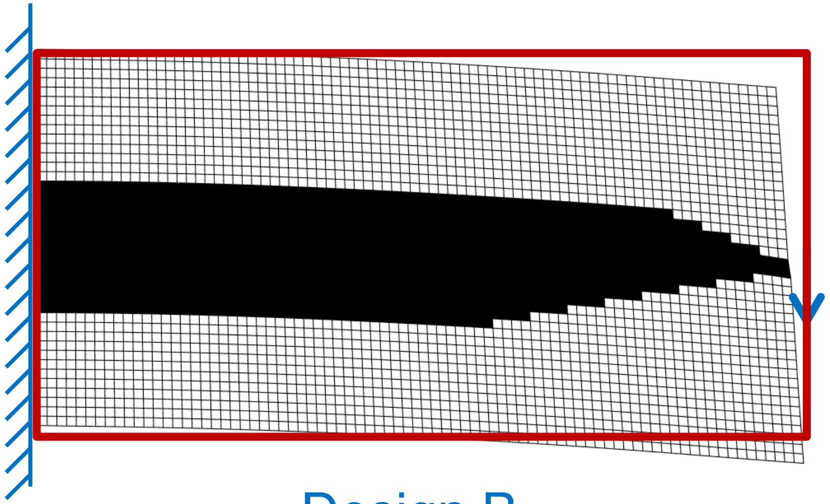
- Which one is the stiffest?



Design C

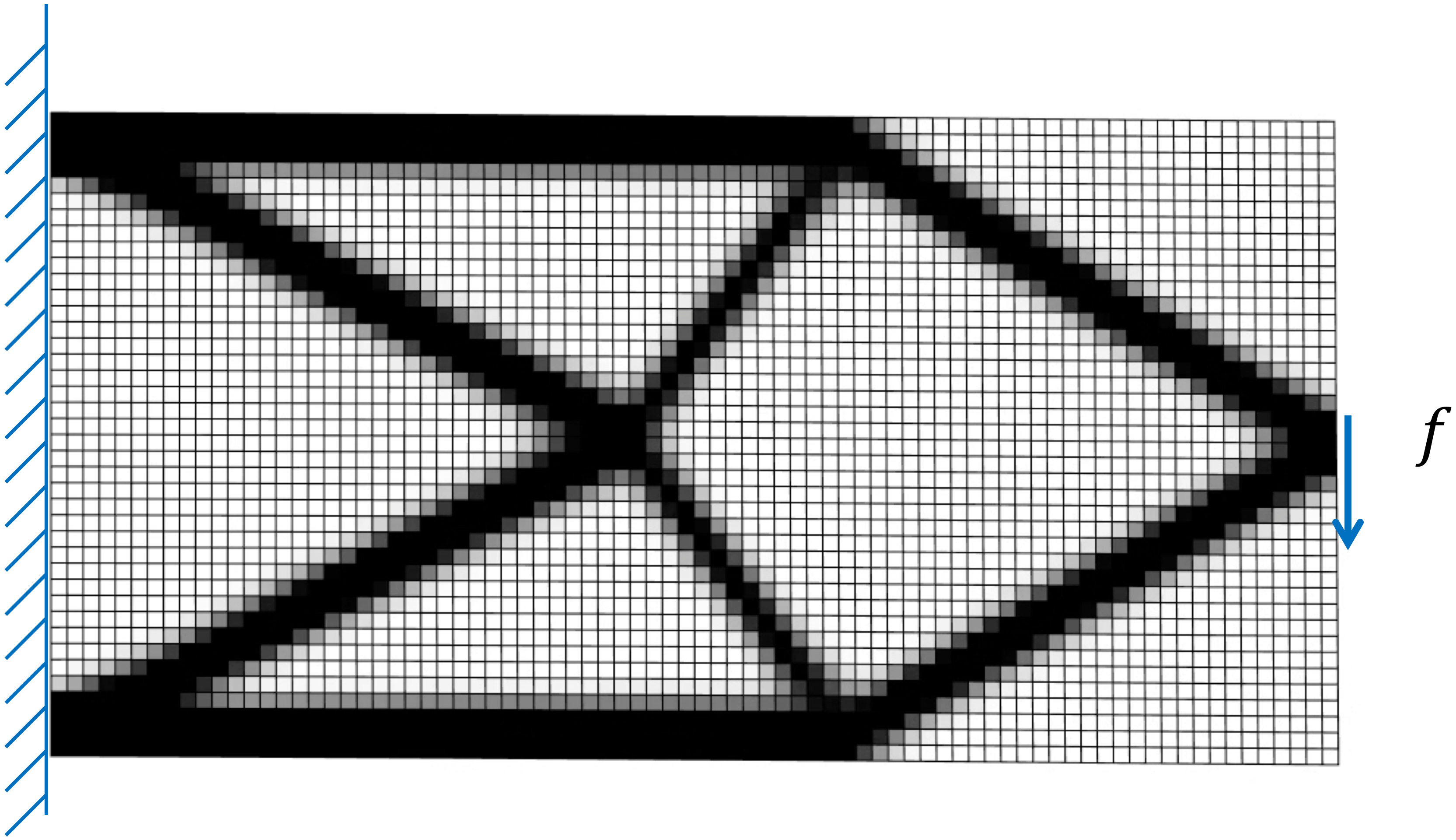


Design A



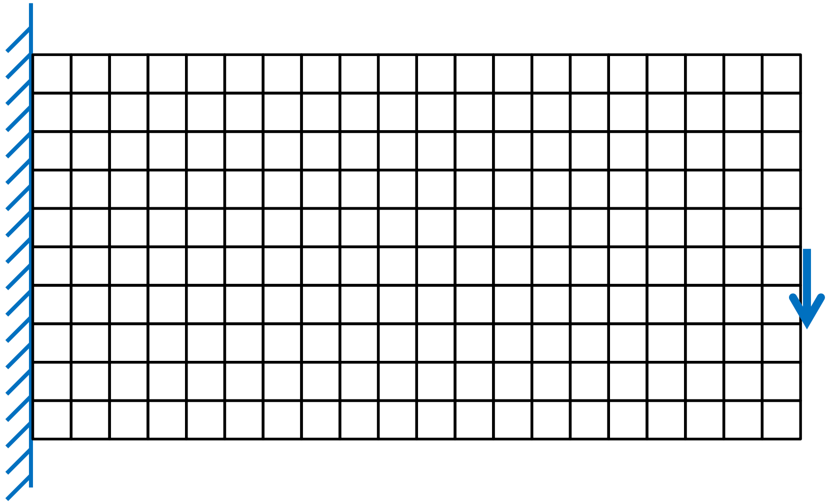
Design B

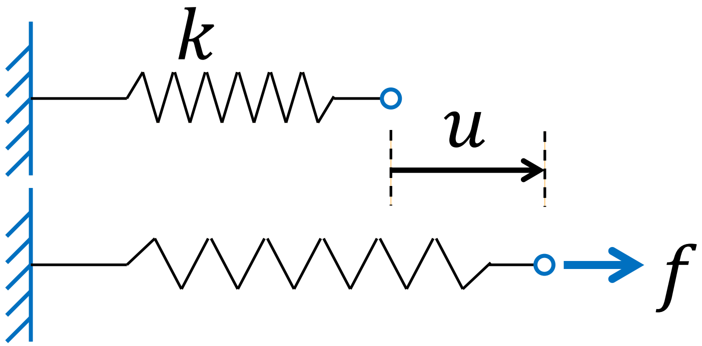
Topology Optimization Animation



Topology Optimization

Minimize:	$\boxed{c = \frac{1}{2} U^T K U}$	$\longleftarrow$	Elastic energy	$c = \frac{1}{2} f u = \frac{1}{2} k u^2$
Subject to:	$KU = F$	$\longleftarrow$	Static equation	$ku = f$





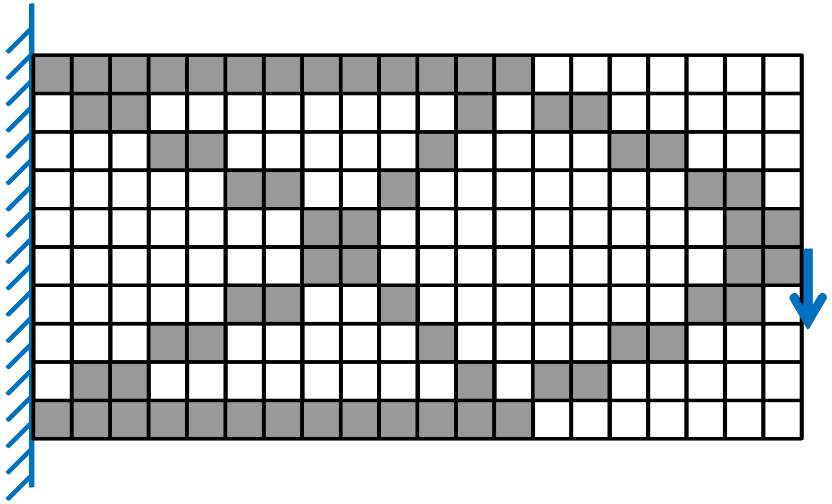
Topology Optimization

Minimize: $c = \frac{1}{2} U^T K U$ — Elastic energy

Subject to: $KU = F$ — Static equation

$$\rho_i = \begin{cases} 1 & \text{(solid)} \\ 0 & \text{(void)} \end{cases}, \forall i$$ Design variables

$$g = \sum_i \rho_i - V_0 \le 0$$ Volume constraint



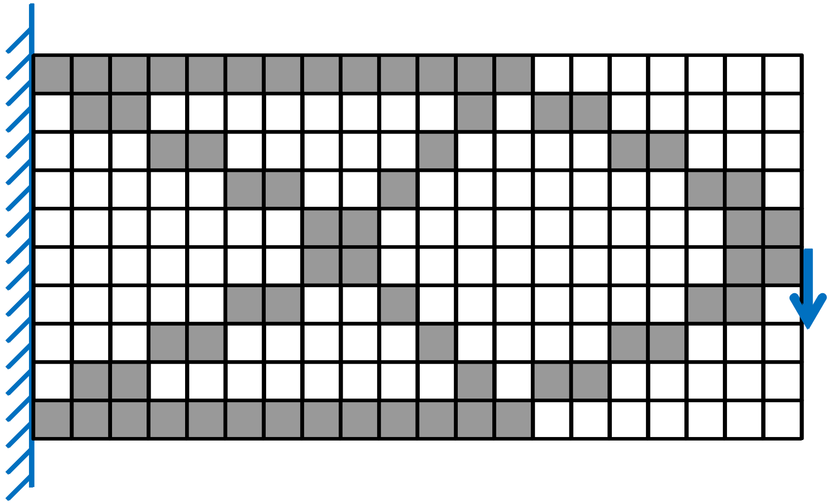
Topology Optimization

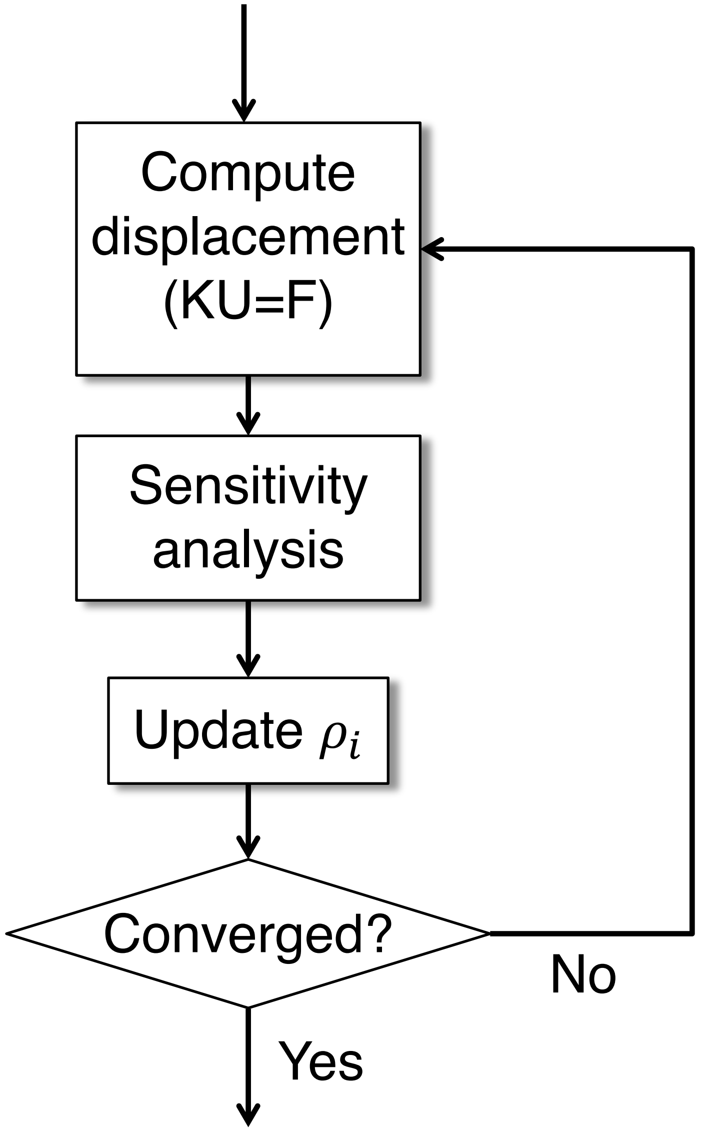
Minimize: $c = \frac{1}{2} U^T K U$

Subject to: $KU = F$

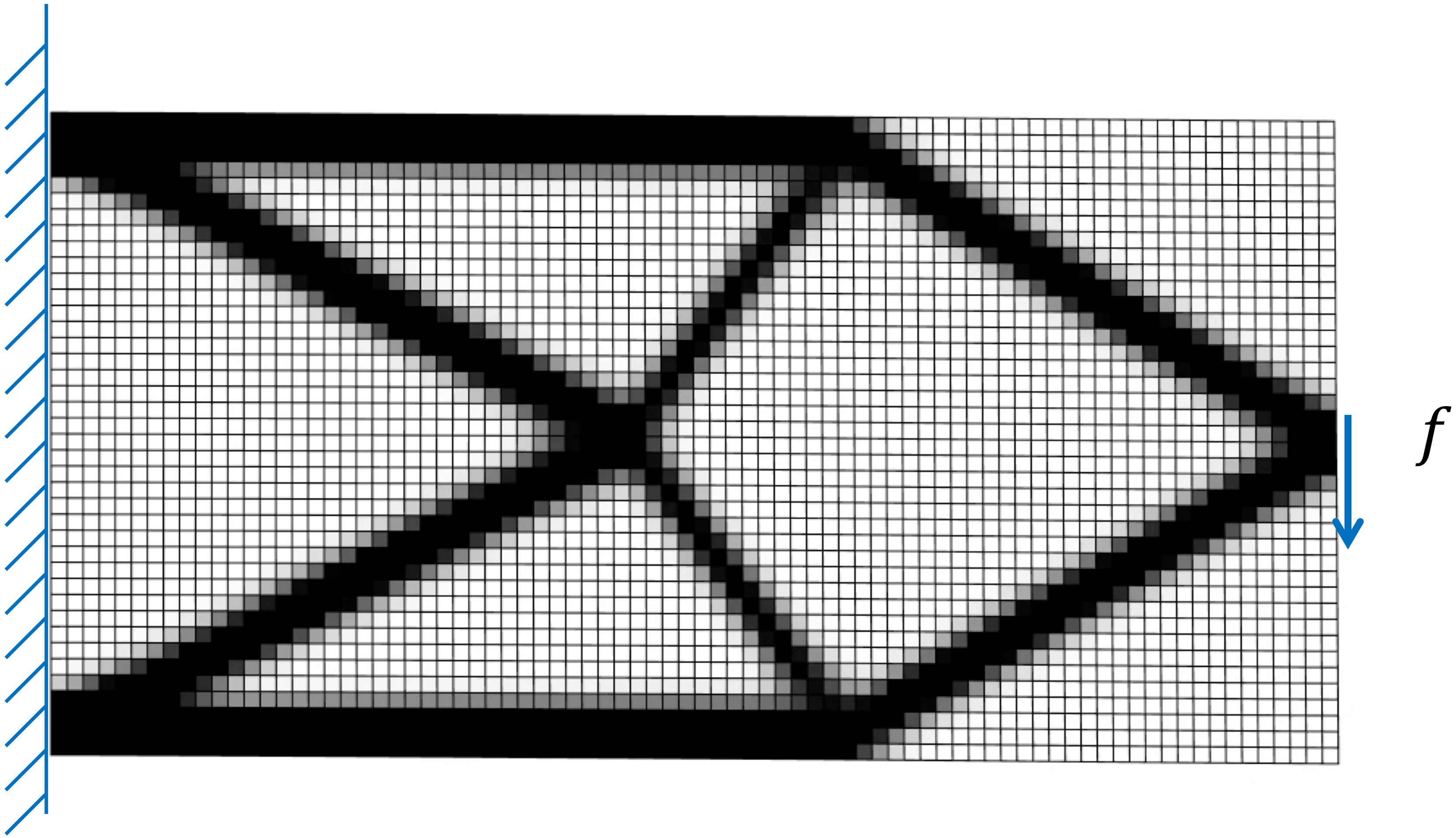
$$\rho_i = \begin{cases} 1 & \text{(solid)} \\ 0 & \text{(void)} \end{cases}, \forall i$$

$$g = \sum_i \rho_i - V_0 \leq 0$$





Topology Optimization Animation



Relaxation: Discrete to Continuous

Minimize: $c = \frac{1}{2} U^T K U$

Subject to: $KU = F$

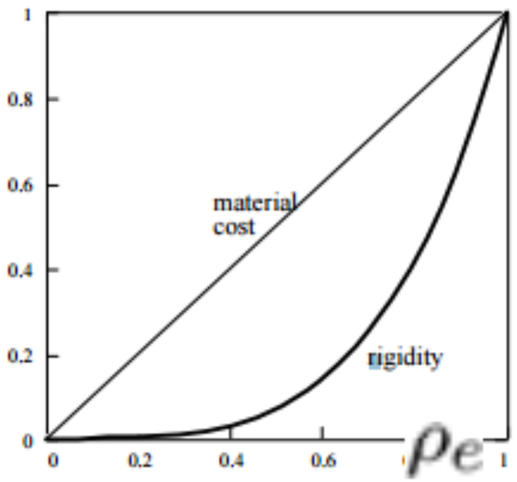
$$\boxed{\rho_i = \begin{cases} 1 & \text{(solid)} \\ 0 & \text{(void)} \end{cases}, \forall i} \quad \Rightarrow \quad \rho_i \in [0\,,1]$$

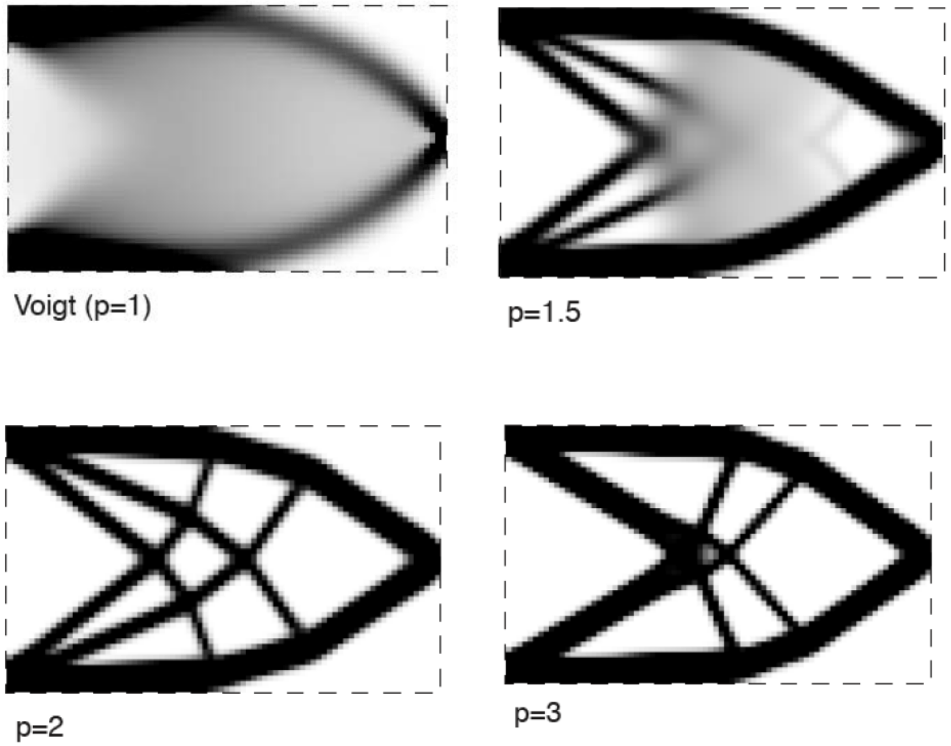
$$\mathrm{g} = \textstyle\sum_i \rho_i - V_0 \leq 0$$

- (Difficult) binary problem → (easier) continuous problem, solved by gradient-based optimization

How to mechanically interpret the intermediate value [0, 1]?

- Linear interpolation of Young's modulus? i.e. $E(\rho) = \rho\bar{E}, \rho \in [0,1]$
  - Mechanically not possible (it doesn't comply with Hashin-Shtrikman bounds)
  - No convergence to black/white structure
- Solid isotropic material with penalization $E(\rho) = \rho^p\bar{E}, p \geq 1$
  - $p = 3$, satisfy Hashin-Shtrikman bounds
  - Promote black/white design





Sensitivity Analysis

- Sensitivity: The derivative of a function with respect to design variables

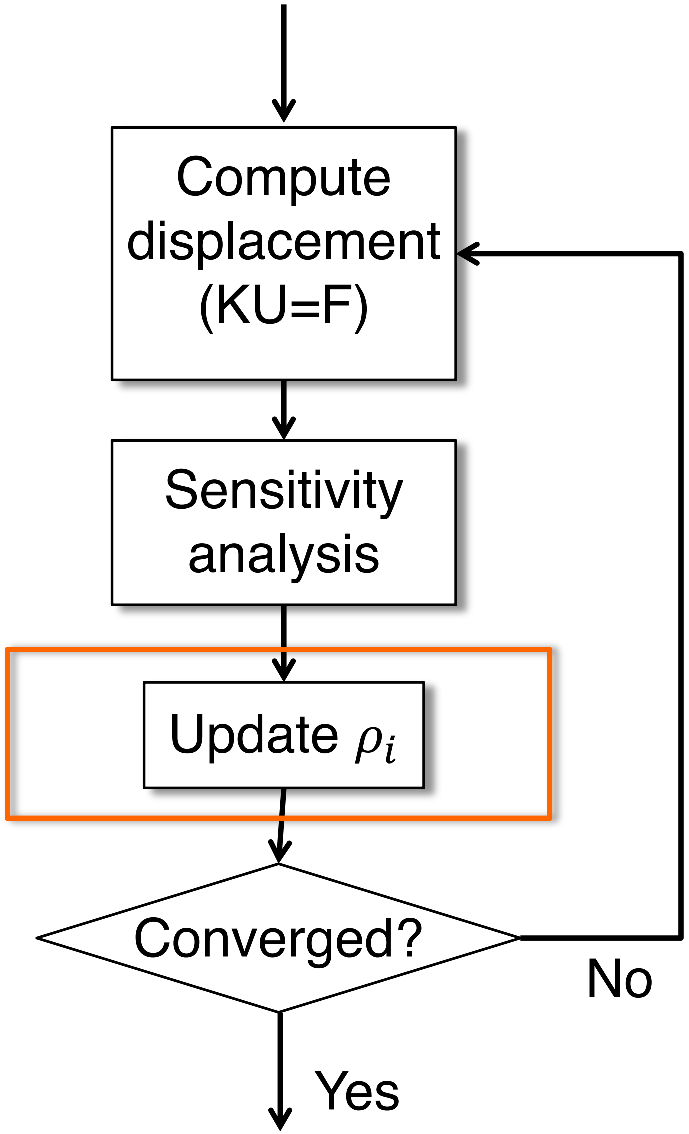
- $\frac{\partial c}{\partial \rho_i} = -\frac{p}{2}\rho_i^{p-1} u_i^T \overline{K} u_i$
  - Smaller than zero
- $\frac{\partial g}{\partial \rho_i} = 1$

Minimize:	$c = \frac{1}{2} U^T K U$
Subject to:	$KU = F$
	$\rho_i \in [0\,,1]$
	$\mathrm{g} = \sum_i \rho_i - V_0 \leq 0$

Design Update

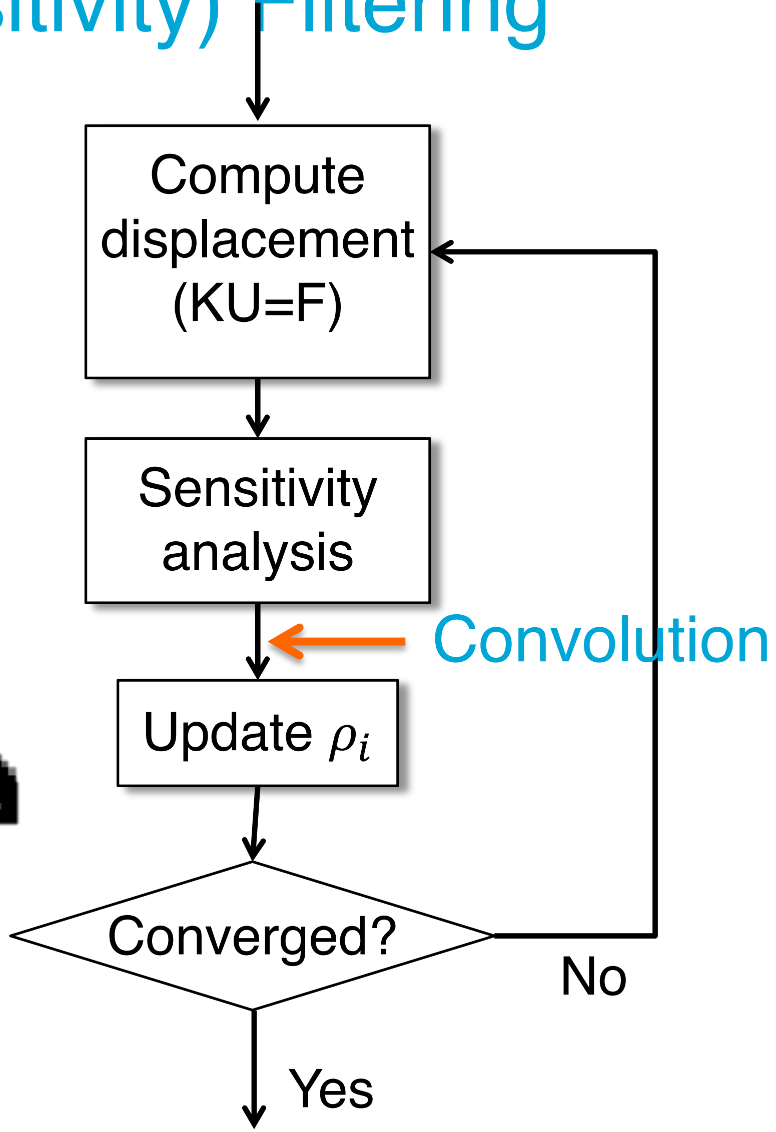
- Mathematical programming
  - Interior point method (IPOPT package)
  - The method of moving asymptotes (MMA)

- Optimality criterion
  - If "$-\frac{\partial c}{\partial \rho_i}$" is large, increase ρ_i
  - Otherwise, decrease ρ_i
  - How to determine large or small?
  - Bisection search for a threshold



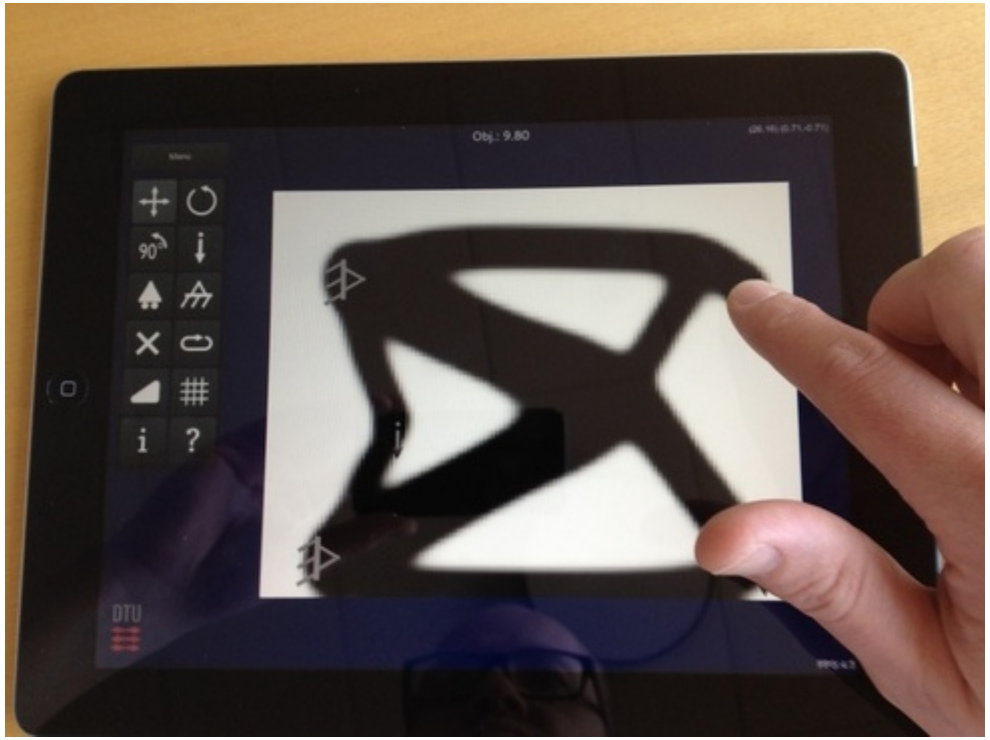
Checkerboard Patterns and (Sensitivity) Filtering

- $\frac{\widetilde{\partial c}}{\partial \rho} = H(\frac{\partial c}{\partial \rho})$ H-convolution operator
- Alternatively, $\tilde{\rho} = H(\rho)$



Demo - TopOpt

- Android, iOS
- www.topopt.dtu.dk



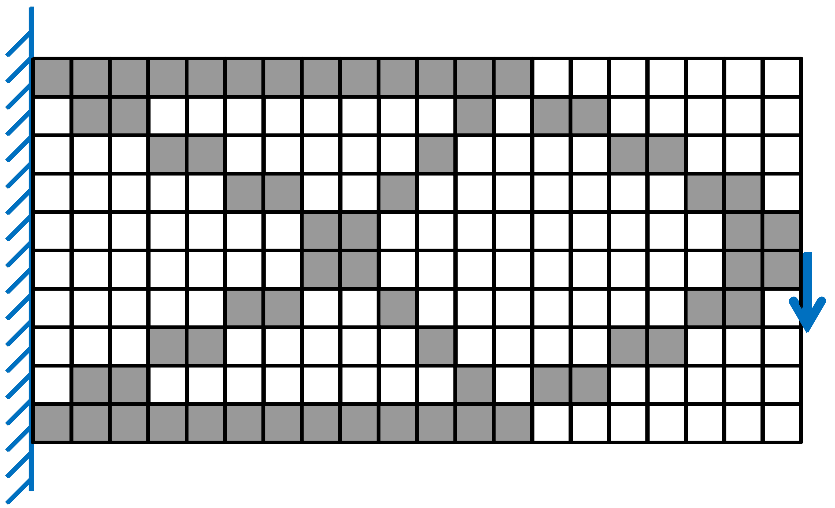
Topology Optimization

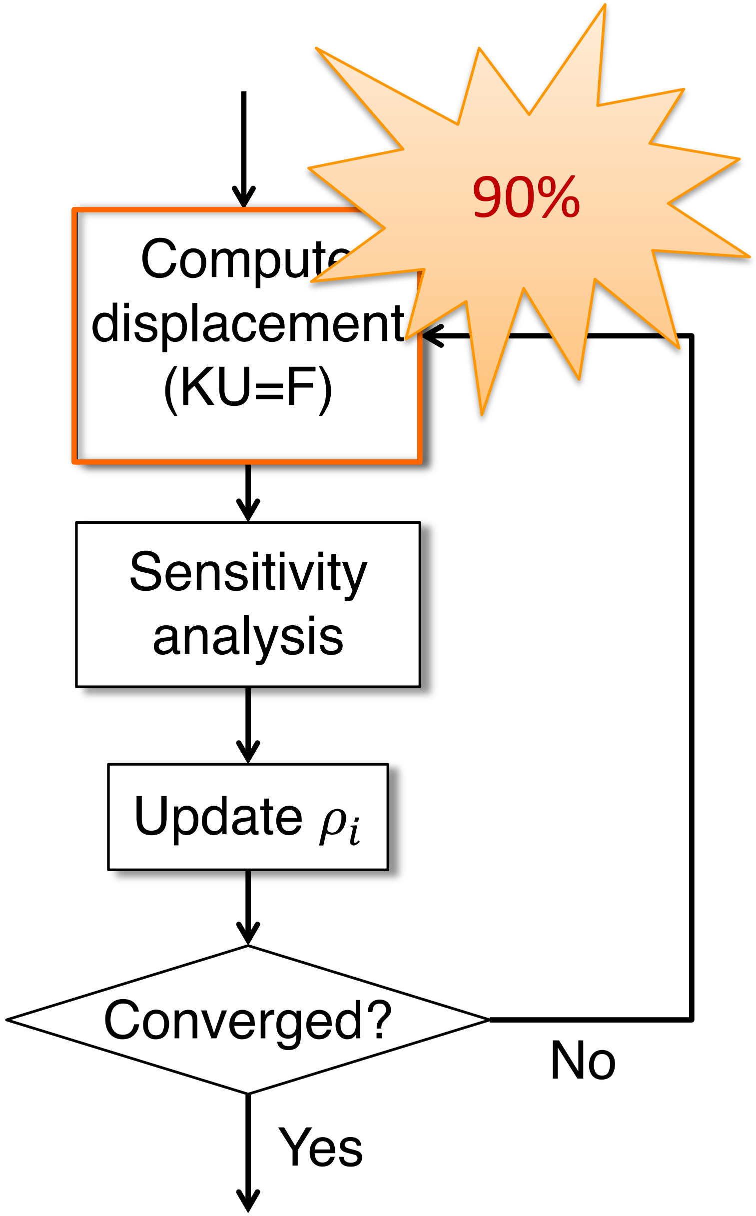
Minimize: $c = \frac{1}{2} U^T K U$

Subject to: $KU = F$

$\rho_i \in [0,1], \forall i$

$g = \sum_i \rho_i - V_0 \leq 0$

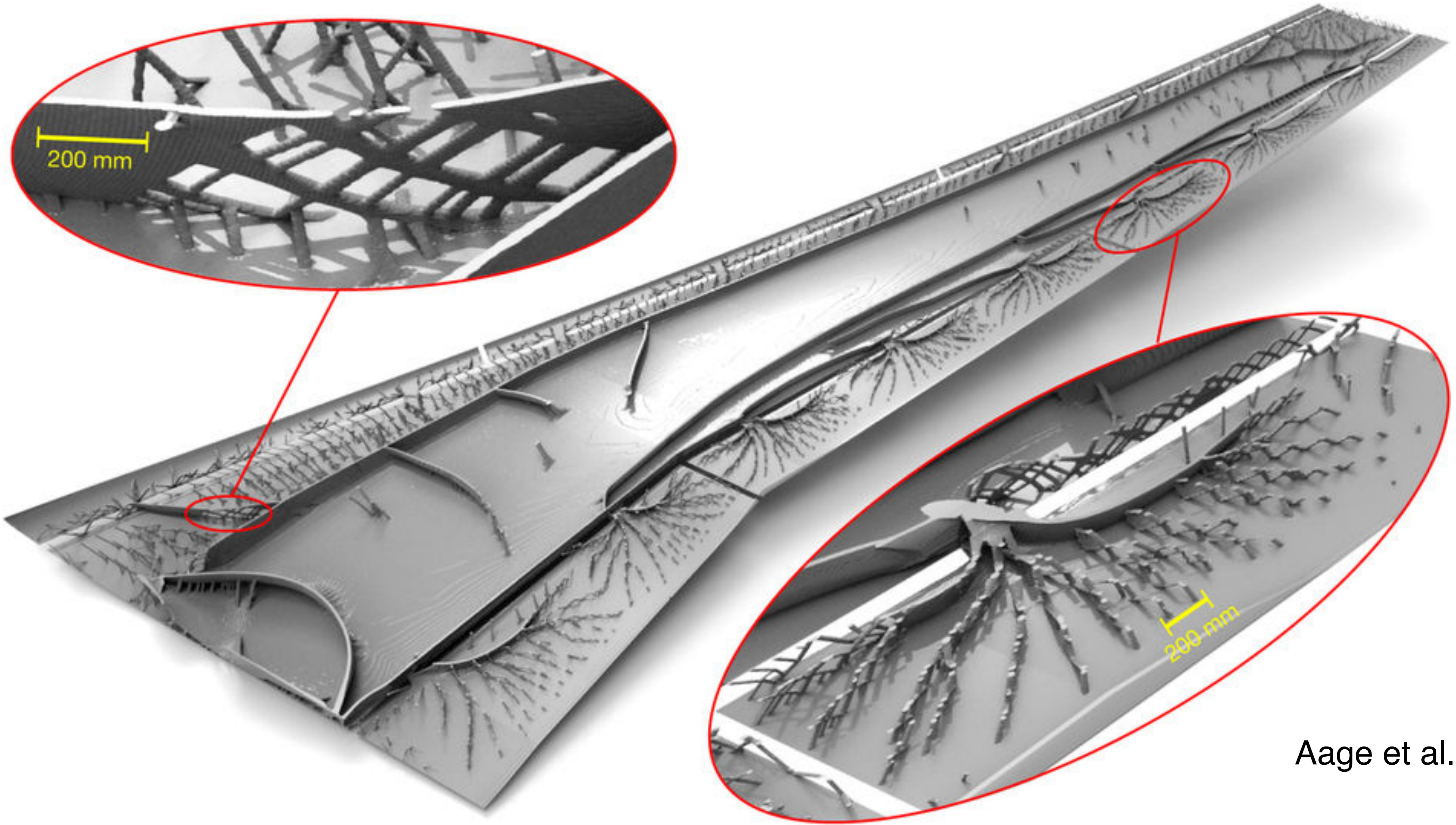




A System for High-Resolution Topology Optimization

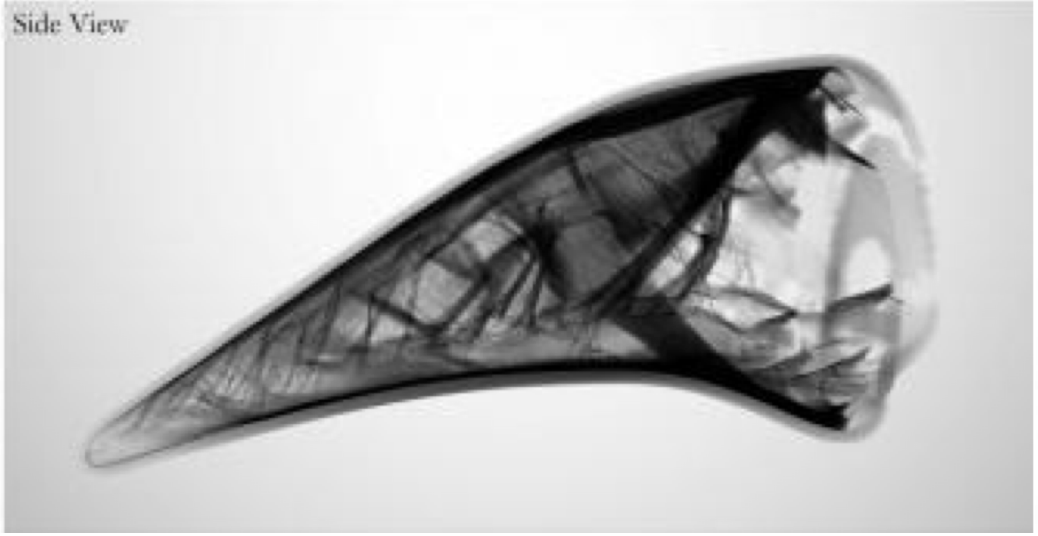
Jun Wu, Christian Dick, Rüdiger Westermann

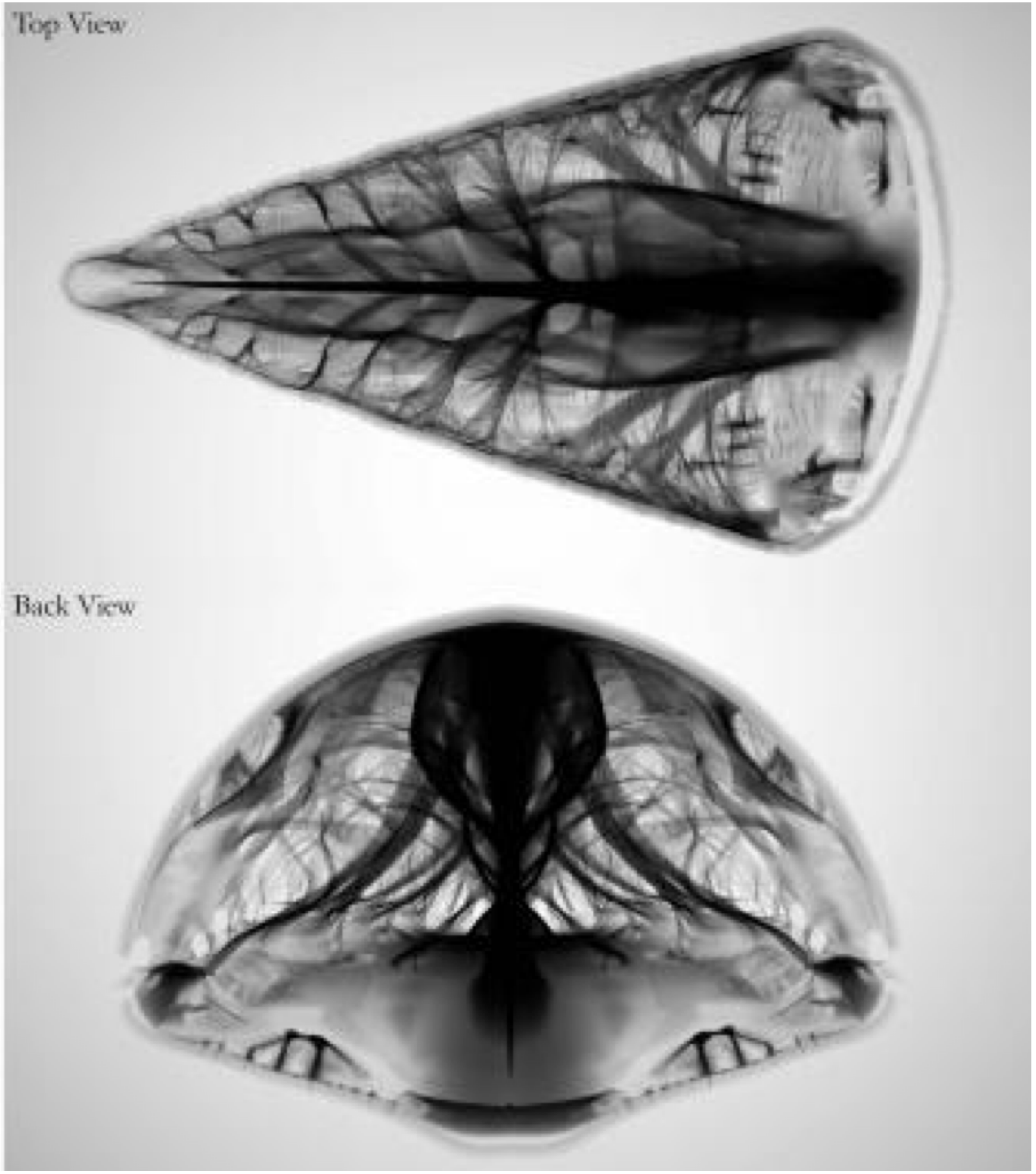
Giga-voxel full-scale wing design, Aage et al. 2017



Aage et al., Nature 2017

- Narrow-Band Topology Optimization on a Sparsely Populated Grid
- Liu et al. 2018

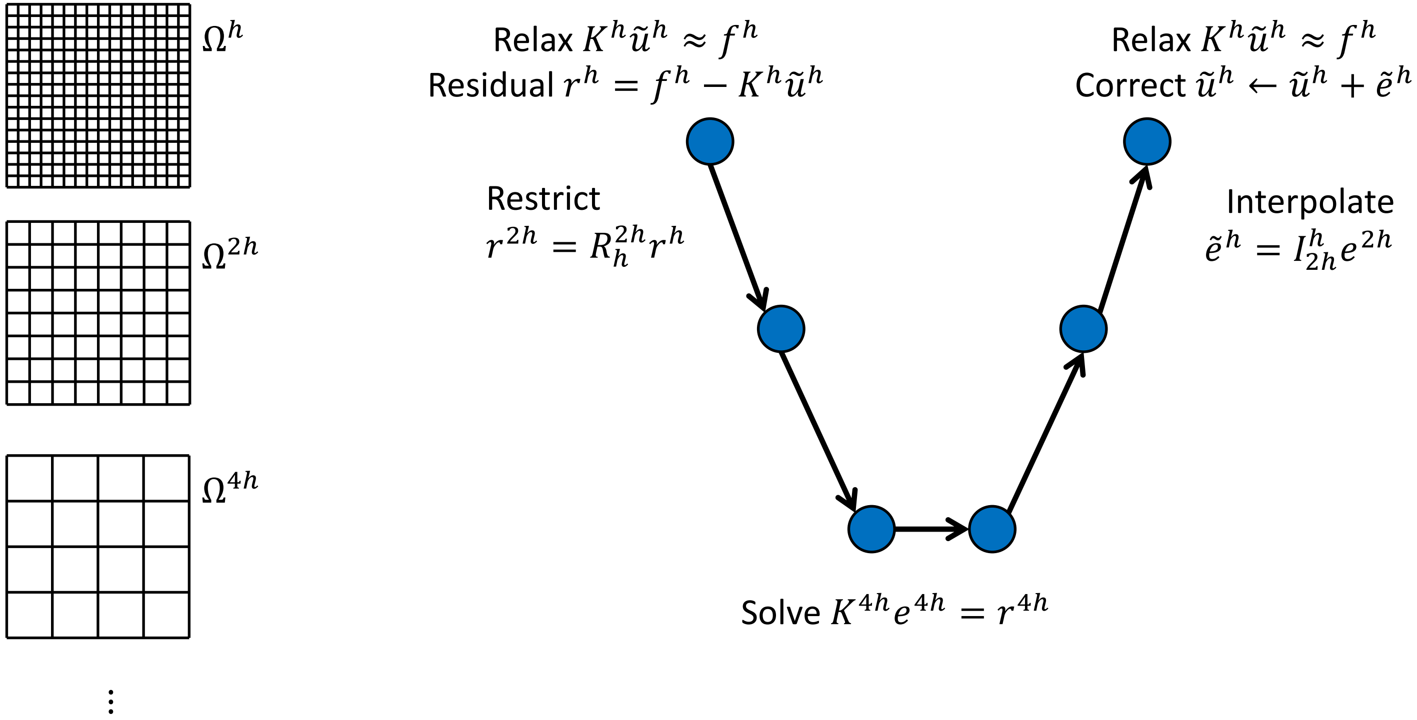




Liu et al., SA 2018

Geometric Multigrid: Solving $Ku = f$

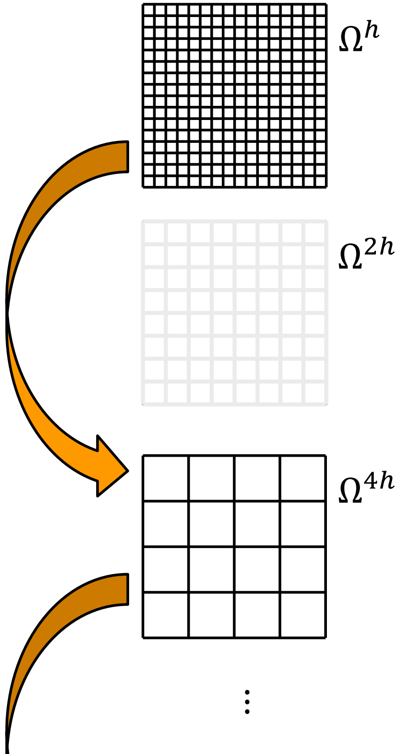
- Successively compute approximations u_m to the solution $\mathrm{u} = \lim_{m \to \infty} u_m$
- Consider the problem on a hierarchy of successively coarser grids to accelerate convergence

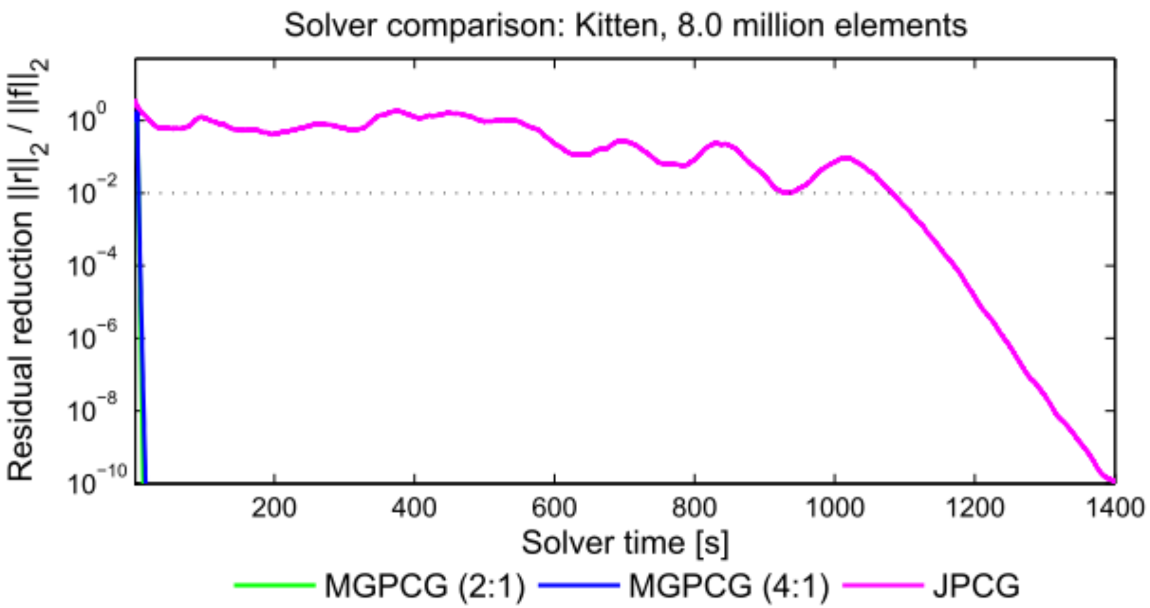


W. Briggs, A multigrid tutorial, 2000

Memory-Efficient Implementation on GPU

- On-the-fly assembly
  - Avoid storing matrices on the finest level
- Non-dyadic coarsening (i.e., 4:1 as opposed to 2:1)
  - Avoid storing matrices on the second finest level

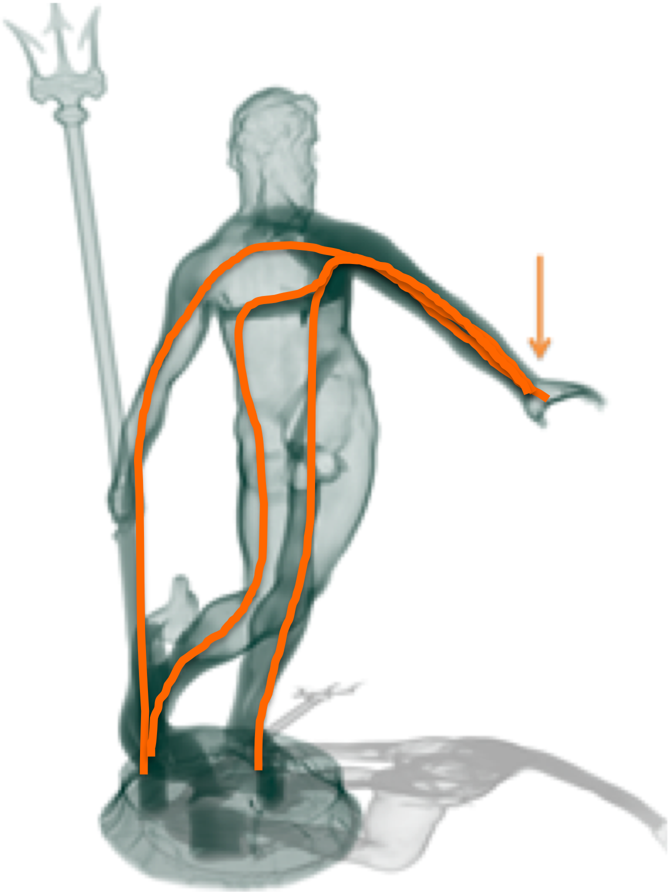


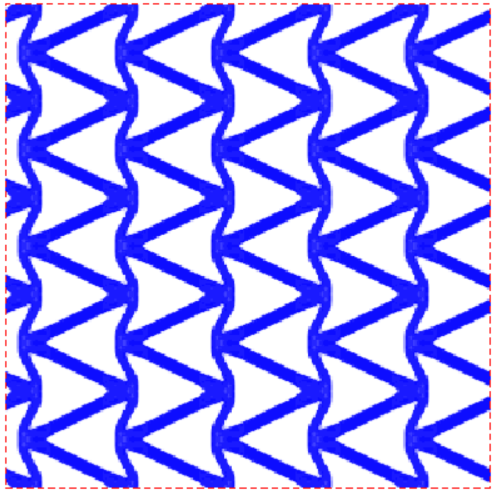


Wu et al., TVCG'2016
Dick et al., SMPT'2011

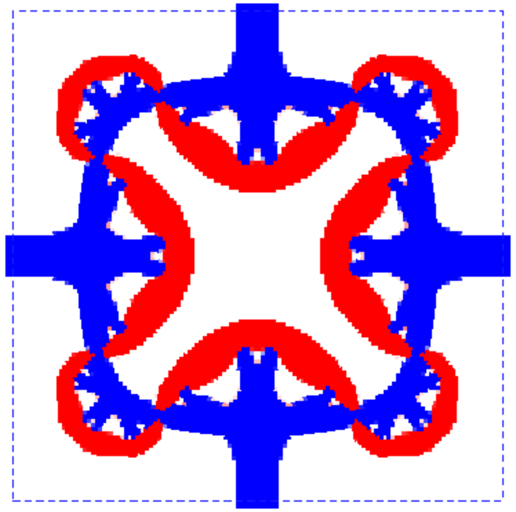
High-Resolution Design

Resolution: 621 × 400 × 1000
#Element 14.2m
Time: 12 minutes

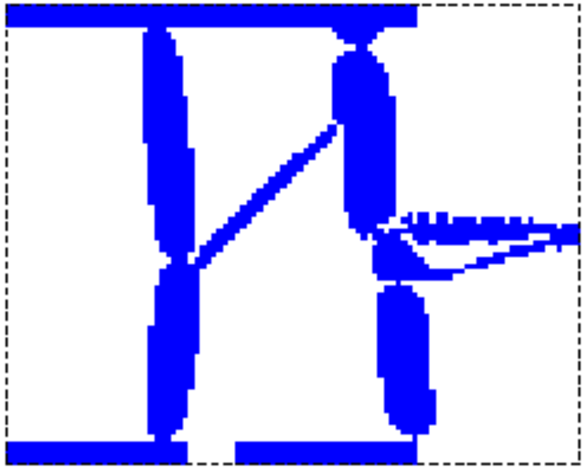




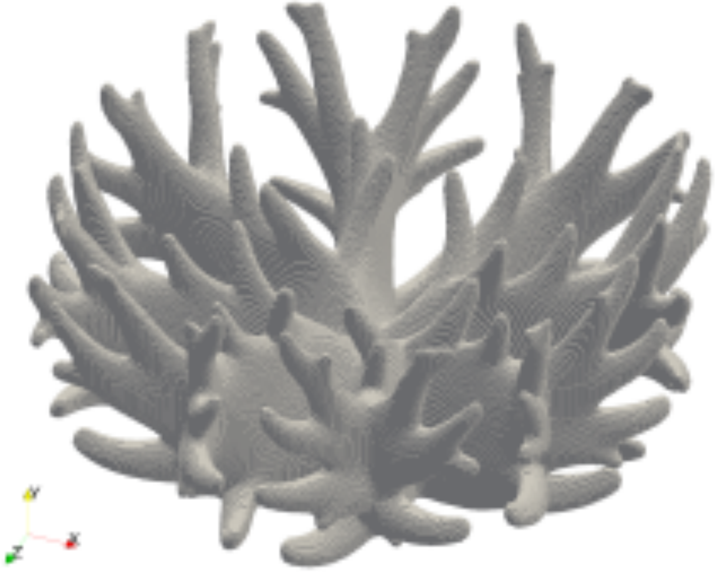
Negative Poisson's ratio
Larsen et al. 1997



Negative thermal expansion
Sigmund &Torquato 1996

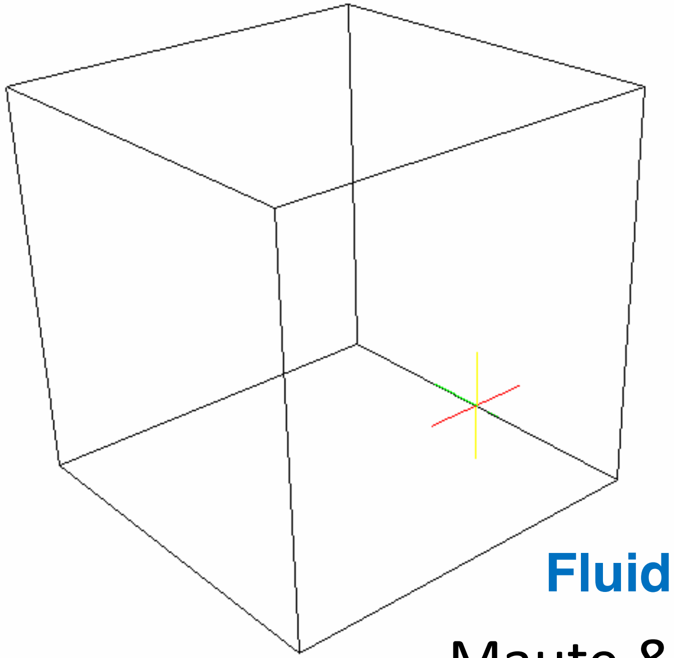


Electric actuator
Sigmund 2000



Natural convection

Alexandersen et al. 2016



Fluid flow

Maute & Pingen

Outline

- Basics of Topology Optimization
- Topology Optimization for Additive Manufacturing

What is additive manufacturing (3D printing)?

- Layered fabrication

allrecipes.com.au

www.mashed.com

Additive Manufacturing: Complexity is free

TU Delft & MX3D, 2015

Joshua Harker

Scott Summit

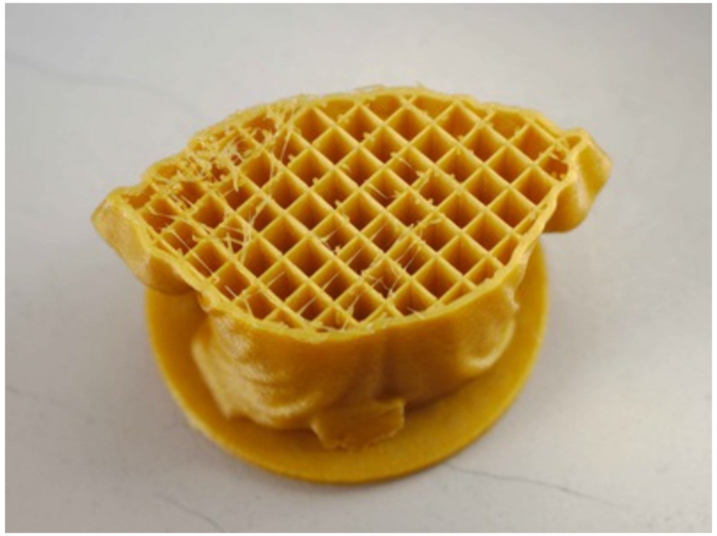
Complexity is free? … Not really!

- Printer resolution: Minimum geometric feature size
- Layer-upon-layer: Supports for overhang region
- Closed cavities in powder-bed based printing
- Infill structures

Supports

Concept Laser GmhH

Infill



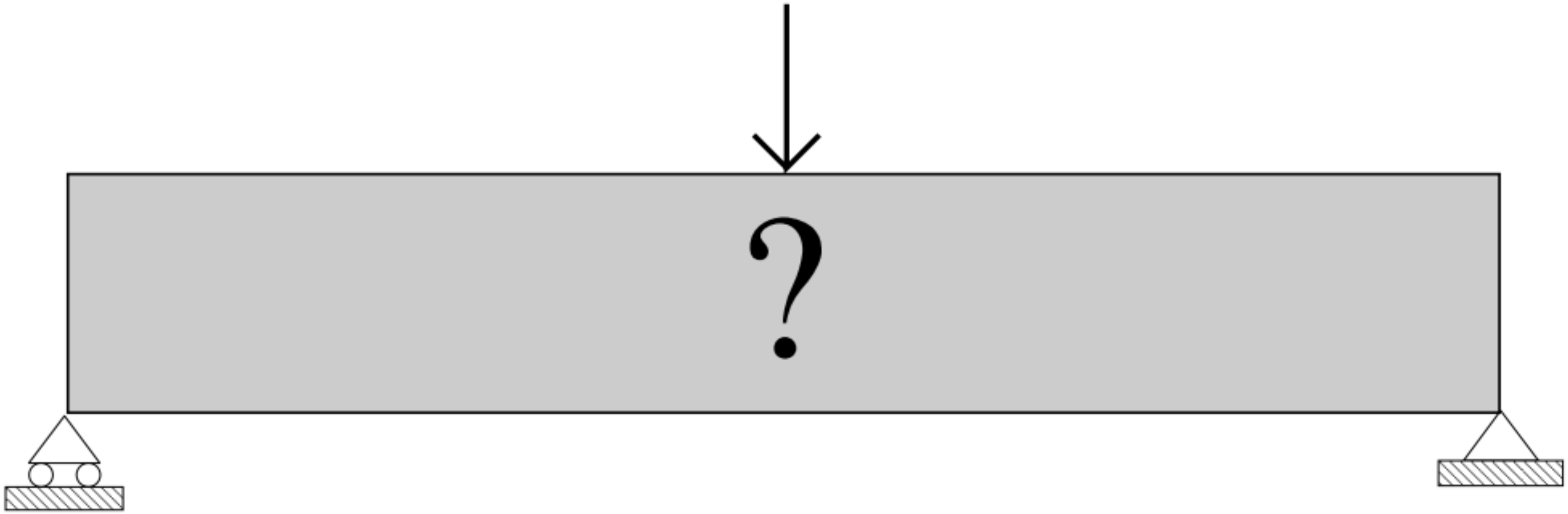
mpi.fs.tum.de

Outline

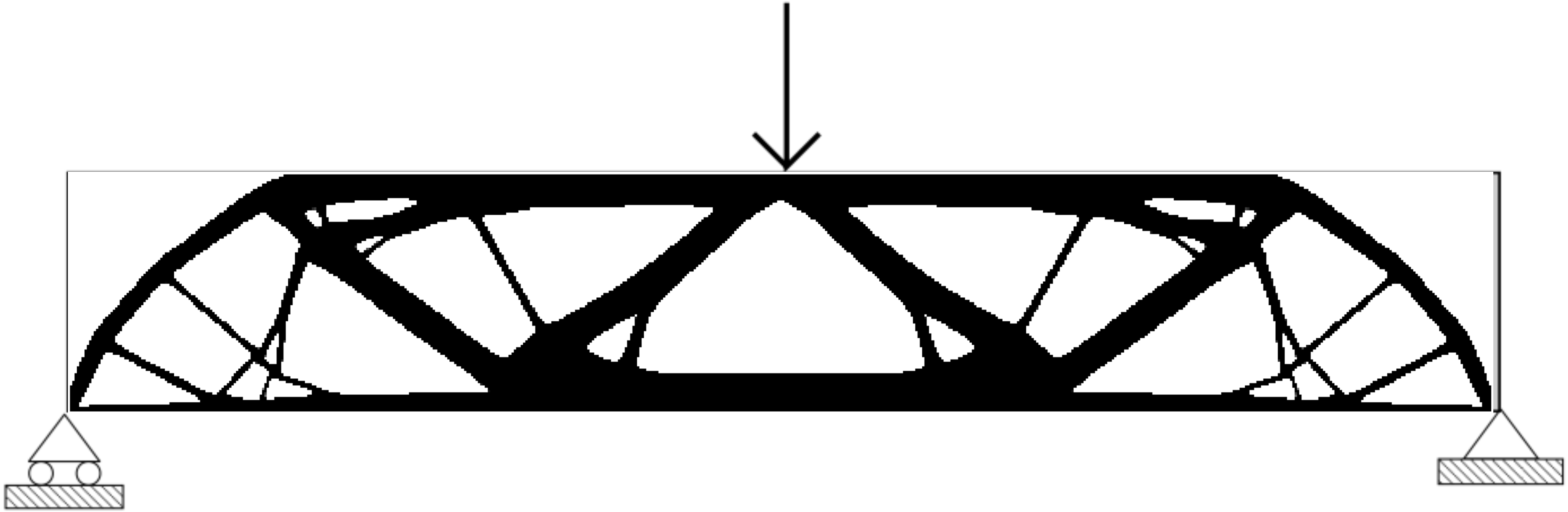
- Basics of Topology Optimization

- Topology Optimization for Additive Manufacturing
  - Geometric feature control by **density filters**
  - Geometric feature control by **alternative parameterizations**

Messerschmidt-Bölkow-Blohm (MBB) beam



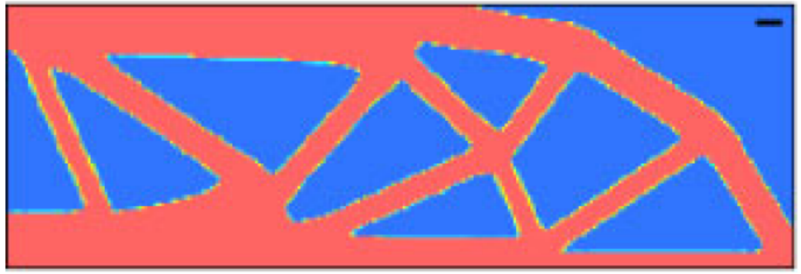
Messerschmidt-Bölkow-Blohm (MBB) beam



Geometric feature control by density filters (An incomplete list)

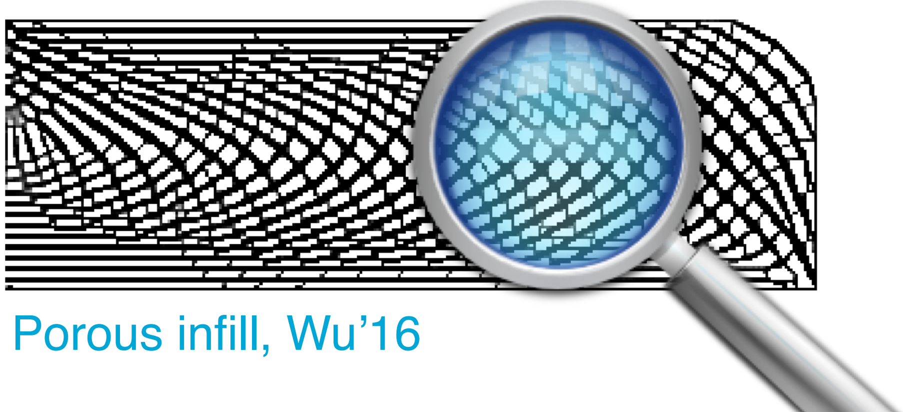
Reference

Minimum feature size, Guest’04



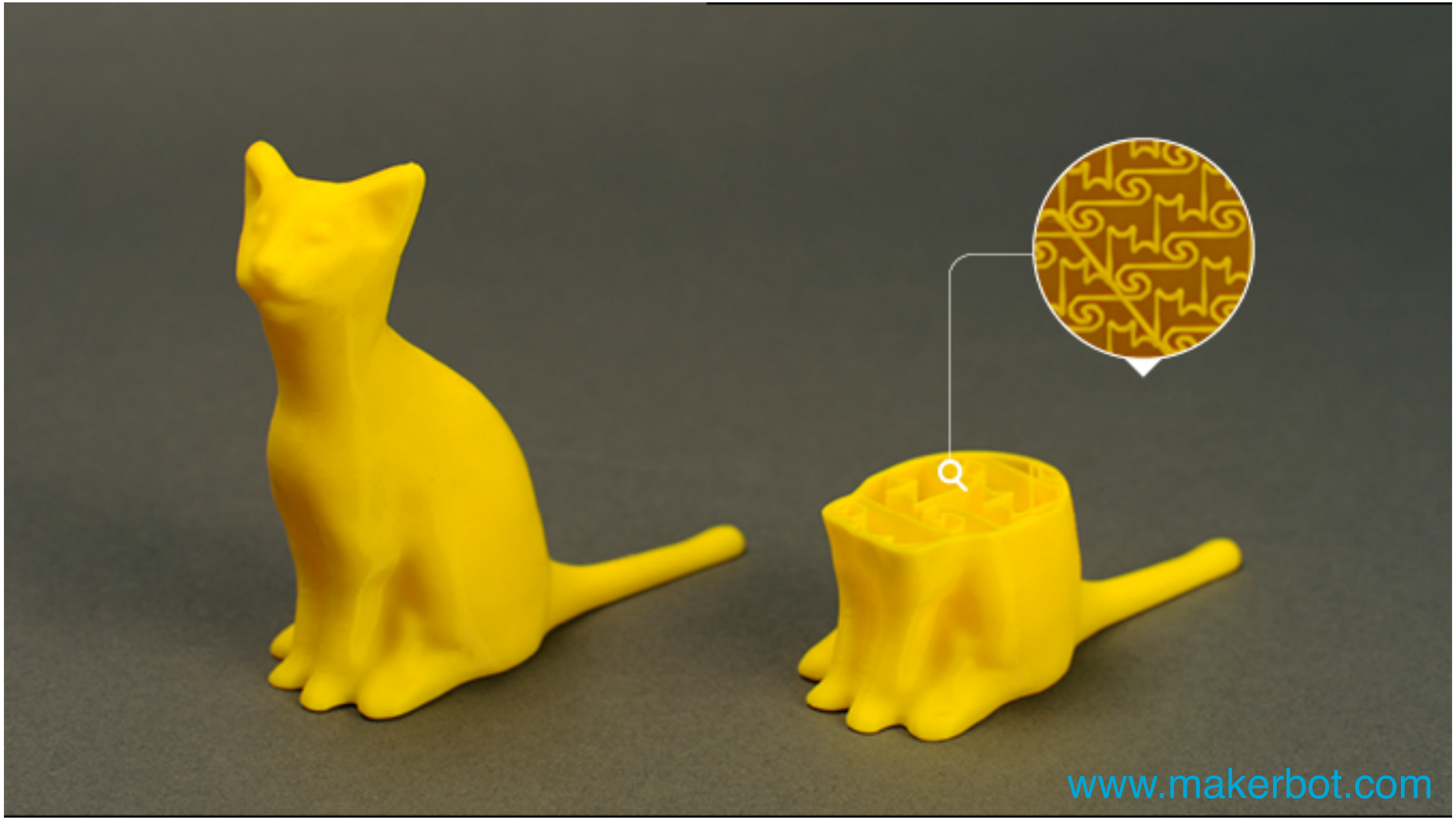
Coating structure, Clausen’15

Self-supporting design, Langelaar’16

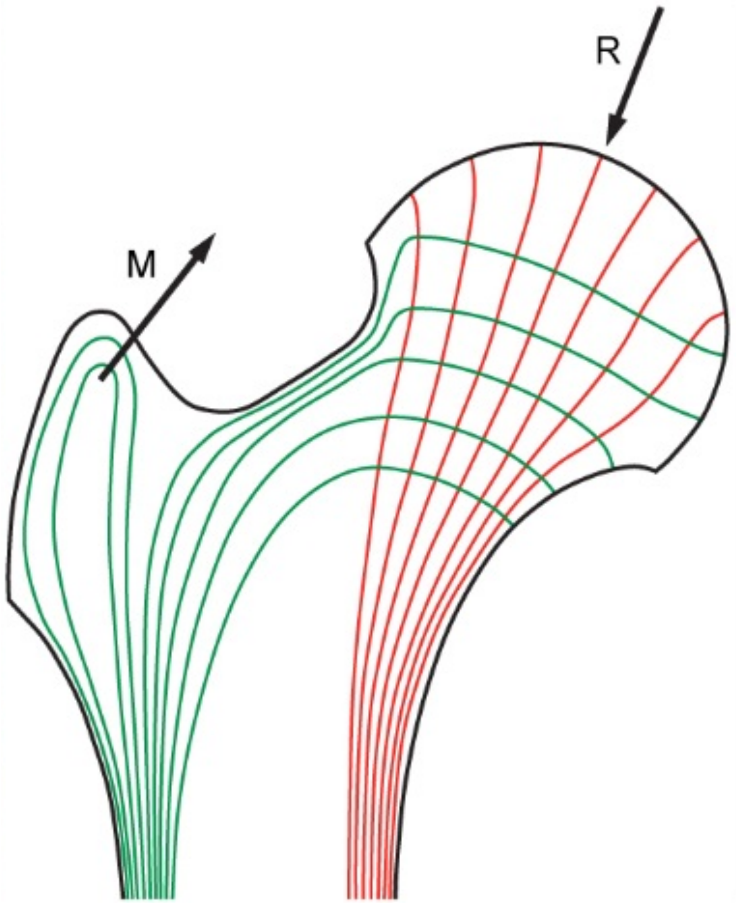


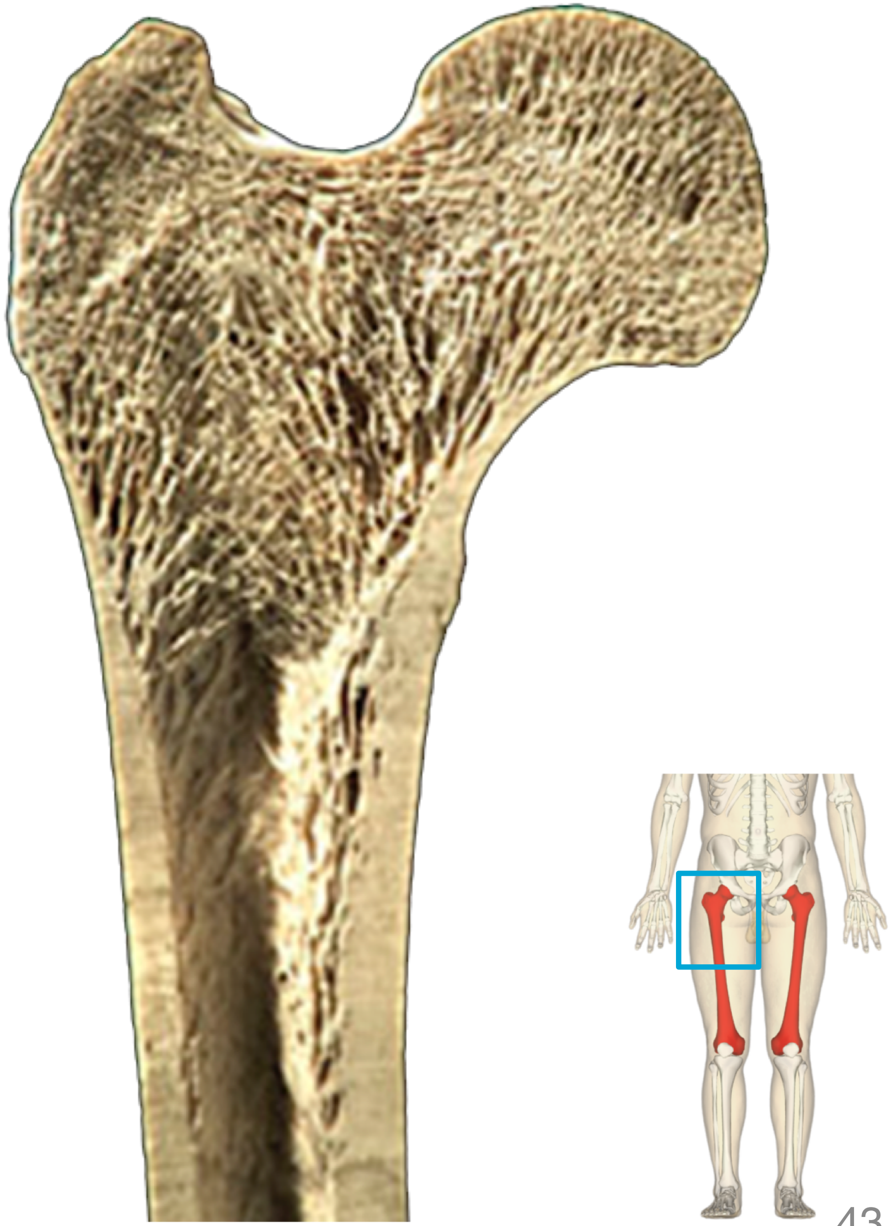
Porous infill, Wu’16

Infill in 3D Printing: Regular Structures



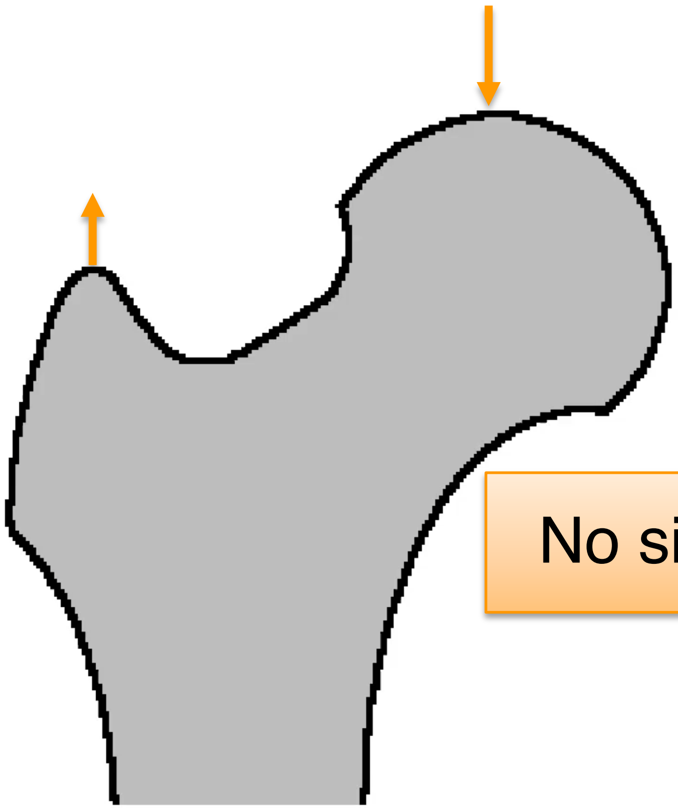
Infill in Bone: Porous Structures





Can we apply the principle of bone to 3D printing?

Topology Optimization Applied to Design Infill



Topology optimization

No similarity in structure

Infill in the bone

Topology Optimization Applied to Design Infill

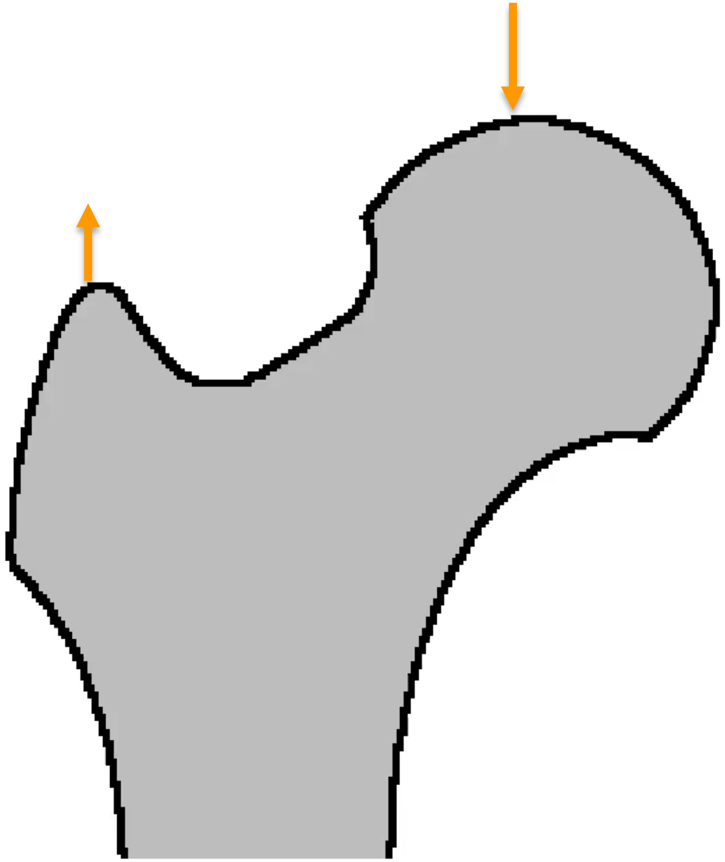
- Materials accumulate to “important” regions
- The total volume $\sum_i \rho_i v_i \leq V_0$ does not restrict local material distribution

Infill by standard topology optimization

Infill in the bone

Bone-like Infill in 2D

Cross-section of a human femur



Approaching Bone-like Structures: The Idea

- Impose **local constraints** to avoid fully solid regions

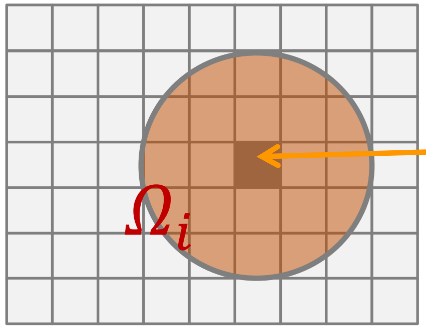
Min: $\quad c = \frac{1}{2} U^T K U$

s.t. : $\quad KU = F$

$\rho_i \in [0,1], \forall i$

$\cancel{\sum_i \rho_i \leq V_0}$

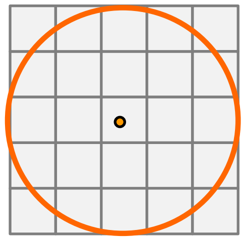
$\hat{\rho}_i \leq \alpha, \forall i$



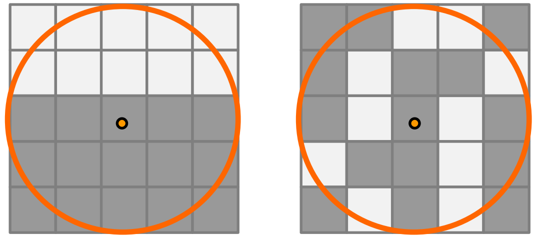
$$\hat{\rho}_i = \frac{\sum_{j \in \Omega_i} \rho_j}{\sum_{j \in \Omega_i} 1}$$

Local-volume measure

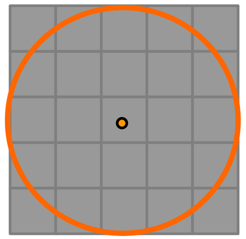
$\hat{\rho}_i = 0.0$



$\hat{\rho}_i = 0.6$



$\hat{\rho}_i = 1.0$



Constraints Aggregation (Reduce the Number of Constraints)

$\boxed{\hat{\rho}_i \leq \alpha, \forall i}$ $\Leftrightarrow$ $\boxed{\max_{i=1,\ldots,n} |\hat{\rho}_i| \leq \alpha}$ $\Leftrightarrow$ $\boxed{\lim_{p\to\infty} \|\rho\|_p = \left(\sum_i (\hat{\rho}_i)^p\right)^{\frac{1}{p}} \leq \alpha}$

Too many constraints!

A single constraint
But non-differentiable

A single constraint
and differentiable
Approximated with p =16

Optimization Process: The same as in the standard topopt

- Impose **local constraints** to avoid fully solid regions

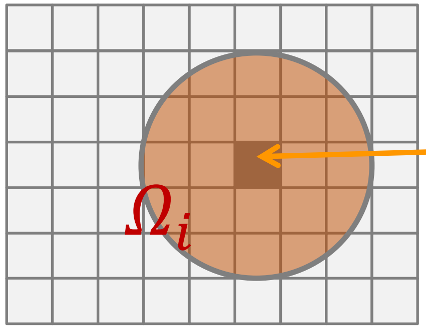
Min: $c = \frac{1}{2} U^T K U$

s.t. : $KU = F$

$\rho_i \in [0,1], \forall i$

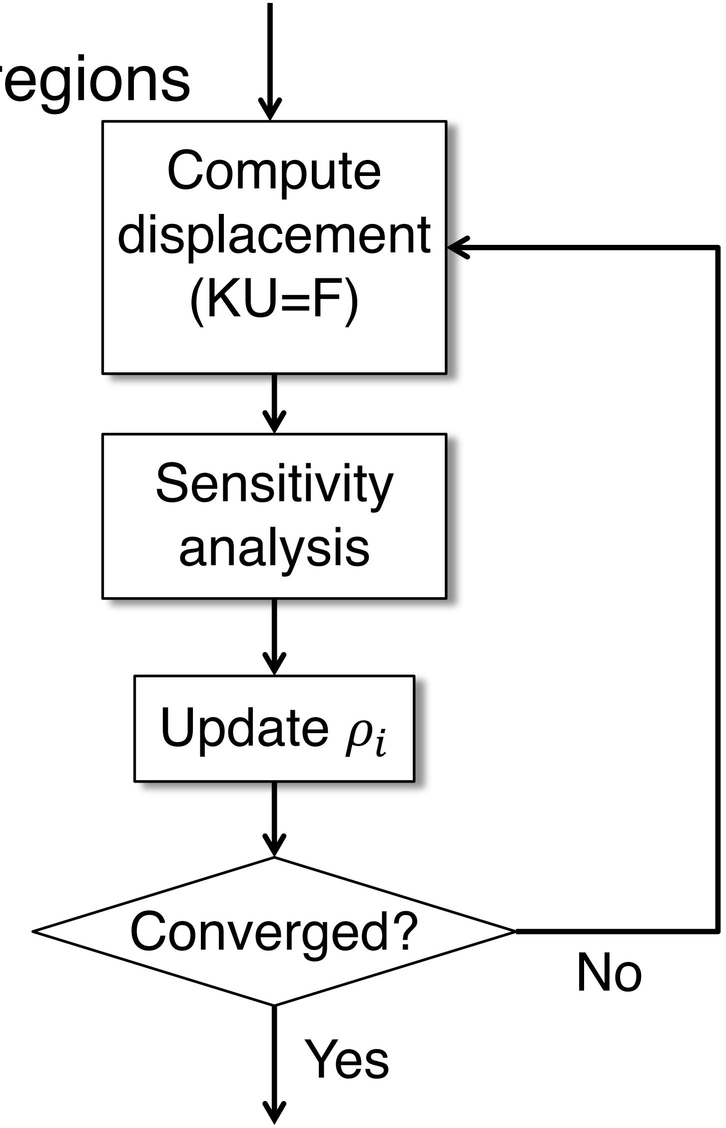
$\cancel{\sum_i \rho_i \le V_0}$

$\hat{\rho}_i \le \alpha, \forall i$

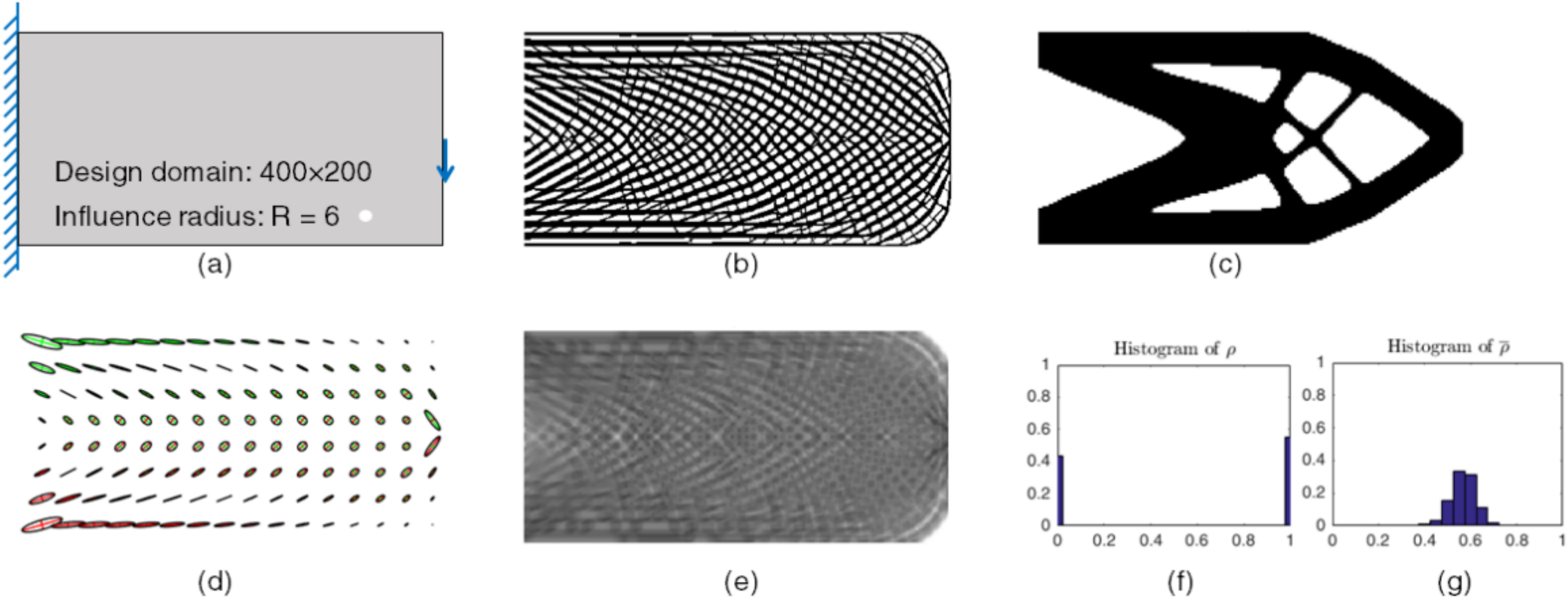


$$\hat{\rho}_i = \frac{\sum_{j \in \Omega_i} \rho_j}{\sum_{j \in \Omega_i} 1}$$

Local-volume measure

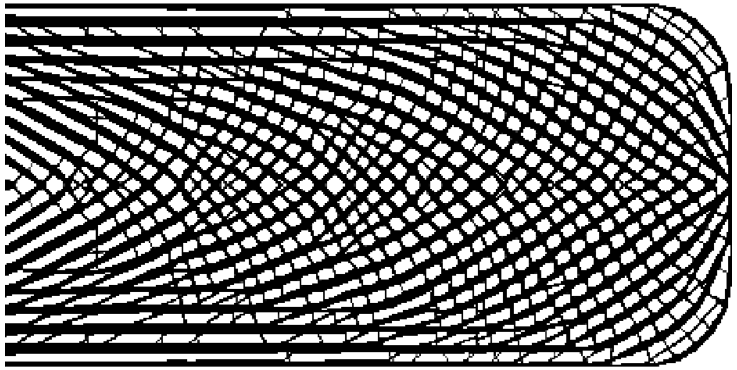


A Test Example

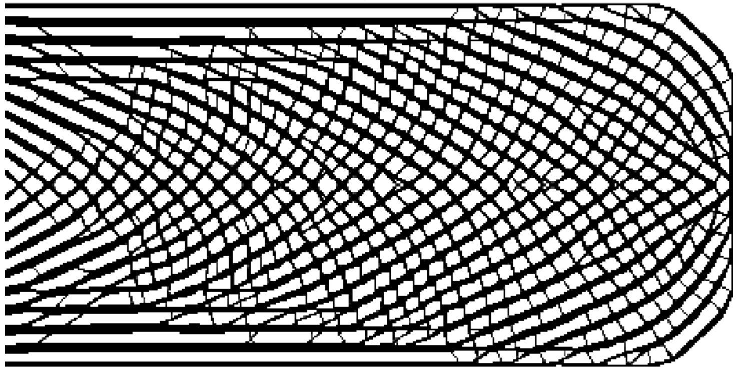


Effects of Filter Radius and Local Volume Upper Bound

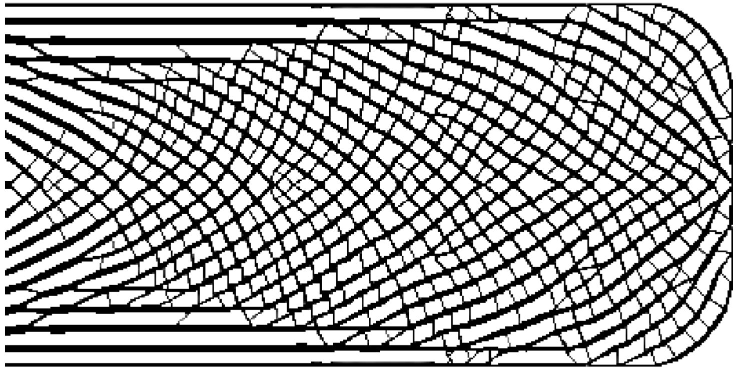
R=6



$(\alpha, c) = (0.6, 76.9)$

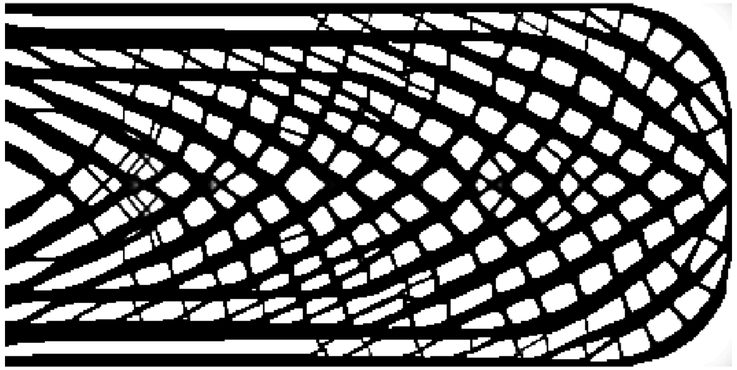


$(0.5, 96.0)$

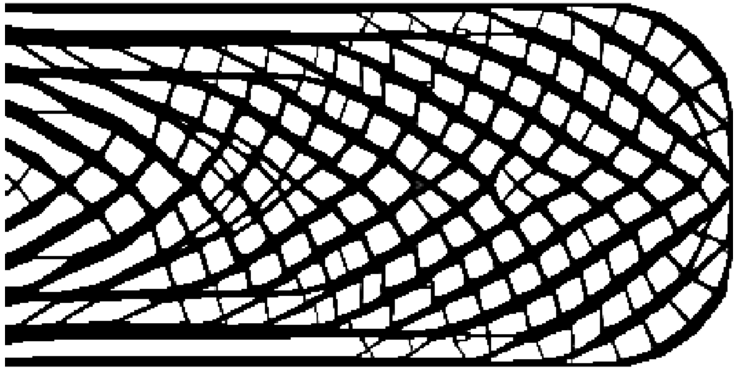


$(0.4, 130.0)$

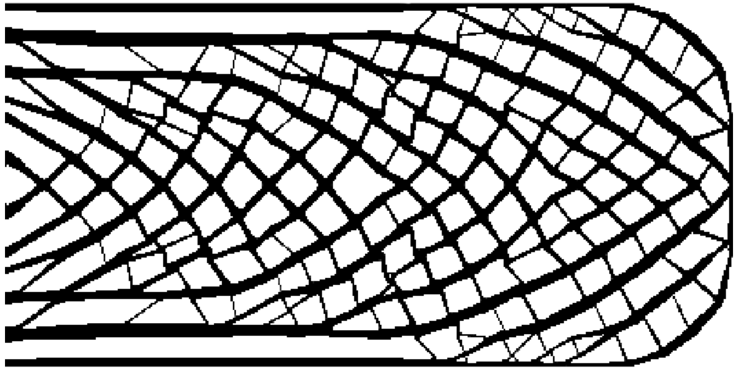
R=12



$(0.6, 73.9)$

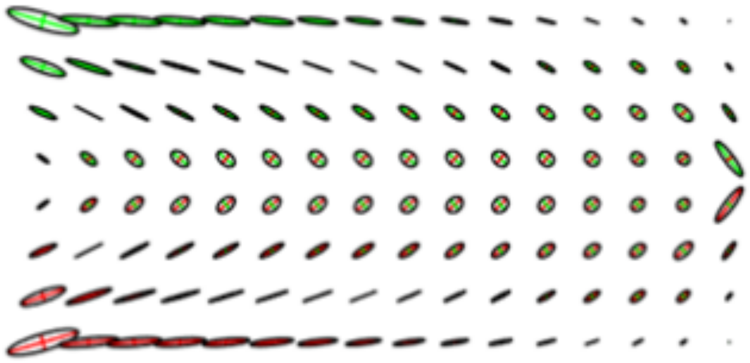


$(0.5, 91.2)$

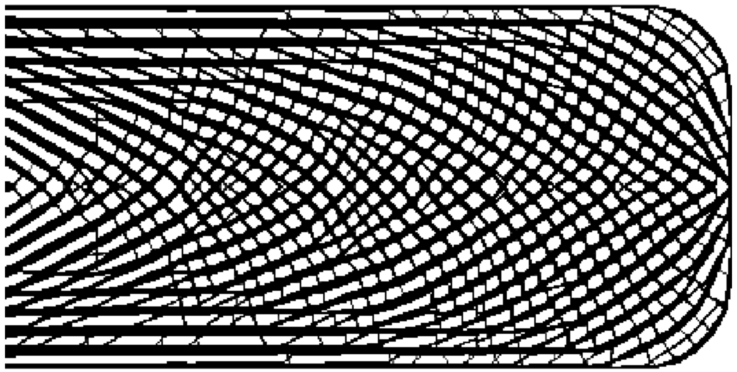


$(0.4, 119.8)$

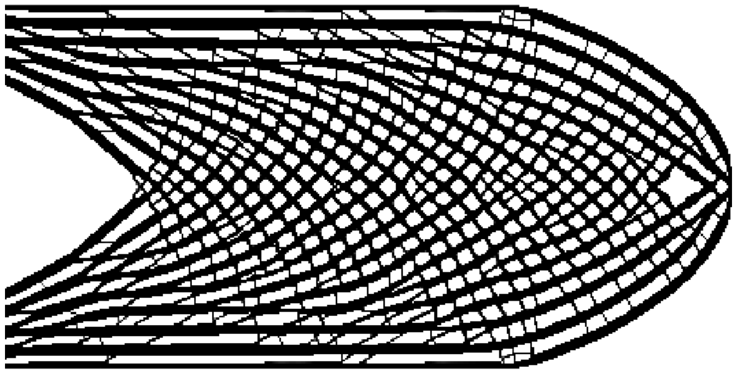
Local + Global Volume Constraints



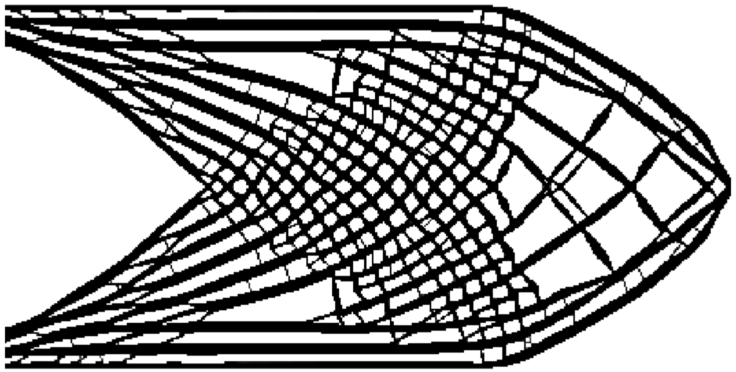
R=6



$(\alpha, \alpha_{total}, c) = (0.6, 0.56, 76.9)$

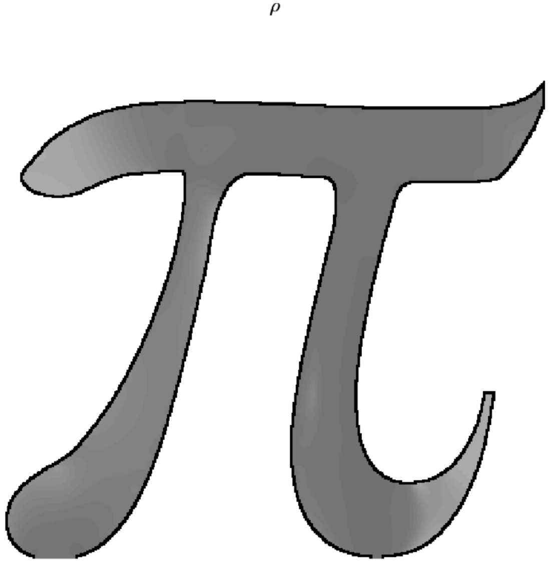


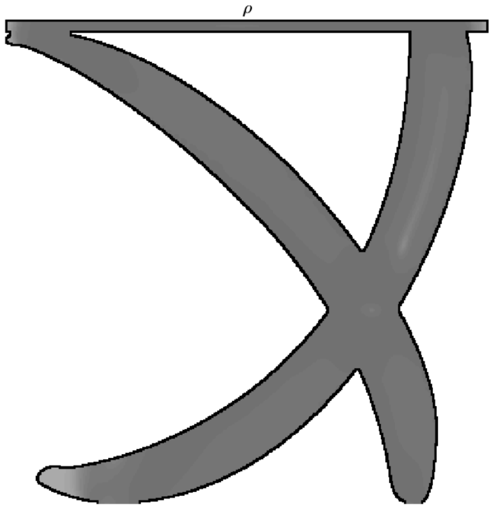
$(0.6, 0.50, 79.1)$

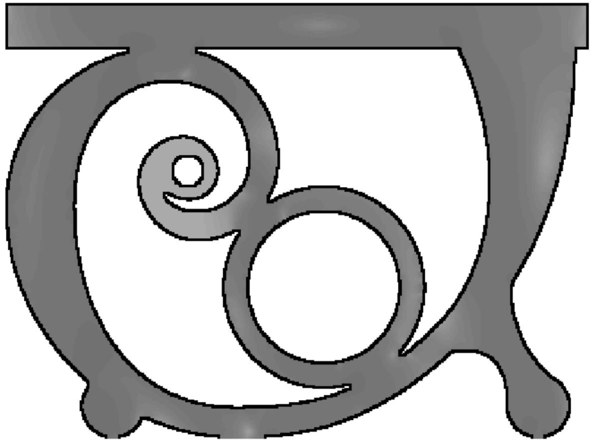


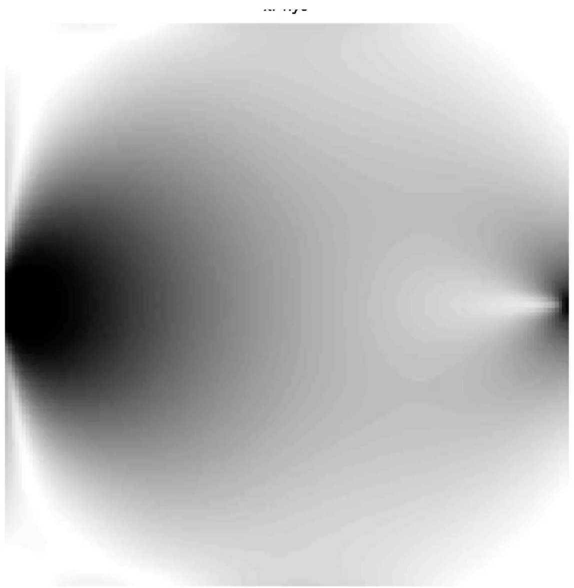
$(0.6, 0.40, 94.0)$

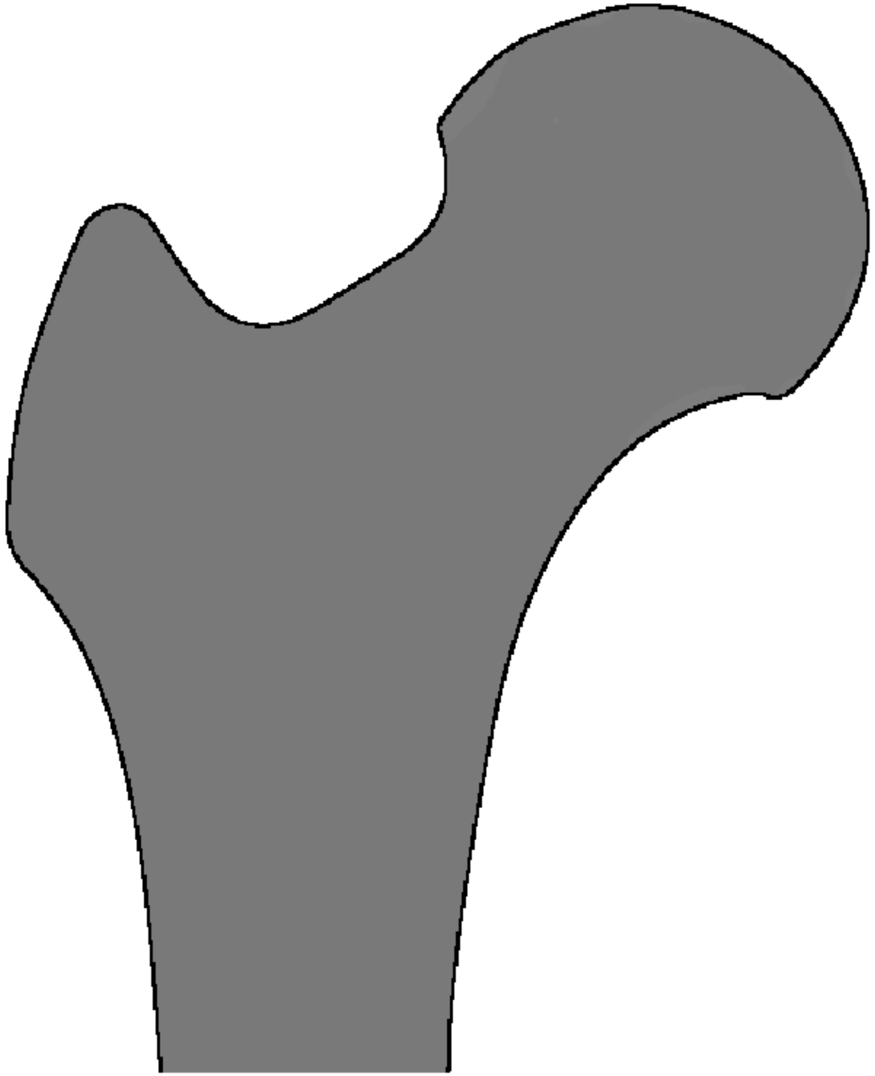
2D Animations







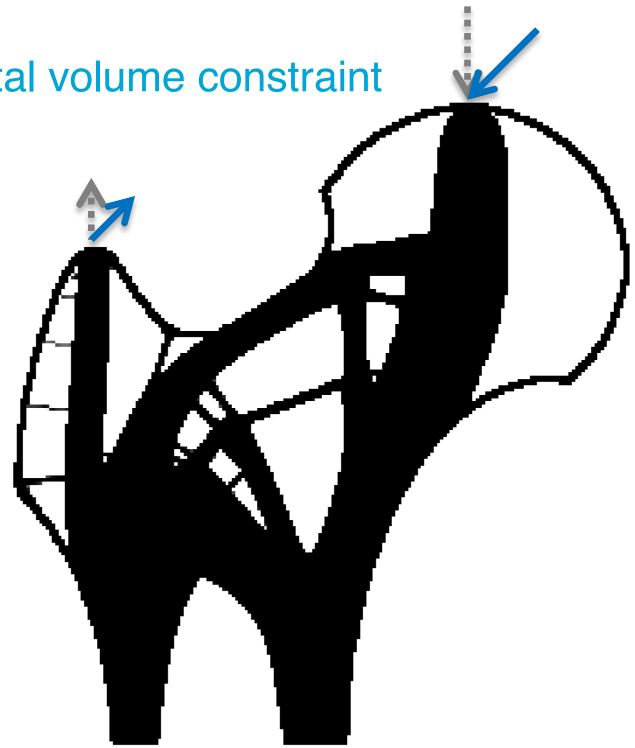




Robustness wrt. Force Variations

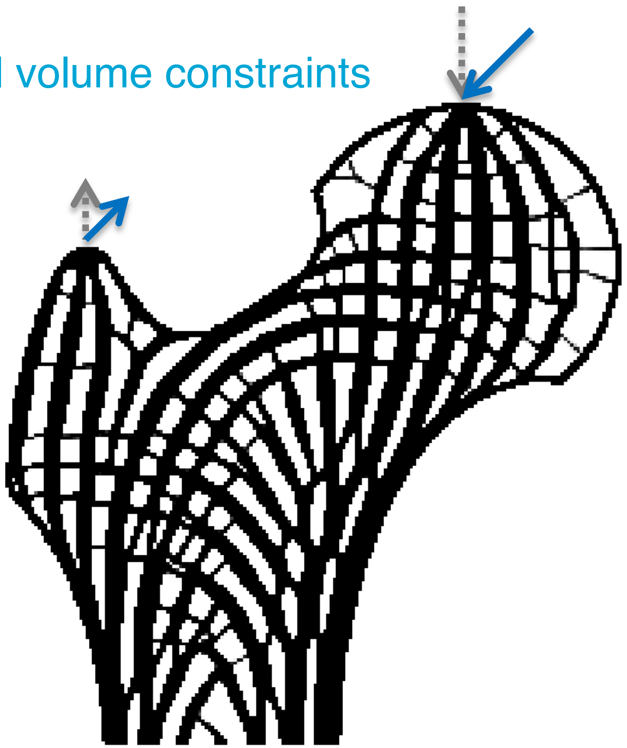
- Porous structures are significantly stiffer (126%) in case of force variations

Total volume constraint



c = 30.54
c'= 45.83

Local volume constraints

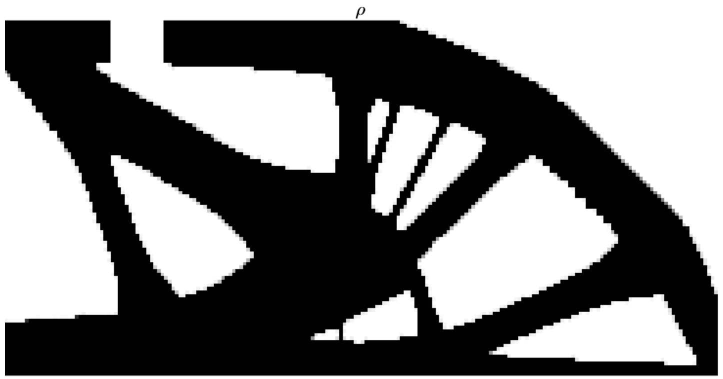


c = 36.72
c' =36.23

Robustness wrt. Material Deficiency

- Porous structures are significantly stiffer (180%) in case of material deficiency

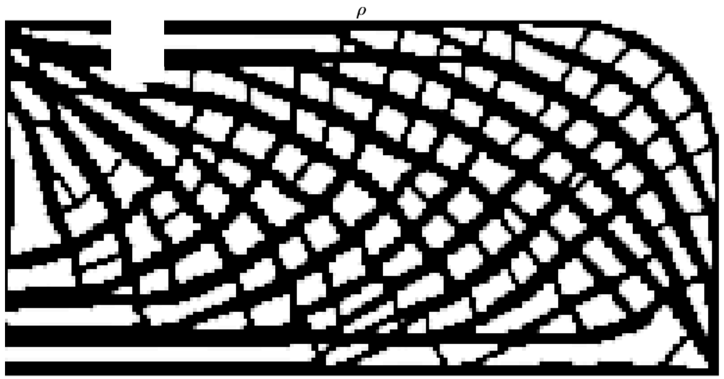
Total volume constraint



c = 76.83

c' =242.77

Local volume constraints

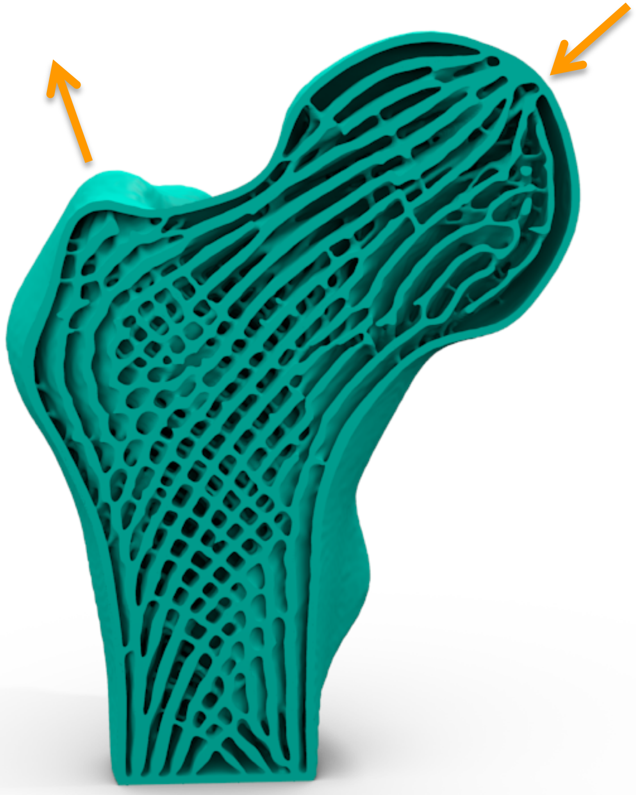


c = 93.48

c'= 134.84

Bone-like Infill in 3D

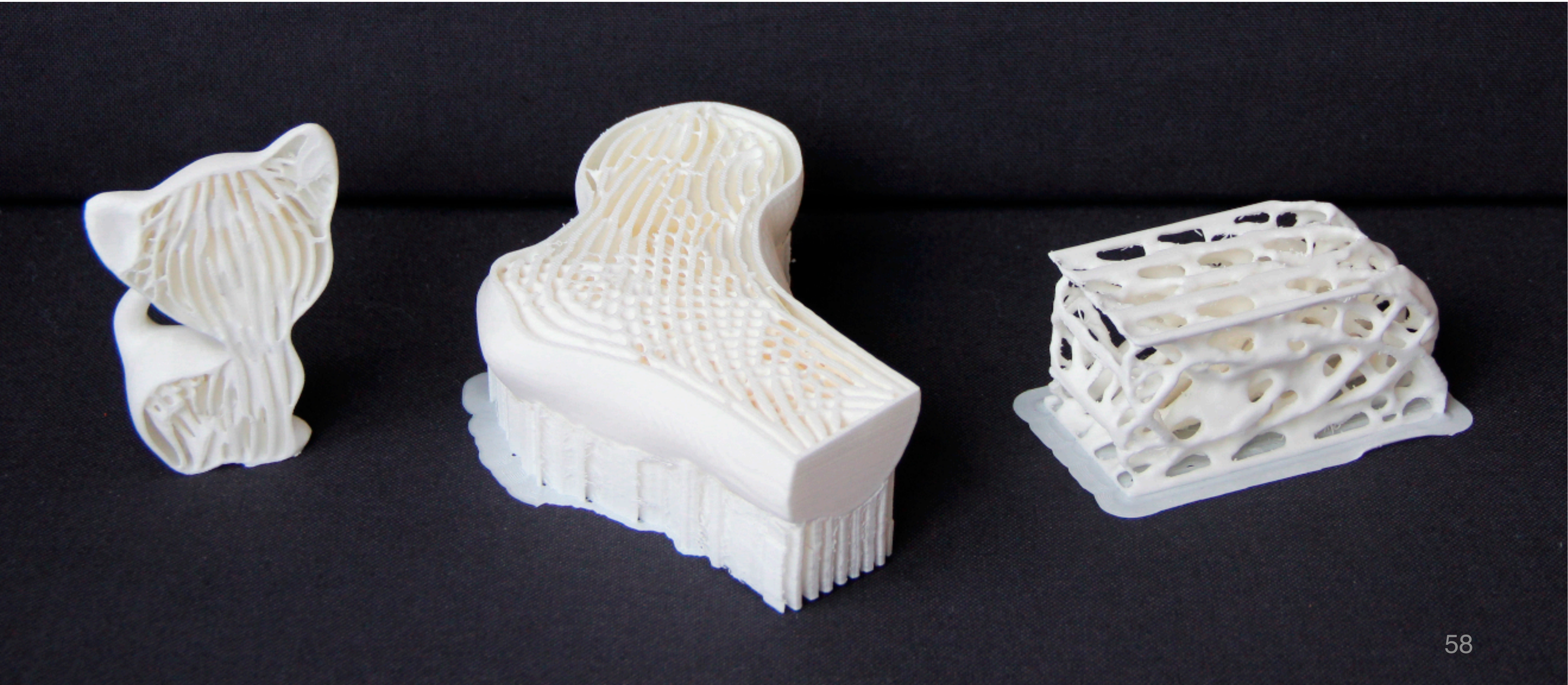
Infill in the bone



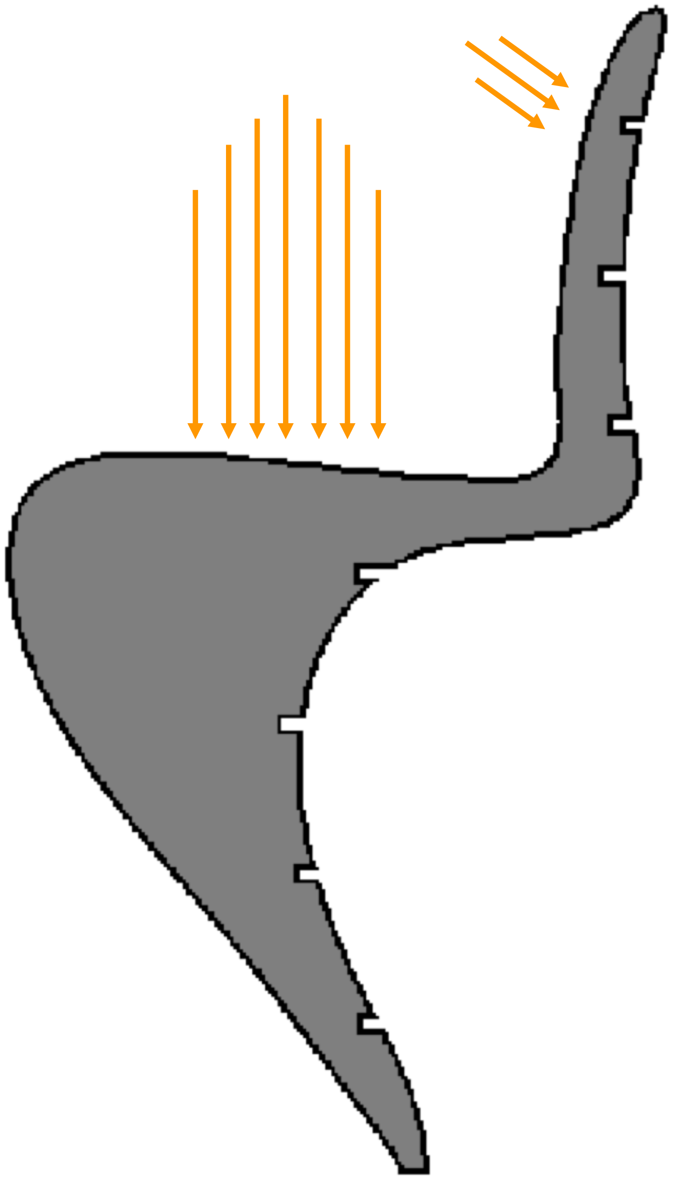
Optimized bone-like infill

Wu et al., TVCG'2017

FDM Prints



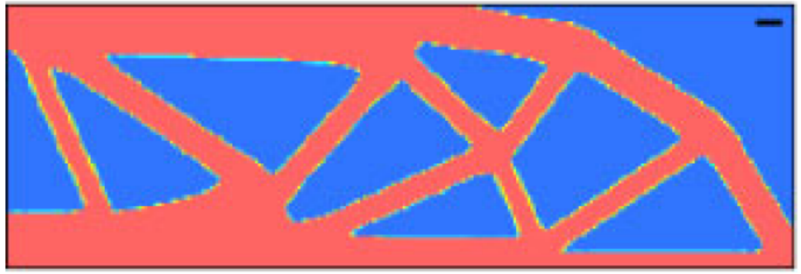
Chair



Geometric feature control by density filters (An incomplete list)

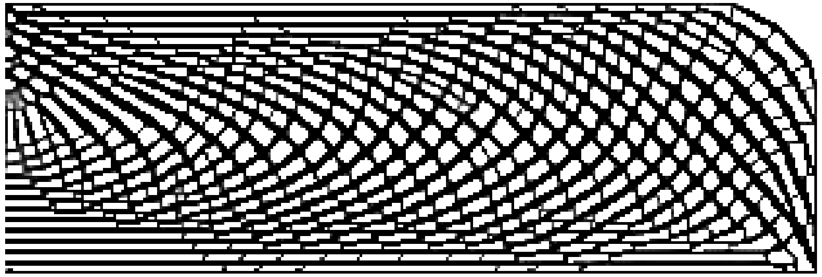
Reference

Minimum feature size, Guest'04



Self-supporting design, Langelaar'16

Coating structure, Clausen'15

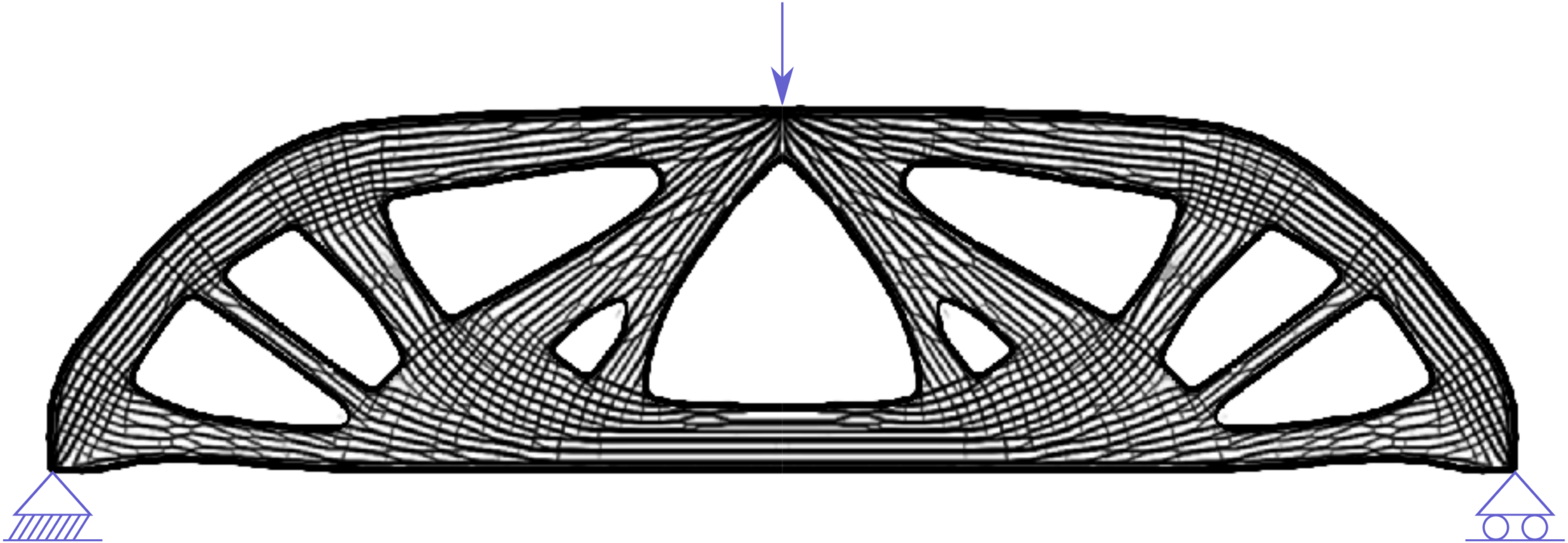


Porous infill, Wu'16

Concurrent Shell-Infill Optimization

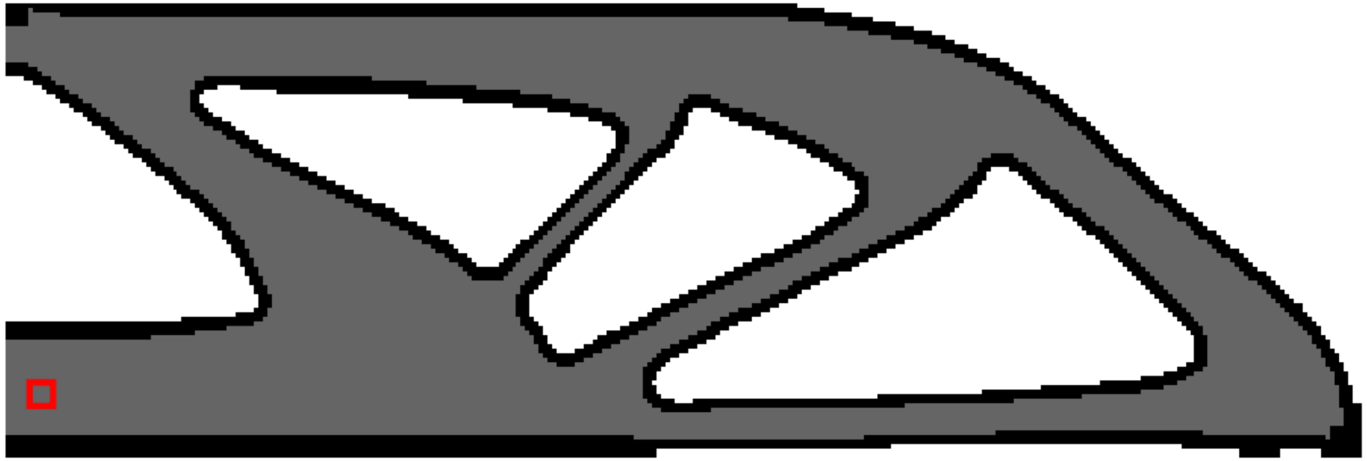
Wu et al., CMAME 2017

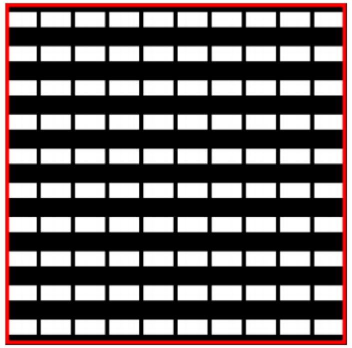
Concurrent Shell-Infill Optimization

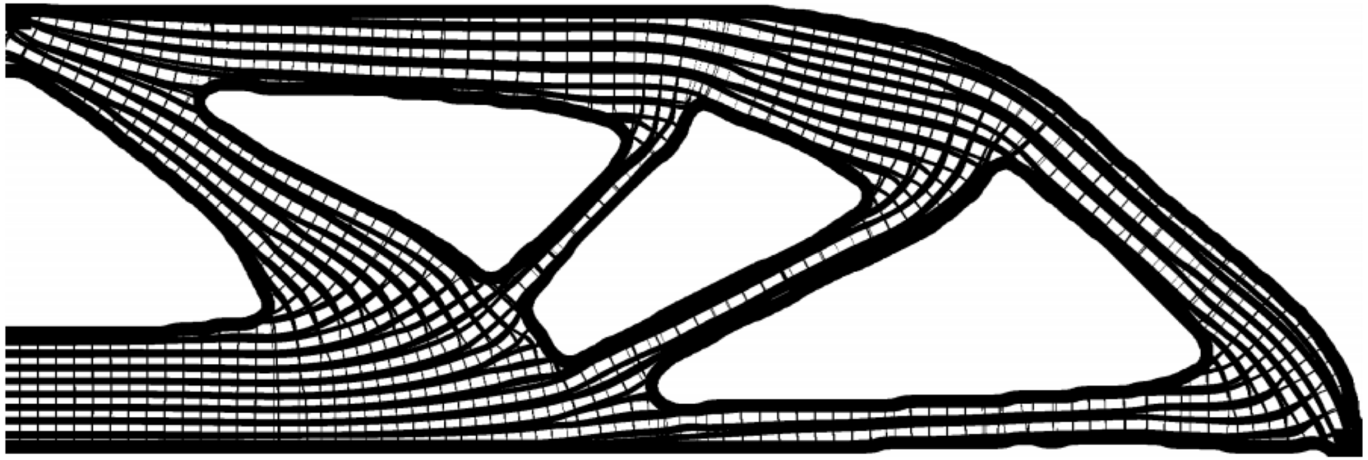


Wu et al., CMAME 2017

Homogenization-based Approach

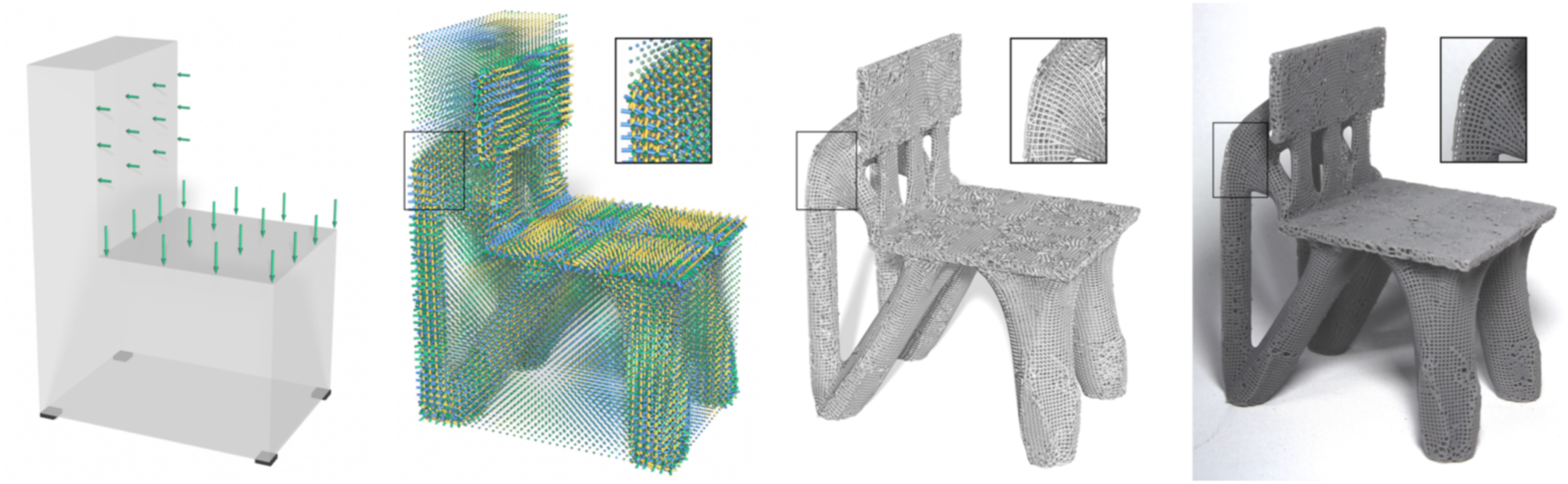






Groen et al. CMAME 2019

3D Conforming Lattice Structures



From left to right: Given a design domain with specified external loads, our method optimizes the distribution of lattice materials for maximizing stiffness. From the optimized, locally-defined lattice configuration, a globally connected lattice structure is compiled, and fabricated by 3D printing.

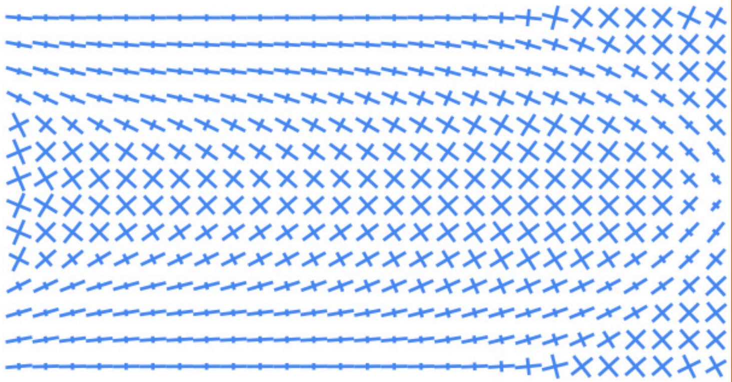
Wu et al. arxiv 2019

TUDelft

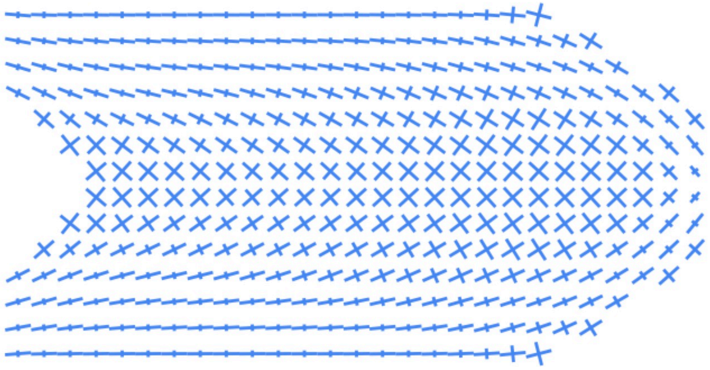
Topology optimization + field-aligned meshing

Design domain

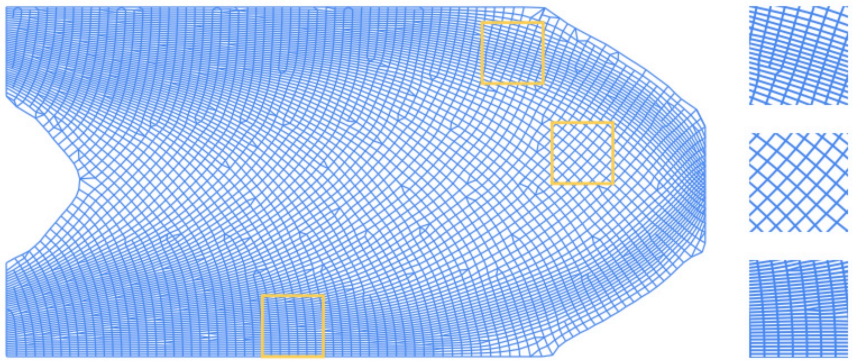
Occupancy of lattice



Orientation and elongation



Occupancy, orientation and elongation

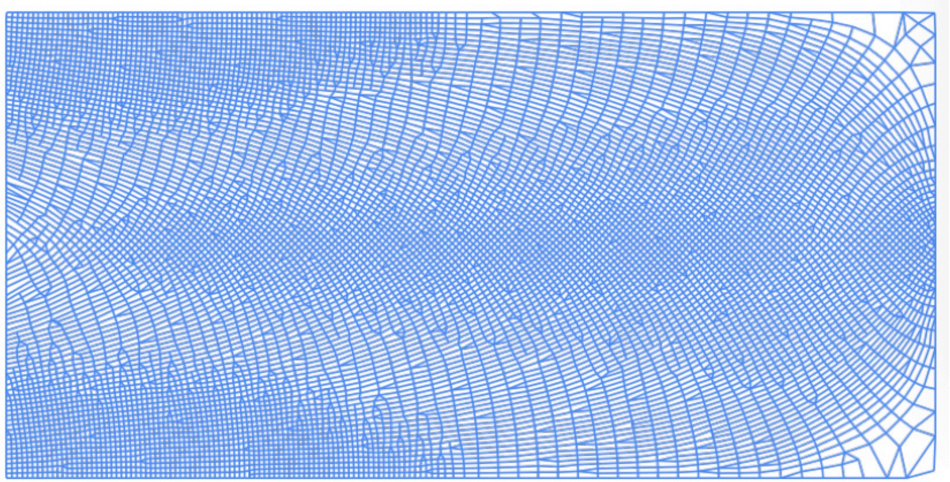


Compiled lattice

Topology optimization + field-aligned meshing

- Lattice by field-aligned meshing vs. by density approach with local volume constraints

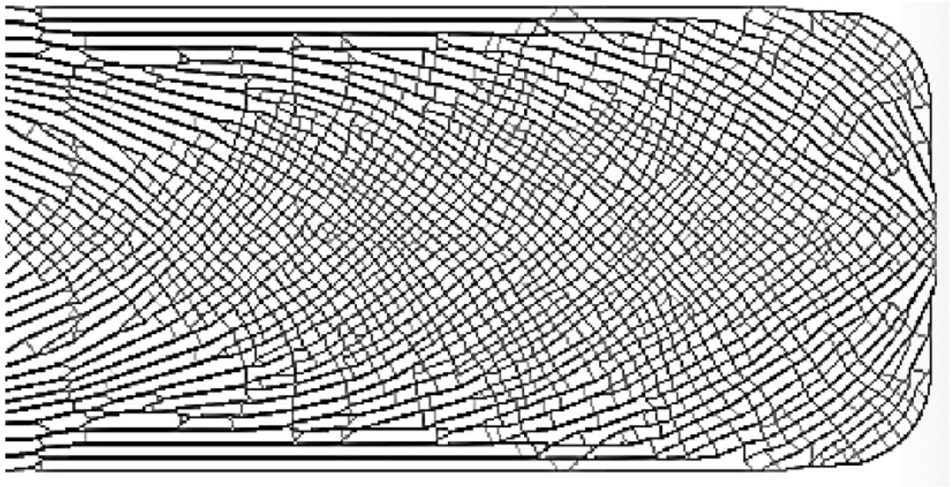
Field-aligned meshing



Compliance: 177.29

Computation time: 1 minute 7 seconds

Density approach



184.64

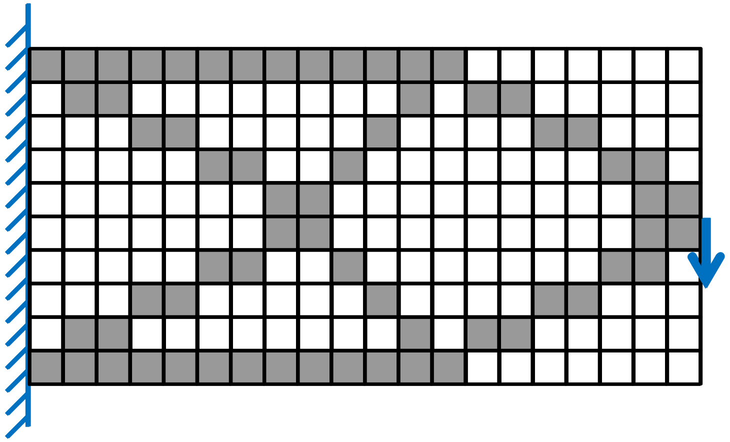
40 minutes

Outline

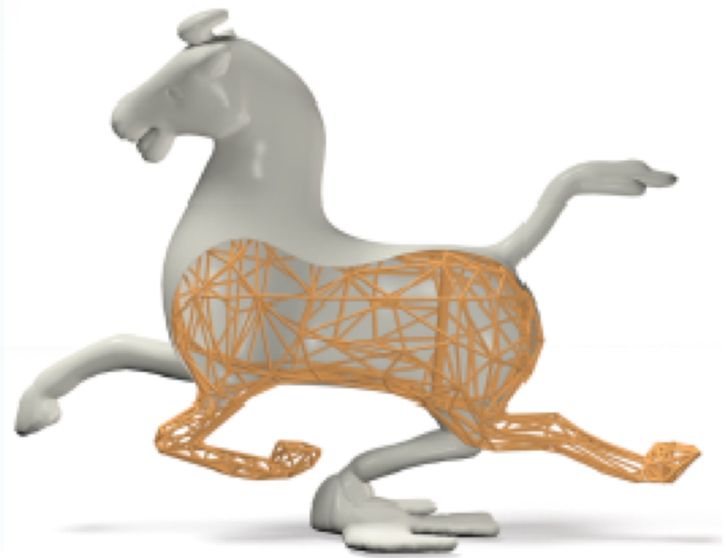
- Basics of Topology Optimization

- Topology Optimization for Additive Manufacturing
  - Geometric feature control by **density filters**
  - Geometric feature control by **alternative parameterizations**

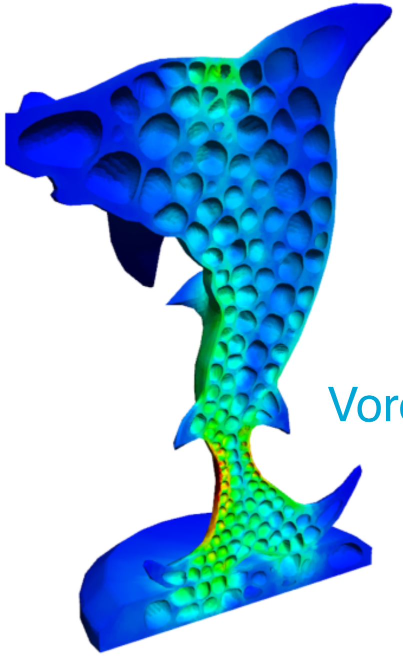
Geometric feature control by alternative parameterizations (An incomplete list)



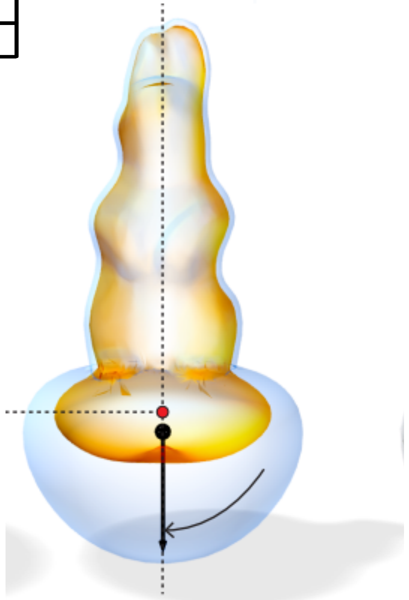
Reference: Voxel discretization



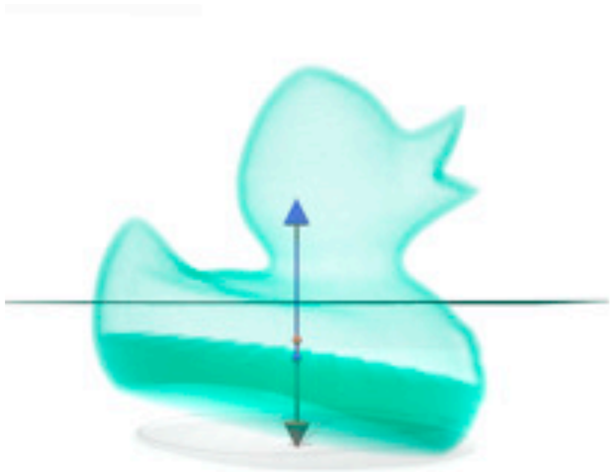
Skin-frame, Wang’13



Voronoi cells, Lu’14



Offset surfaces, Musialski’15

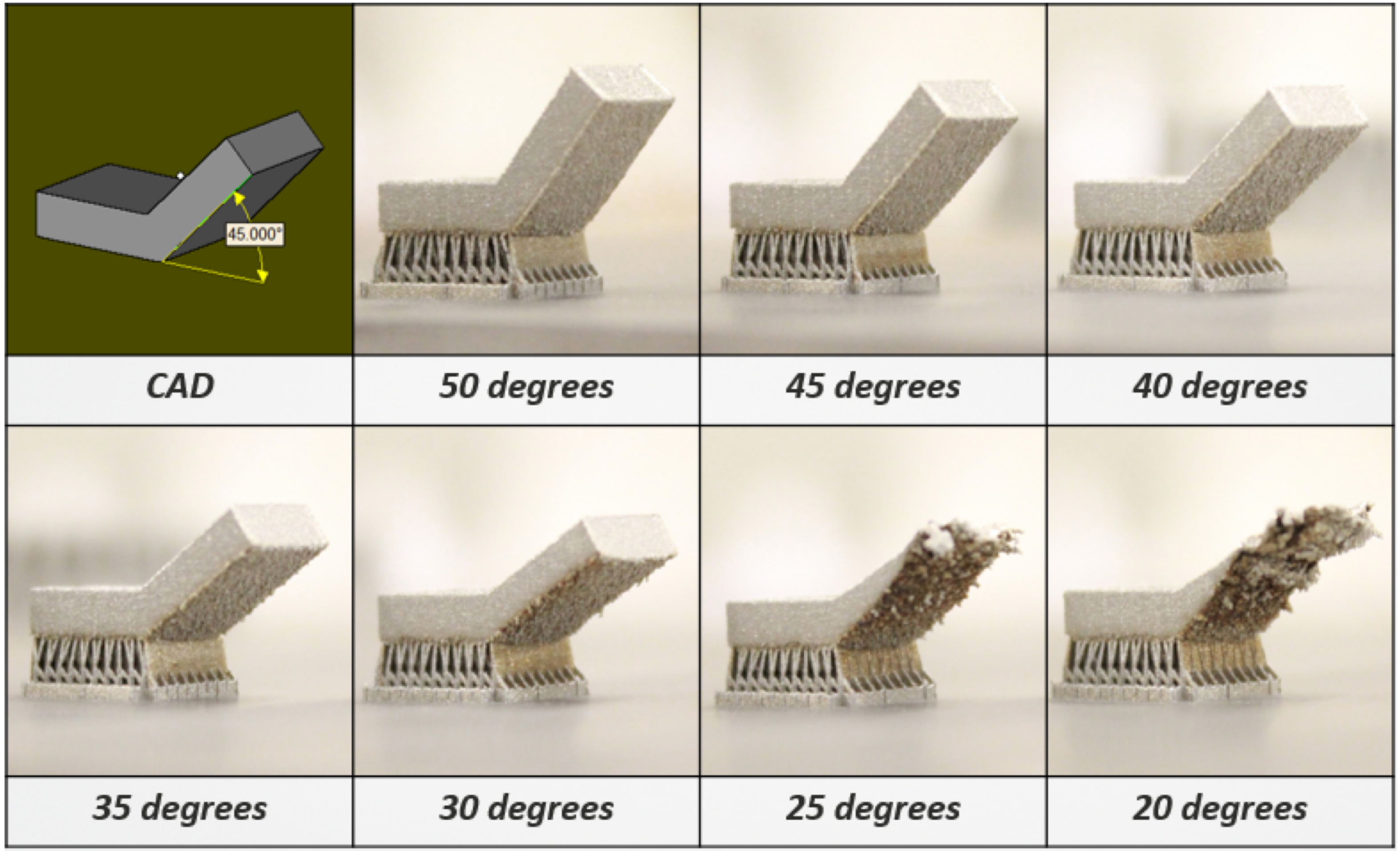


Ray representation, Wu’16

Adaptive rhombic, Wu’18

Overhang in Additive Manufacturing

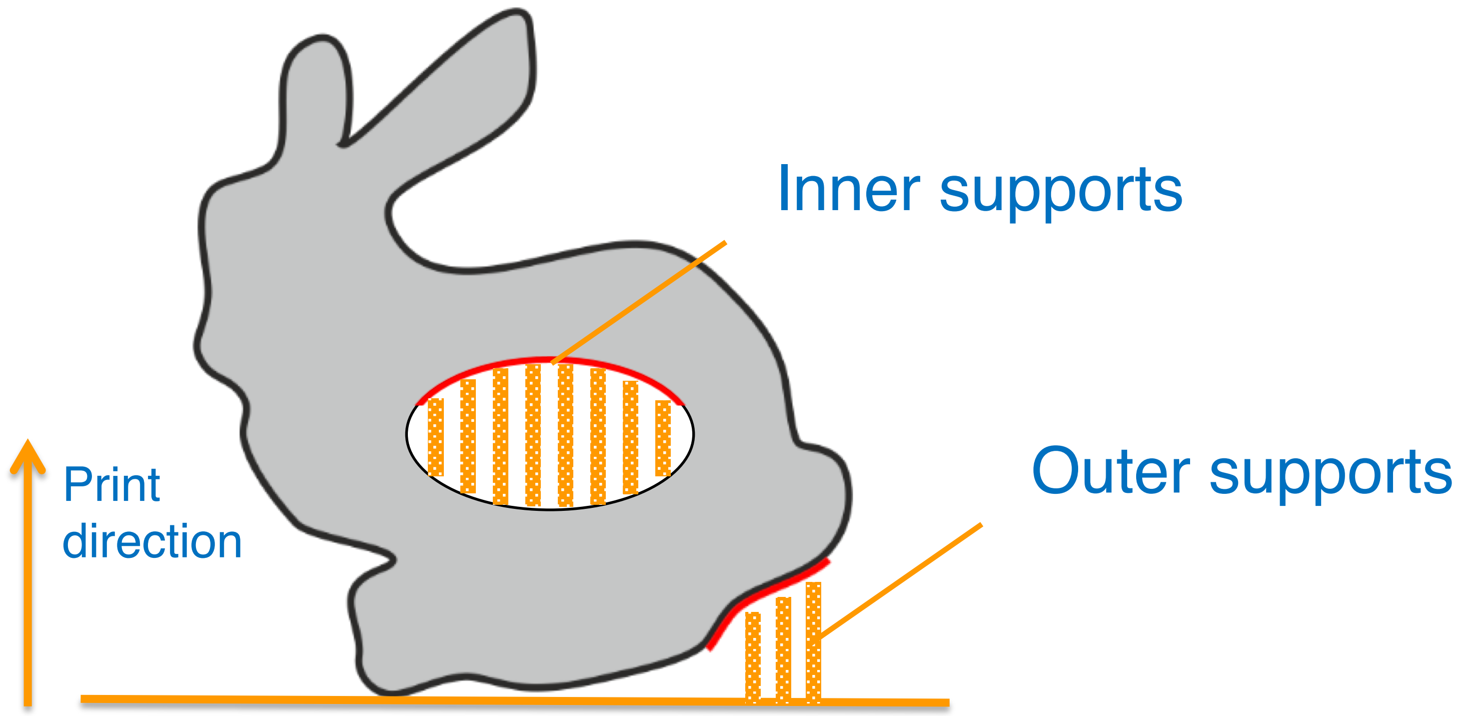
- Support structures are needed beneath overhang surfaces



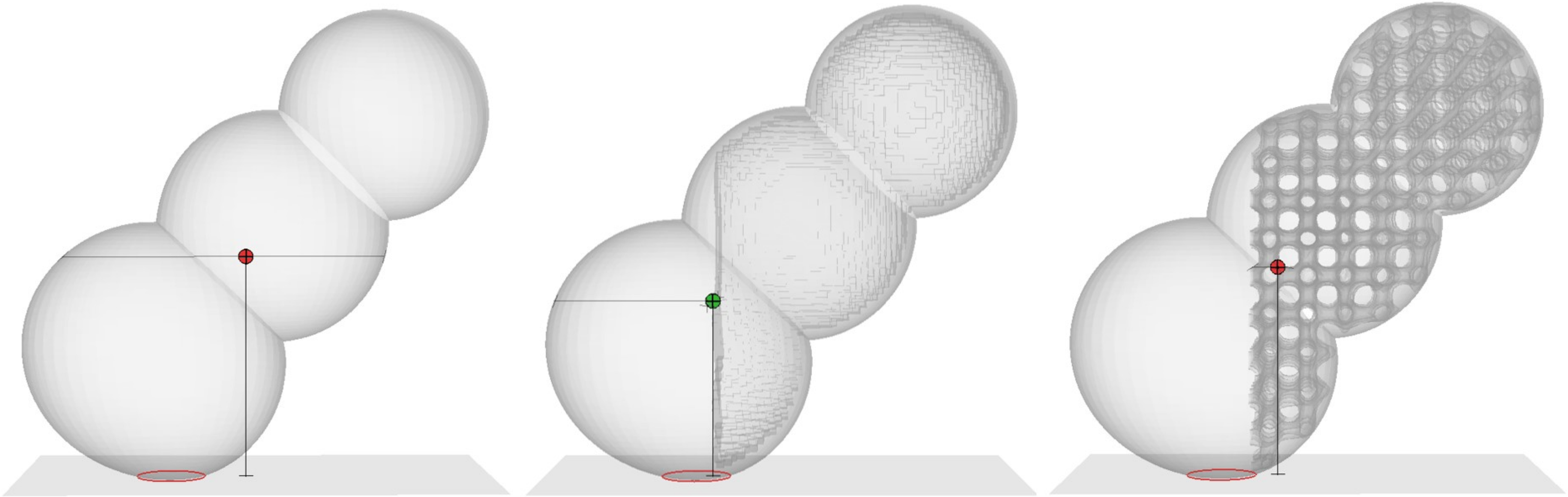
https://www.protolabs.com/blog/tag/direct-metal-laser-sintering/

Support Structures in Cavities

- Post-processing of inner supports is problematic



Infill & Optimization Shall Integrate



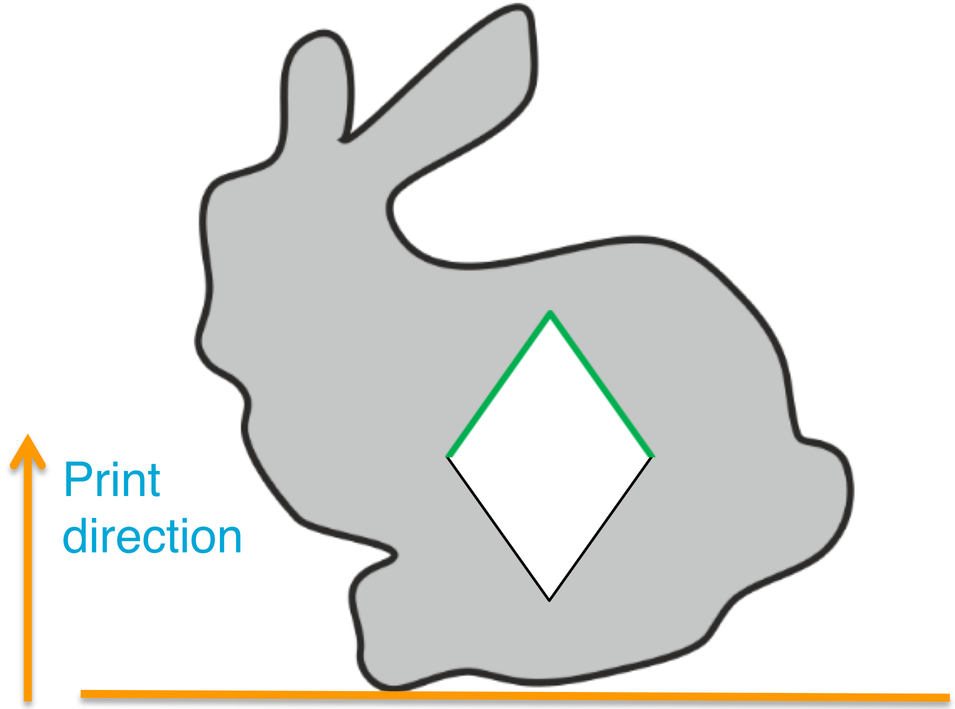
Solid, Unbalanced

Optimized, Balanced

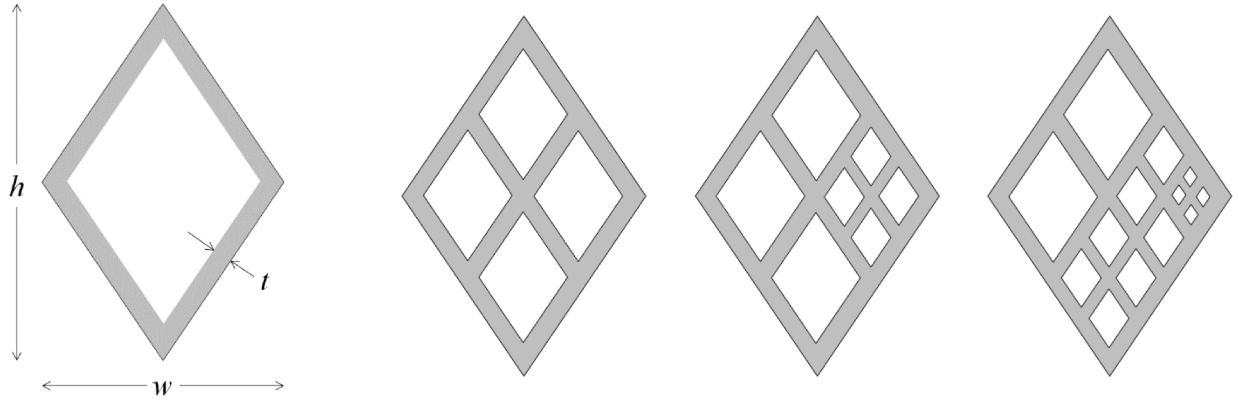
With infill, Unbalanced

The Idea

- Rhombic cell: to ensure self-supporting
- Adaptive subdivision: as design variable in optimization

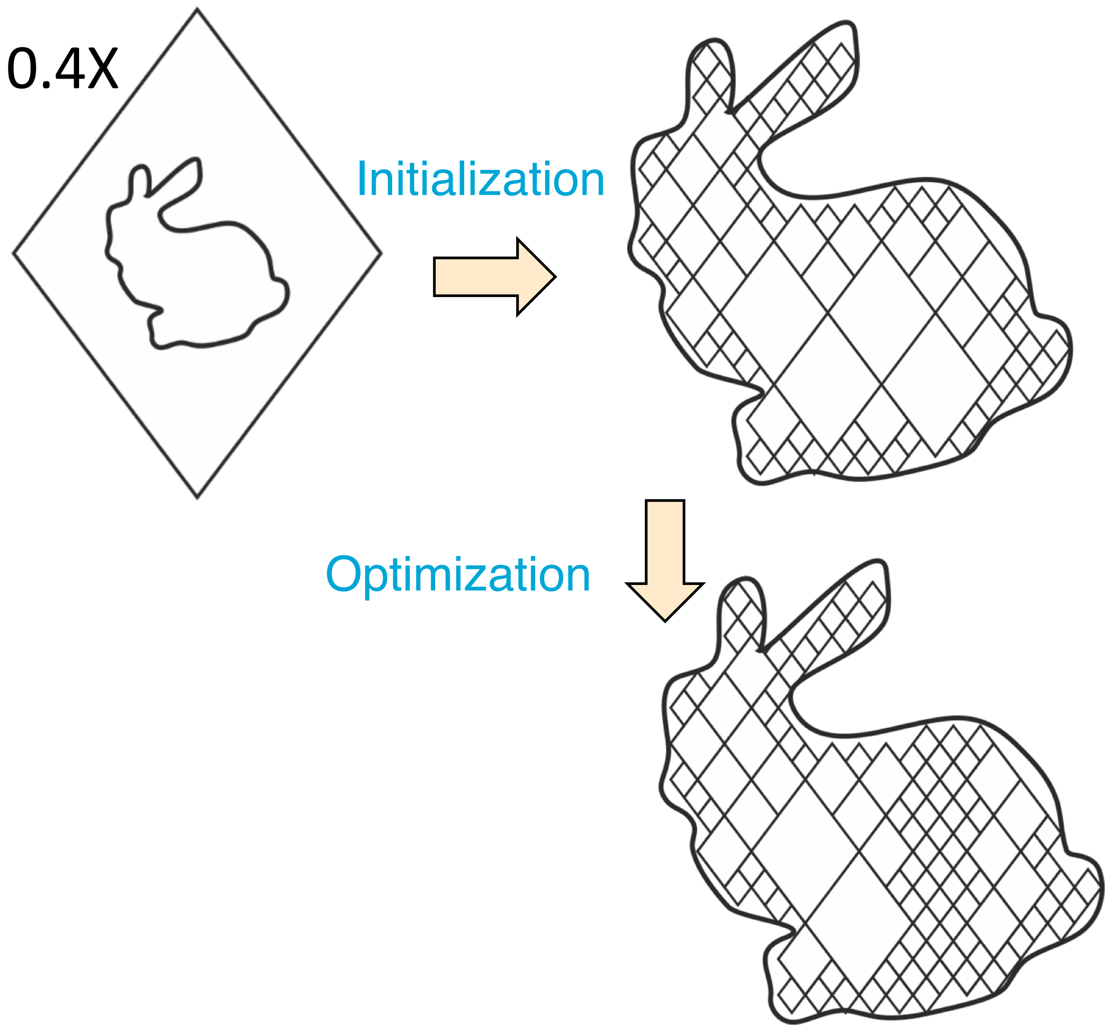


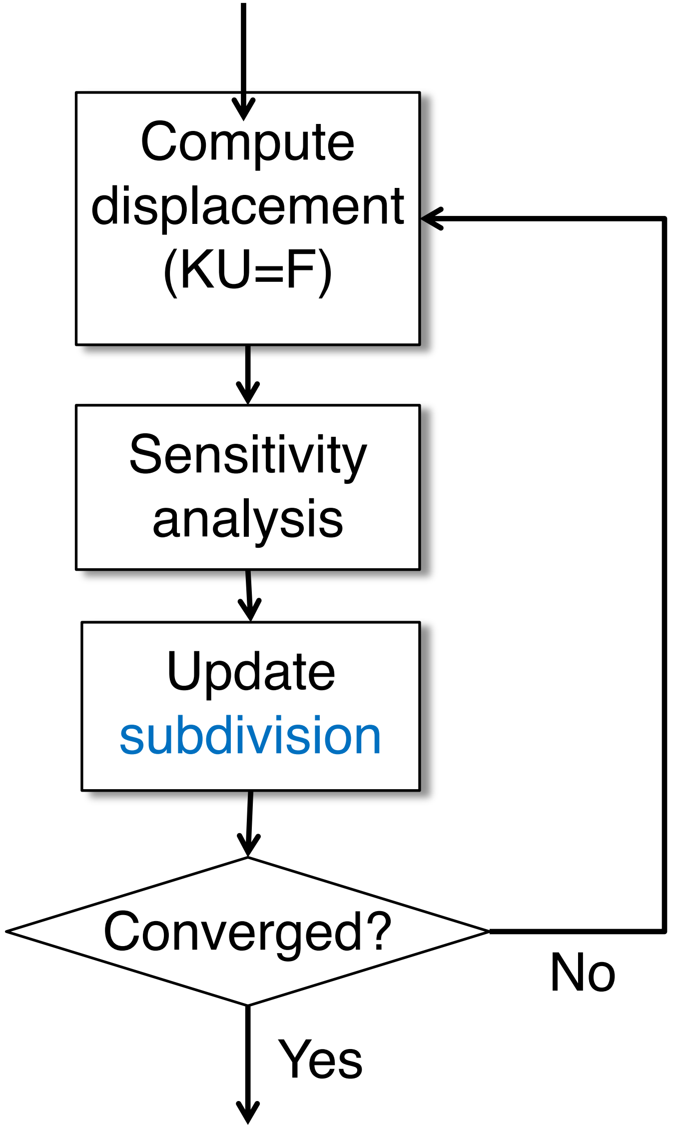
Rhombic cell



Adaptive subdivision

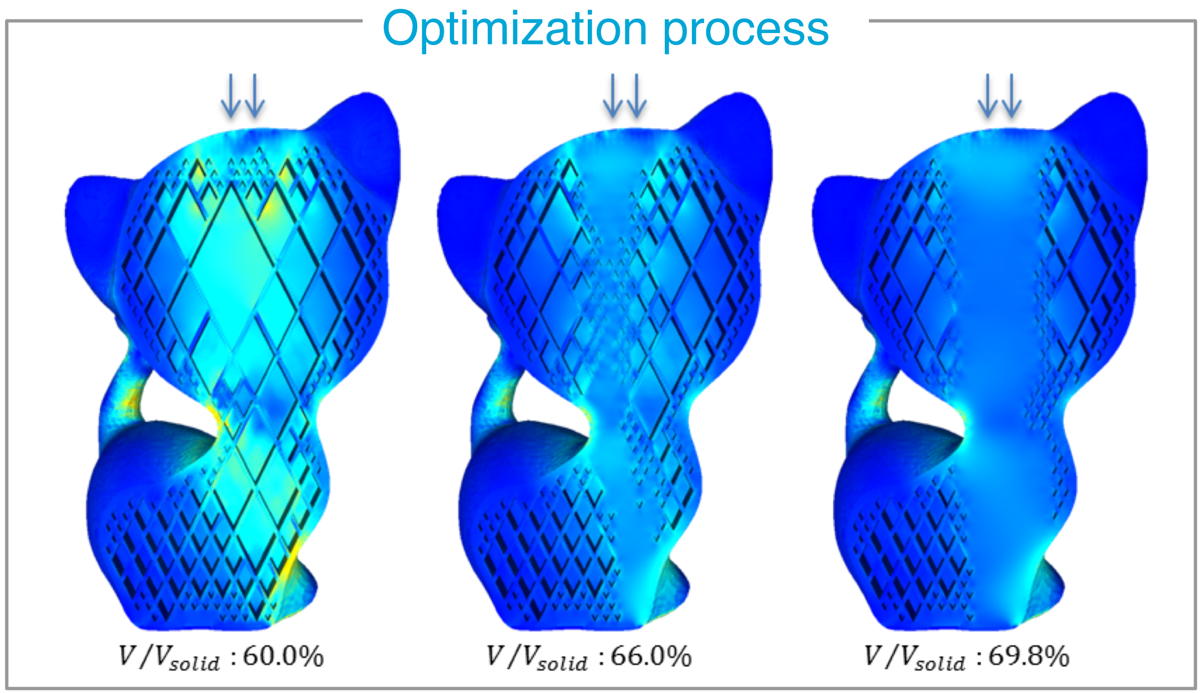
Self-Supporting Rhombic Infill: Workflow

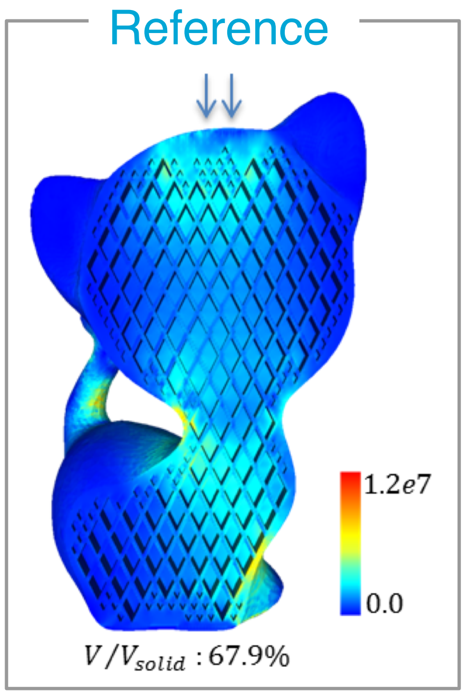




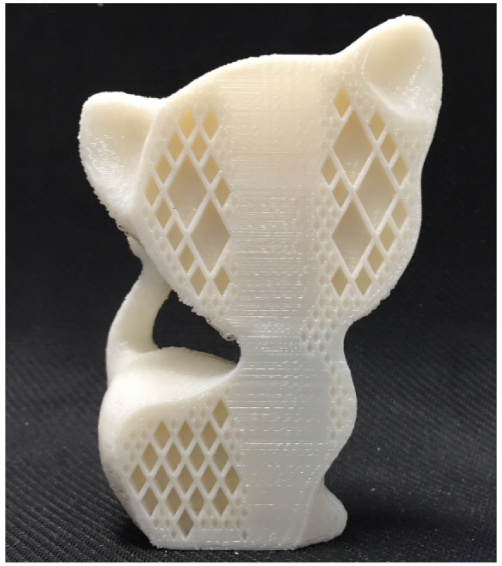
Self-Supporting Rhombic Infill: Results

- Optimized mechanical properties, compared to regular infill
- No additional inner supports needed



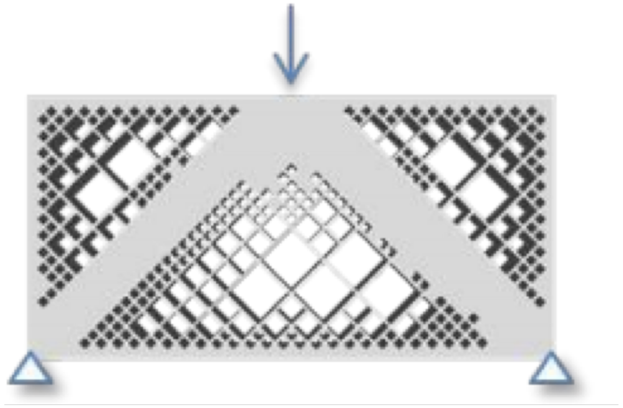


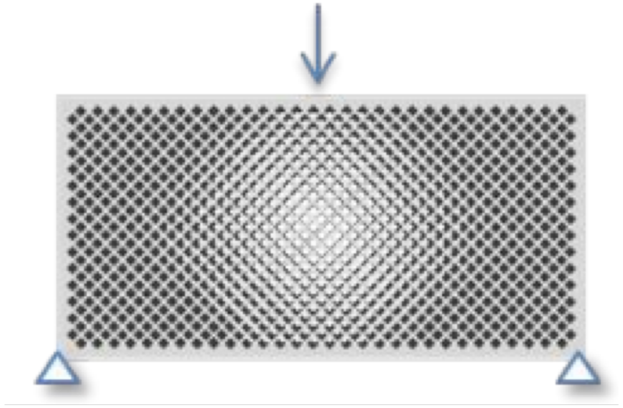
Print



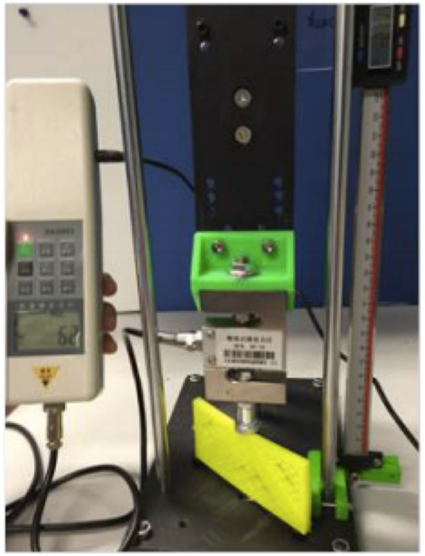
Wu et al., CAD'2016

Mechanical Tests





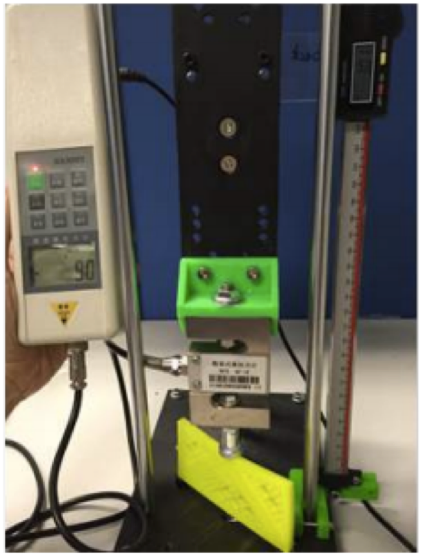
Under same force (62 N)



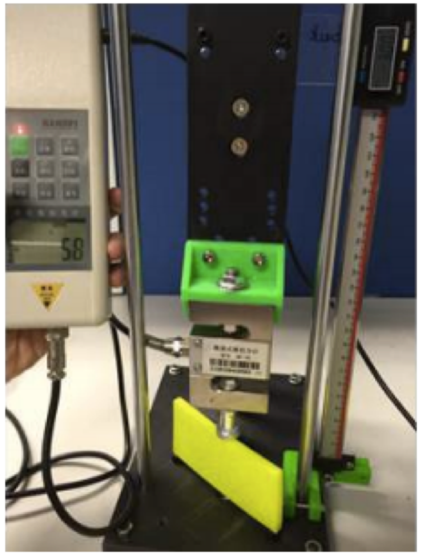
Dis.
2.11 mm

Dis.
4.08 mm

Under same displacement (3.0 mm)



Force
90 N



Force
58 N

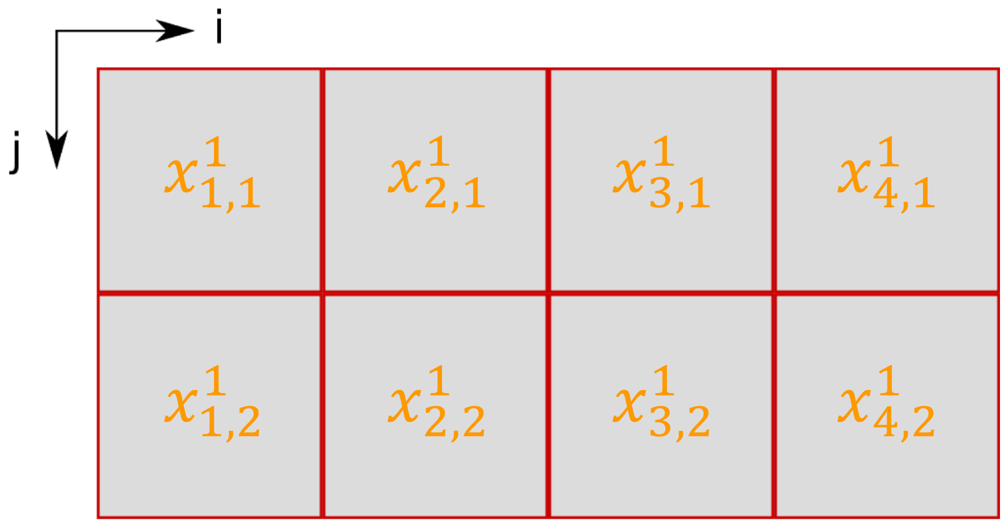
Questions

- Can we re-formulate it as a continuous problem?
- The continuous formulation possibly comes to a more optimal solution?
- To verify how good or bad the greedy approach is.

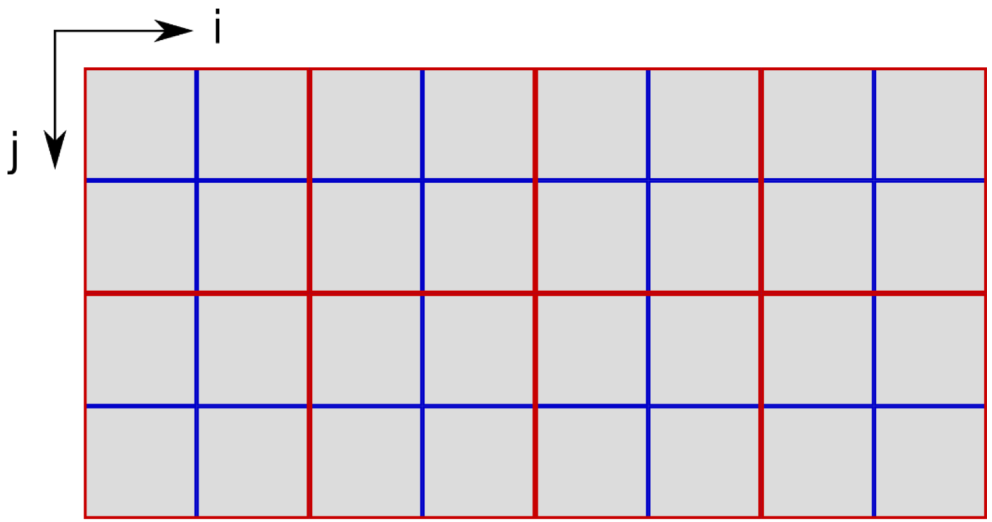
Method

Multi-level Design Variables $x_{i,j}^{k}$

- Design variables on the k-th level $x_{i,j}^{k} \in [0,1]$



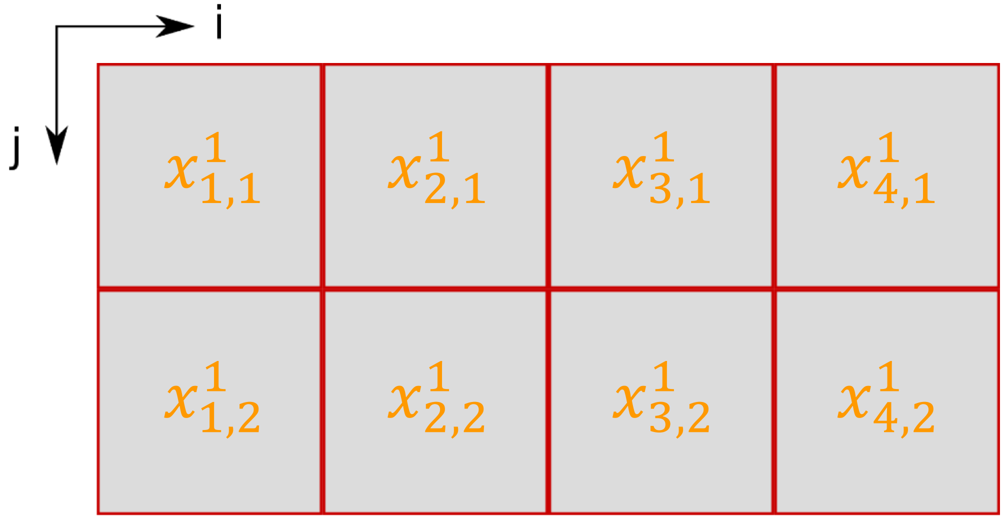
Level 1: 4×2



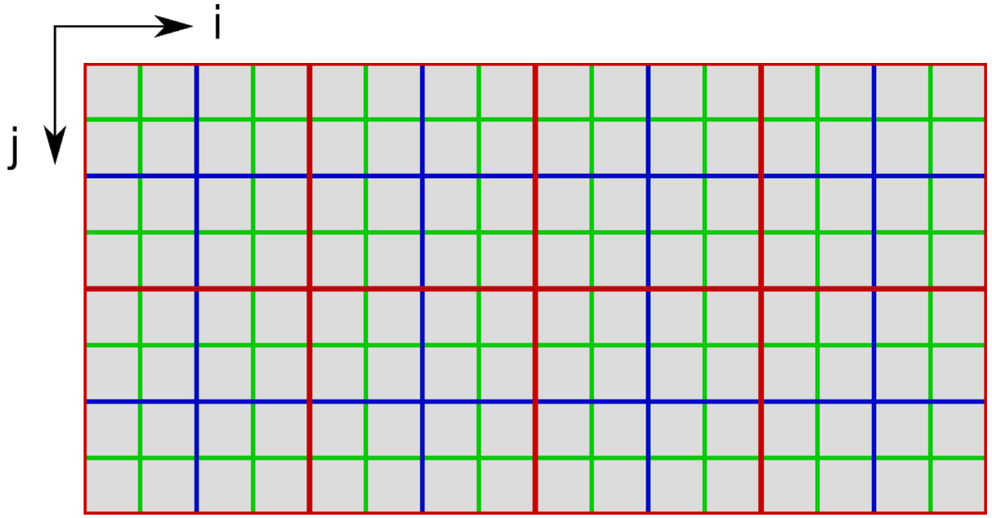
Level 2: 8×4

Multi-level Design Variables $x_{i,j}^k$

- Design variables on the k-th level $x_{i,j}^k \in [0,1]$



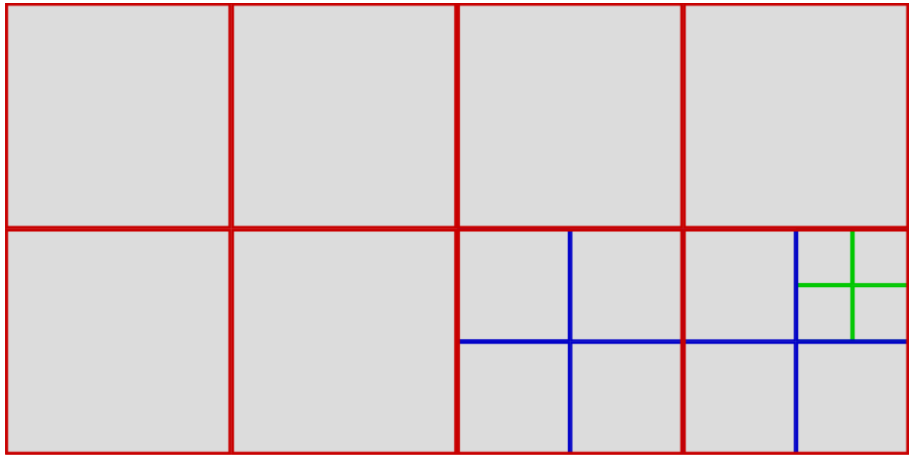
Level 1: 4×2



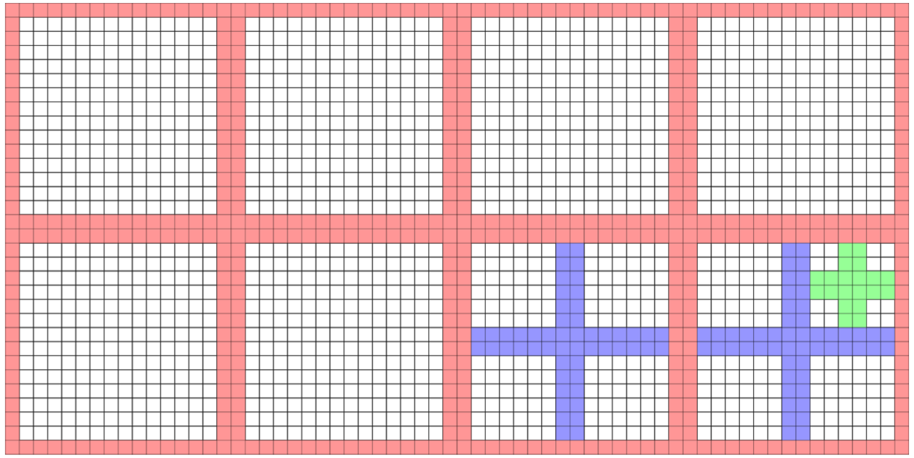
Level 3: 16×8

Finite Elements ρ

- Uniform finite elements, mapped from the quadtree grid
  - Density of corresponding finite elements $\rho = x_{i,j}^{k} \in [0,1]$



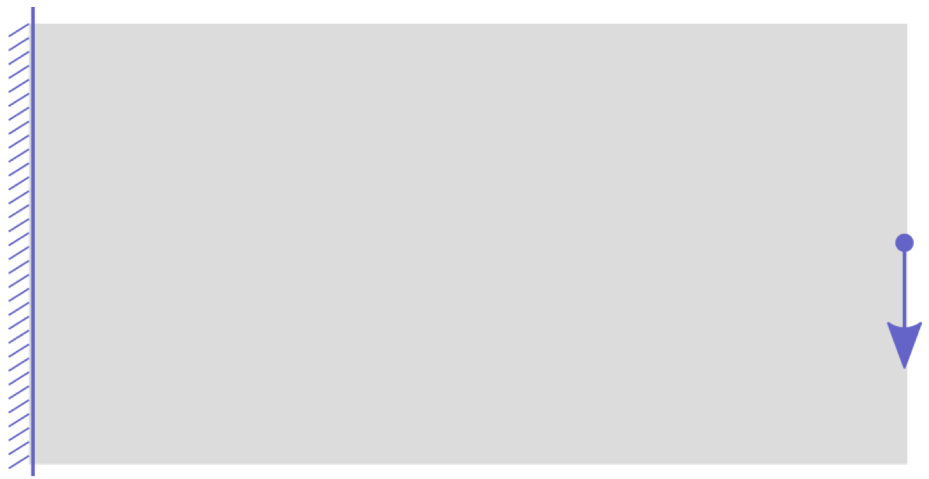
Quadtree grid



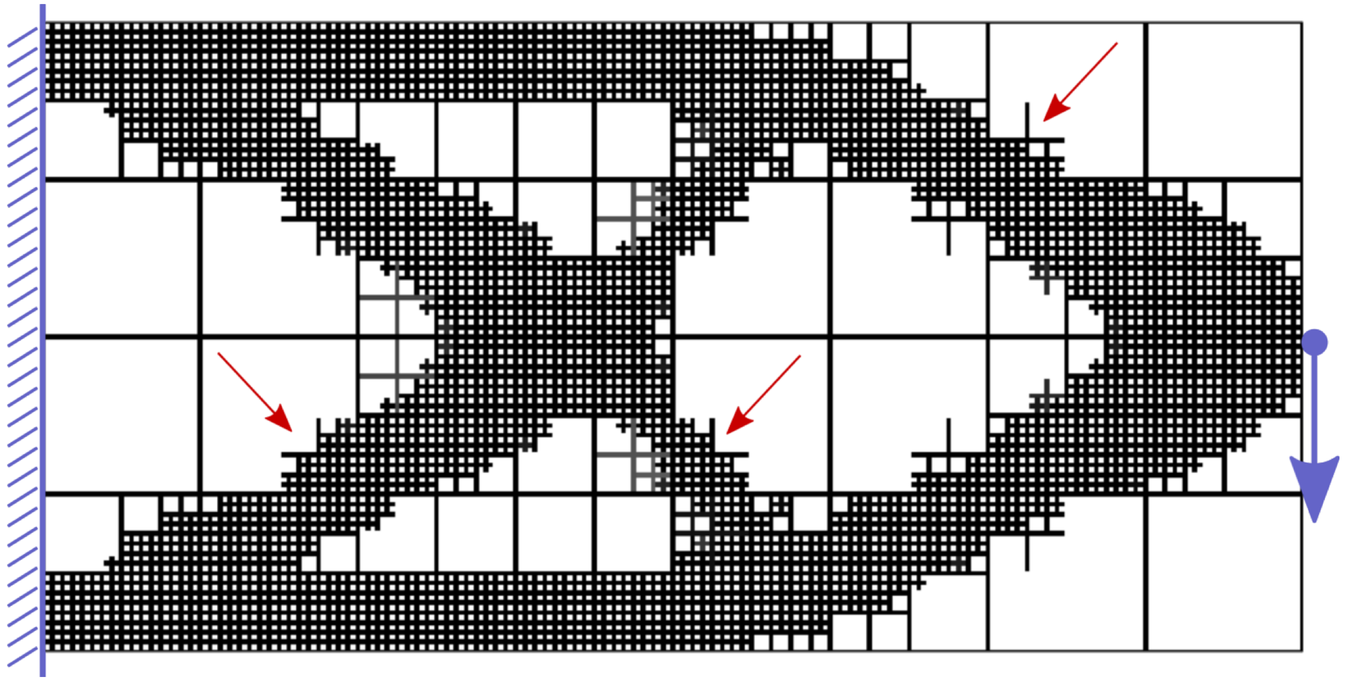
Finite element grid

Optimization Problem: Minimum Compliance

- $\min_{\boldsymbol{x}}: c = \boldsymbol{U}^T \boldsymbol{K}(\boldsymbol{\rho})\boldsymbol{U}$
- s.t. $\boldsymbol{K}(\boldsymbol{\rho})\boldsymbol{U} = \boldsymbol{F}$
- $V(\boldsymbol{\rho}) = \sum_{\forall e} \rho_e v_e \leq V^*$
- $x_{i,j}^k \in [0,1]$
- $\boldsymbol{\rho} = \sum_{k=0}^{\bar{k}} \boldsymbol{T}^k \boldsymbol{x}^k$

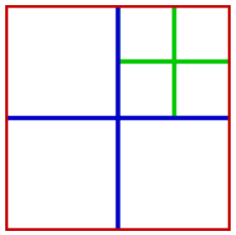


Result 1

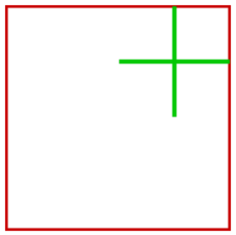


Refinement Filter

- Dependency among two levels $\tilde{x}^{k}_{i,j} = \min(x^{k}_{i,j}, x^{k-1}_{i-1,j-1})$



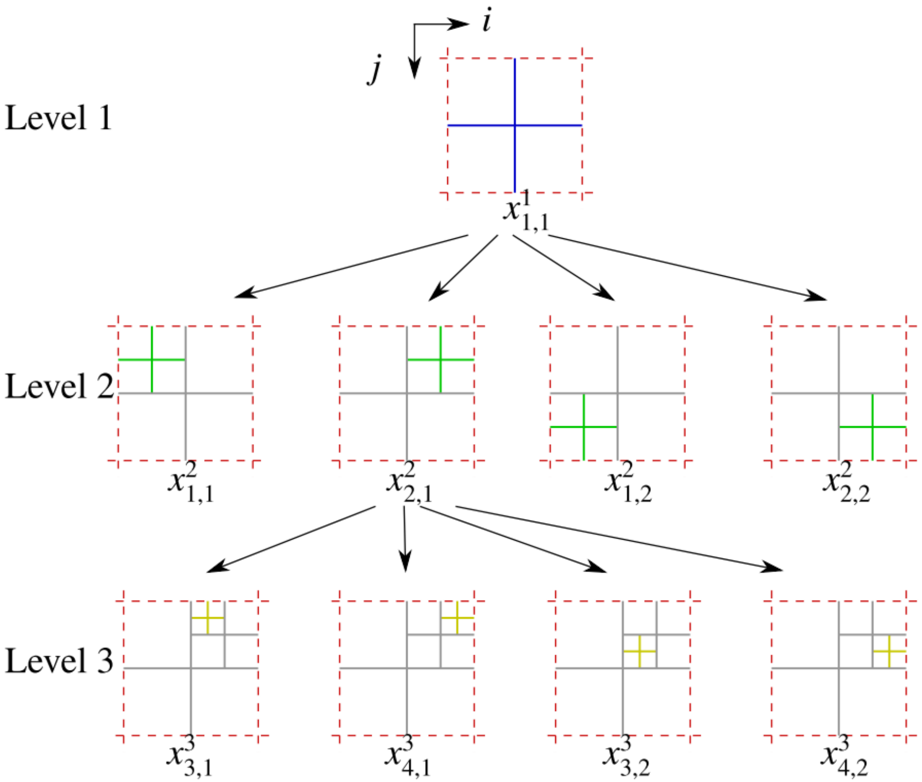
Valid refinement



Invalid refinement

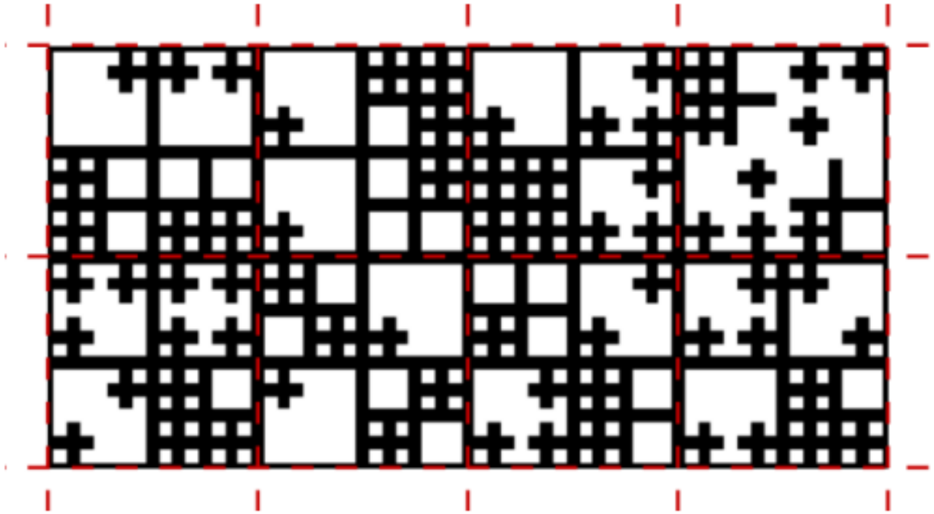
Refinement Filter

- Dependency among two levels $\tilde{x}_{i,j}^{k} = \min(x_{i,j}^{k}, x_{i-1,j-1}^{k-1})$
- Among multiple levels $\tilde{x}_{i,j}^{k} = \min(x_{i,j}^{k}, x_{i-1,j-1}^{k-1}, \ldots, x_{i-k+1,j-k+1}^{1})$

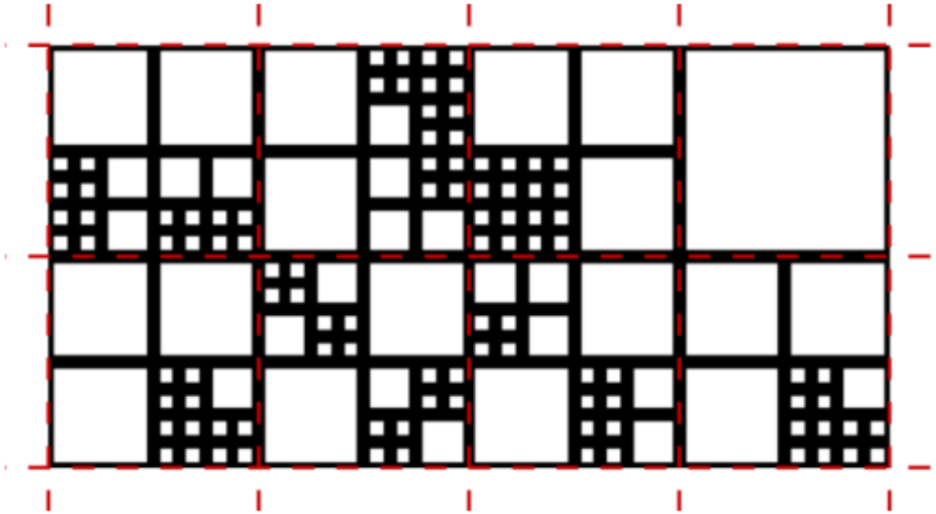


Refinement Filter

- Dependency among two levels $\tilde{x}_{i,j}^{k} = \min(x_{i,j}^{k}, x_{i-1,j-1}^{k-1})$
- Among multiple levels $\tilde{x}_{i,j}^{k} = \min(x_{i,j}^{k}, x_{i-1,j-1}^{k-1}, \ldots, x_{i-k+1,j-k+1}^{1})$



Invalid refinement



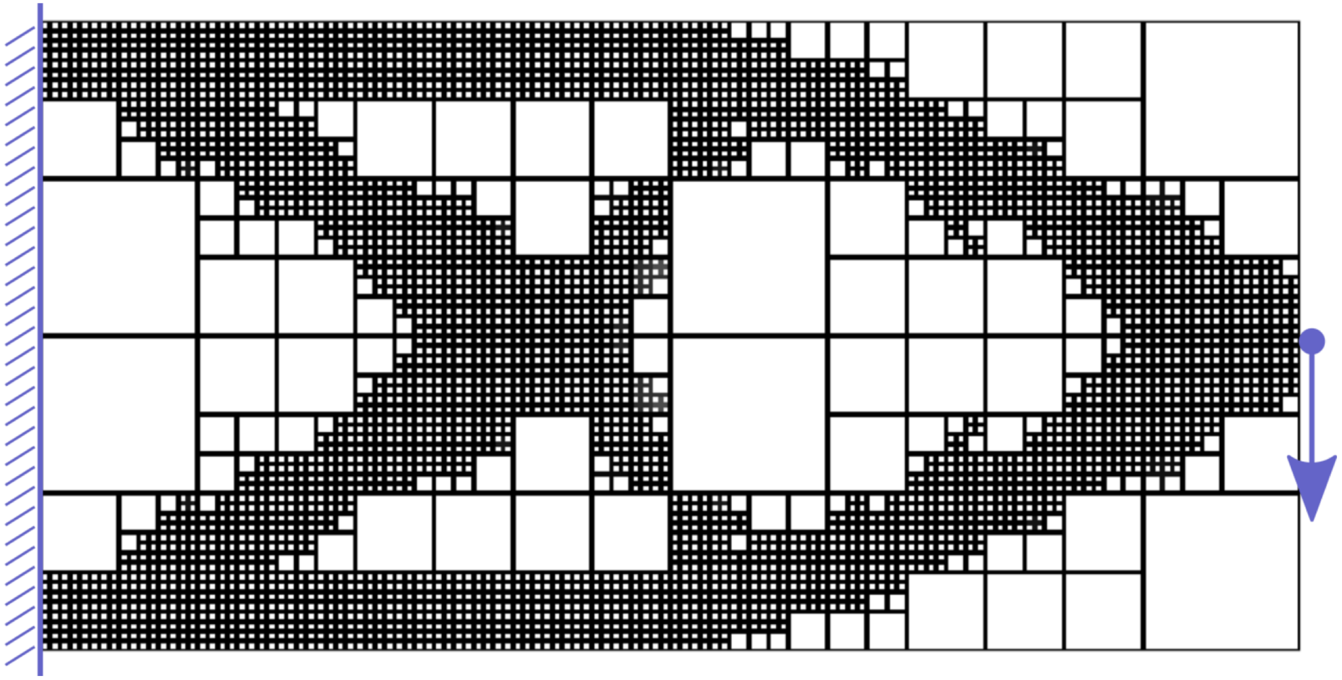
Valid refinement

Refinement Filter

- Dependency among two levels $\tilde{x}_{i,j}^{k} = \min(x_{i,j}^{k}, x_{i-1,j-1}^{k-1})$
- Among multiple levels $\tilde{x}_{i,j}^{k} = \min(x_{i,j}^{k}, x_{i-1,j-1}^{k-1}, \dots, x_{i-k+1,j-k+1}^{1})$
- Continuous approximation

$$\tilde{x}_{i,j}^{k} \approx \left\| \left(x_{i,j}^{k}, x_{i-1,j-1}^{k-1}, \dots, x_{i-k+1,j-k+1}^{1} \right) \right\|_{p_n}$$

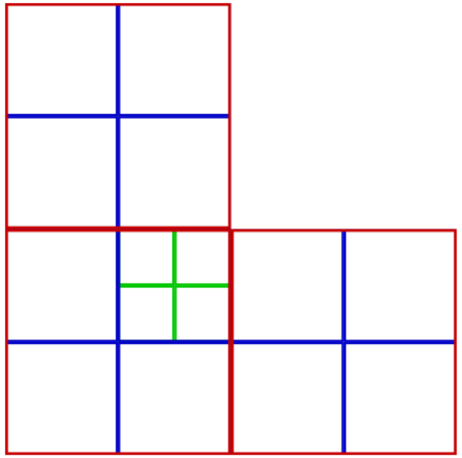
Result 2



Refinement Filter: Balanced Quadtree

- Dependency among two levels $\tilde{x}_{i,j}^{k} = \min(x_{i,j}^{k}, x_{i^{-1},j^{-1}}^{k-1})$
- Dependency among two levels for balanced quadtree

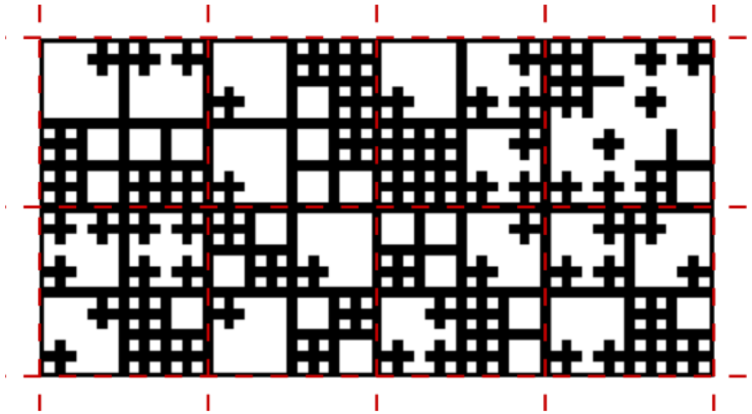
$$\tilde{x}_{i,j}^{k} = \min(x_{i,j}^{k}, x_{i^{-1},j^{-1}}^{k-1}, x_{i^{-1}\pm 1,j^{-1}\pm 1}^{k-1})$$



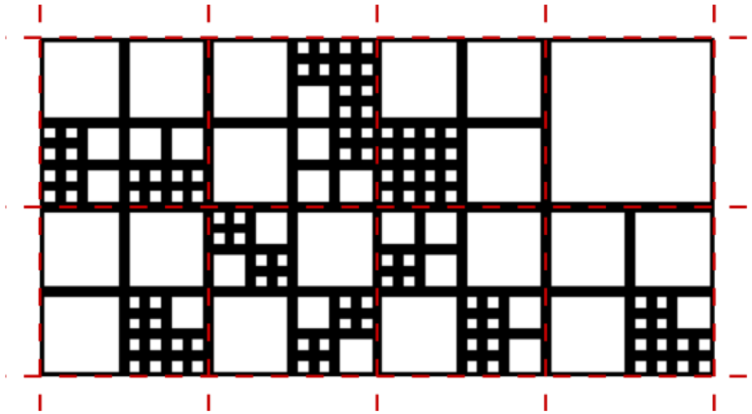
Refinement Filter: Balanced Quadtree

- Dependency among two levels $\tilde{x}_{i,j}^{k} = \min(x_{i,j}^{k}, x_{i^{-1},j^{-1}}^{k-1})$
- Dependency among two levels for balanced quadtree

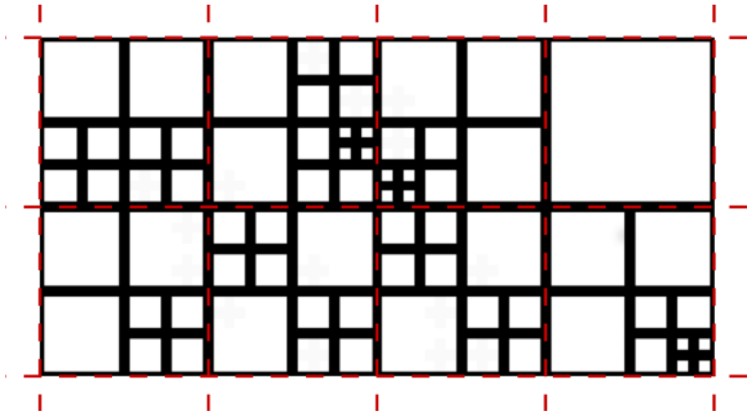
$$\tilde{x}_{i,j}^{k} = \min(x_{i,j}^{k}, x_{i^{-1},j^{-1}}^{k-1}, x_{i^{-1}\pm 1,j^{-1}\pm 1}^{k-1})$$



Invalid refinement

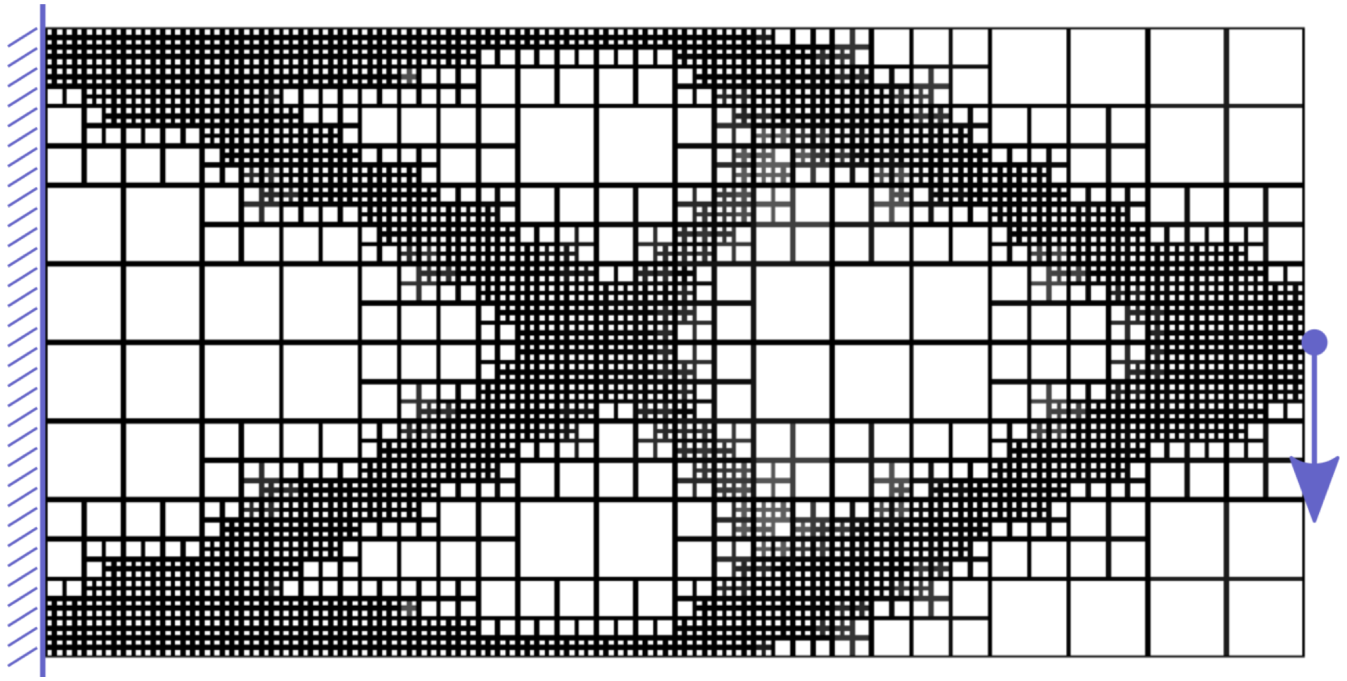


Quadtree refinement

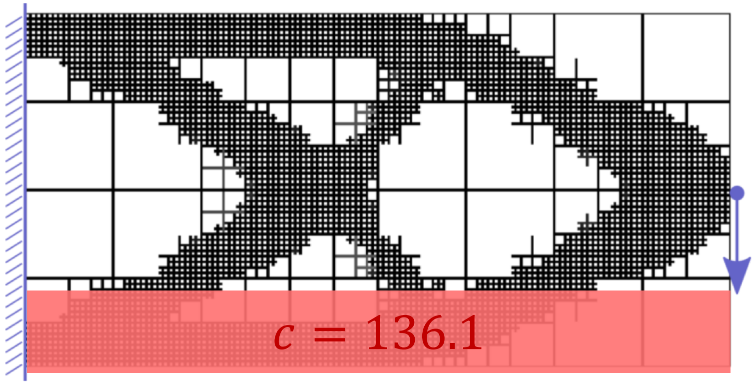


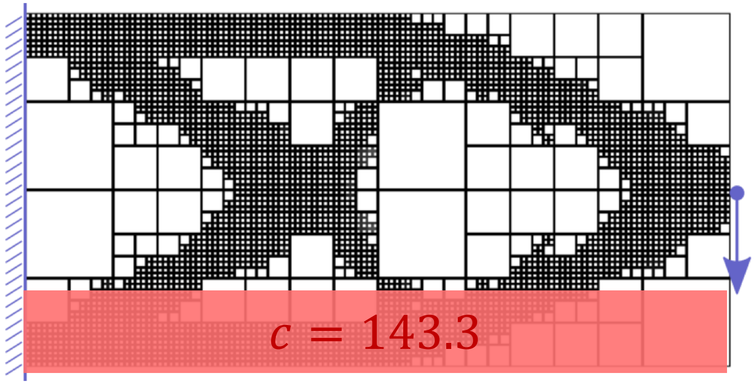
Balanced quadtree

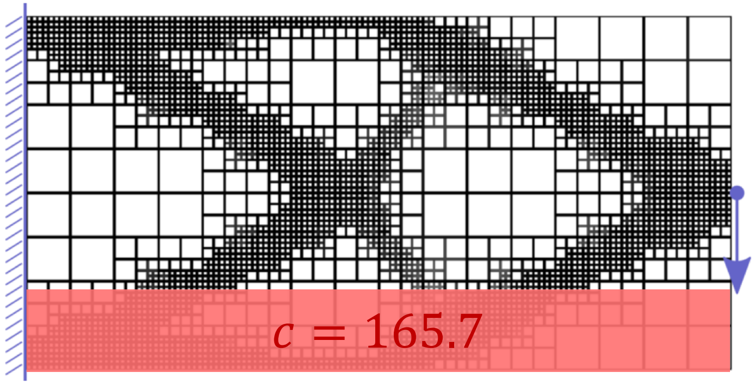
Refinement Filter: Balanced Quadtree



Comparison: Designed force

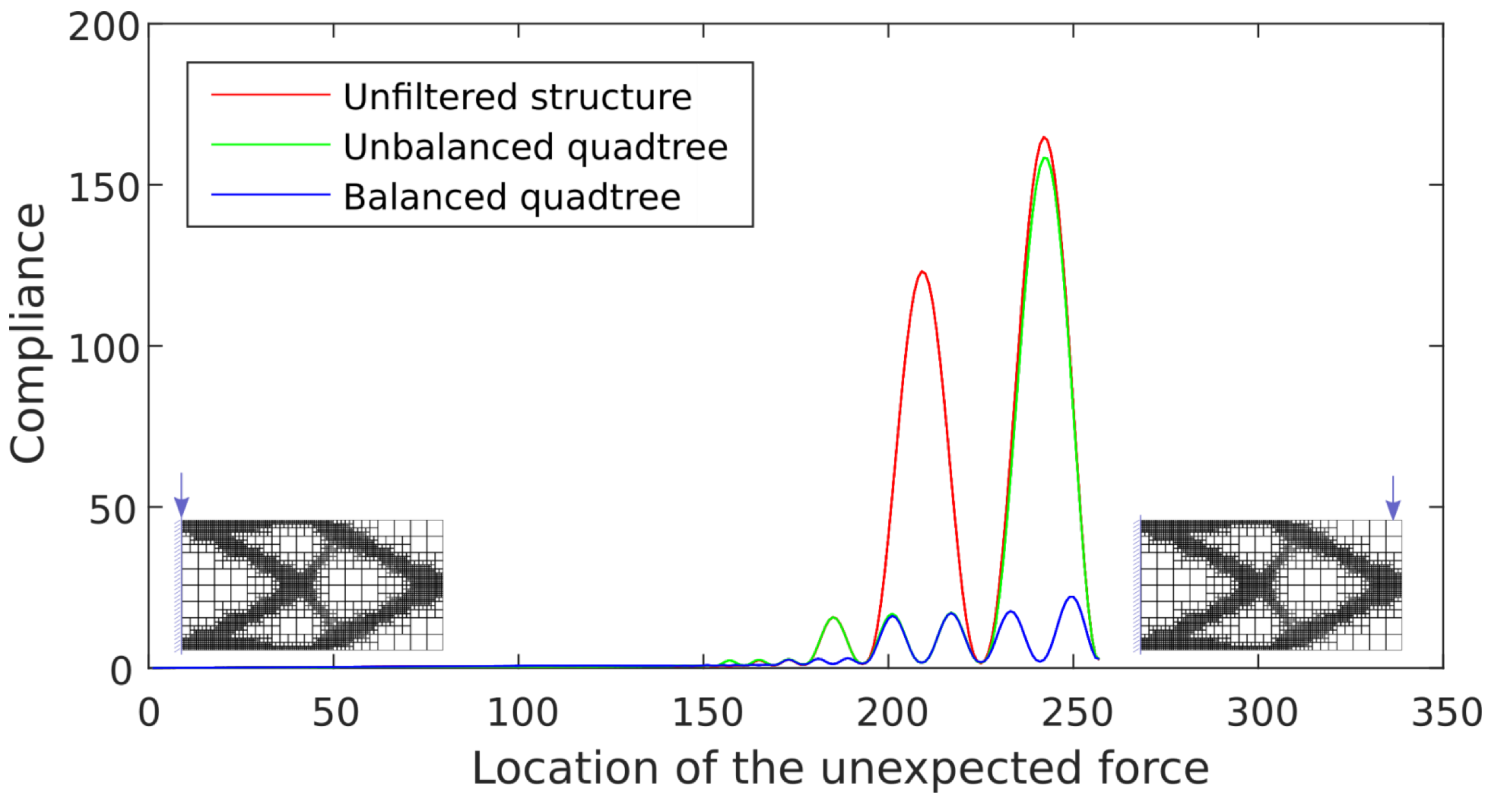






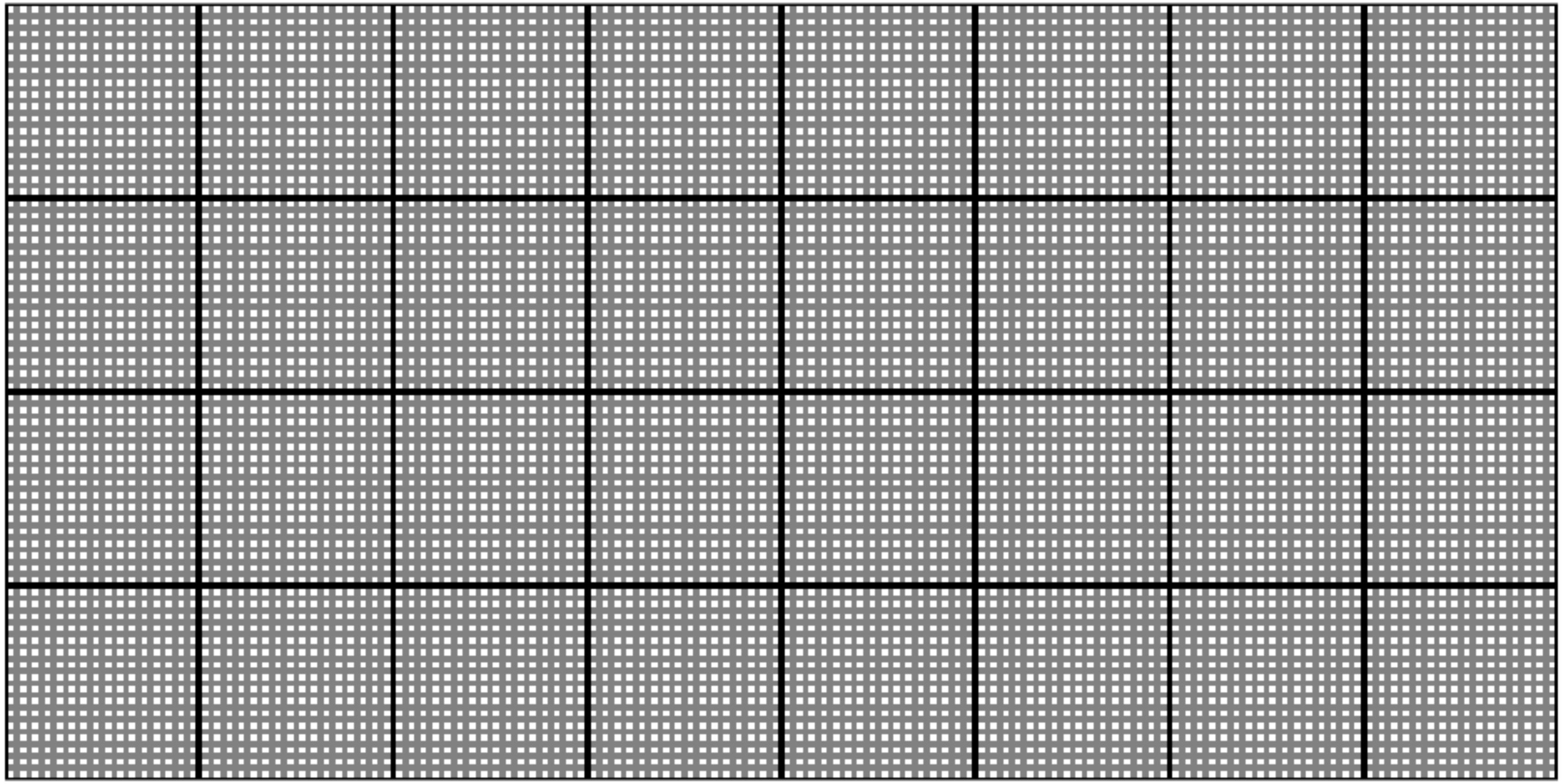
Unexpected force

- Quadtree is robust to unexpected force

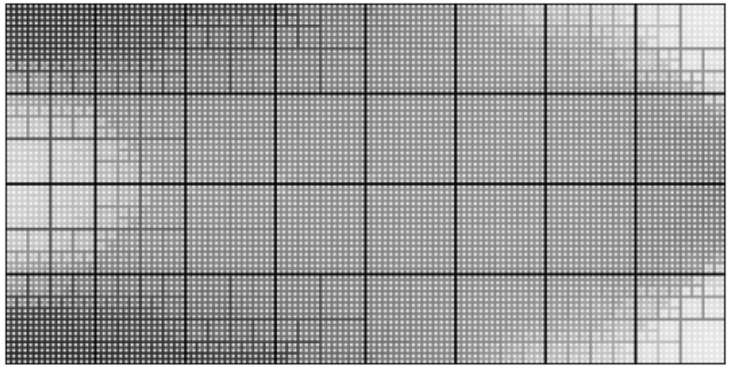


Results

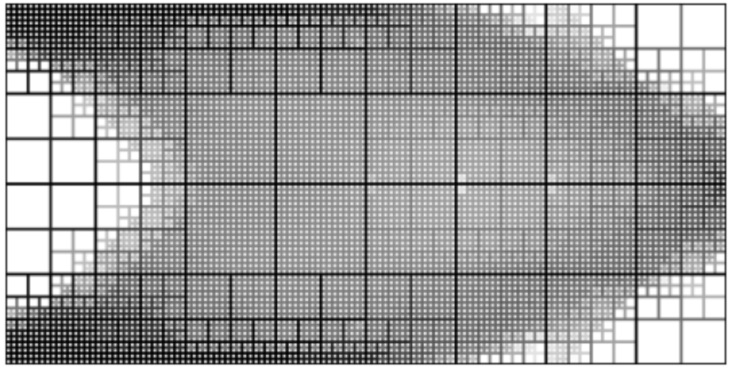
Results: Animation



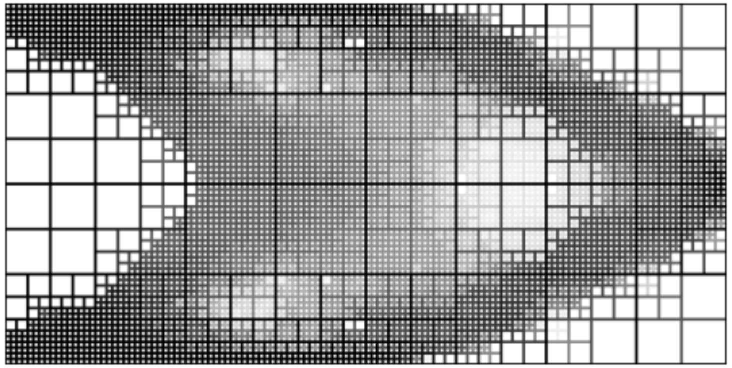
Results: Convergence



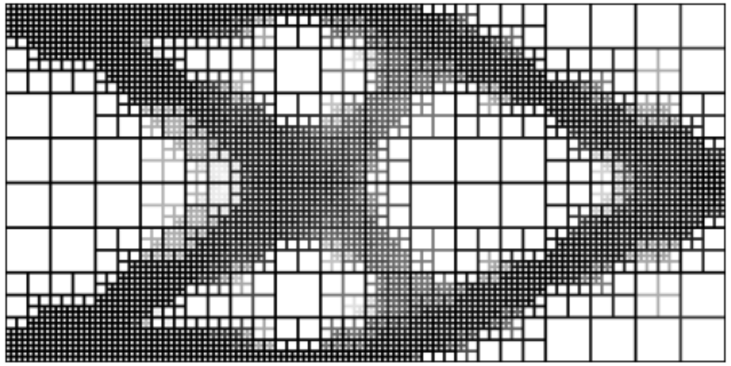
(a) It. 40, c = 317.2, s = 0.631



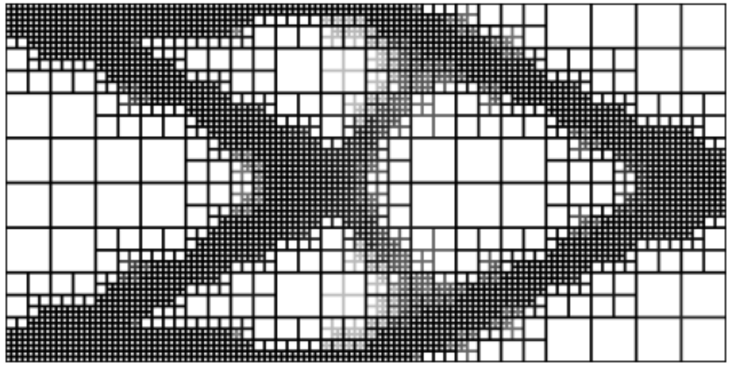
(b) It. 100, c = 249.8, s = 0.500



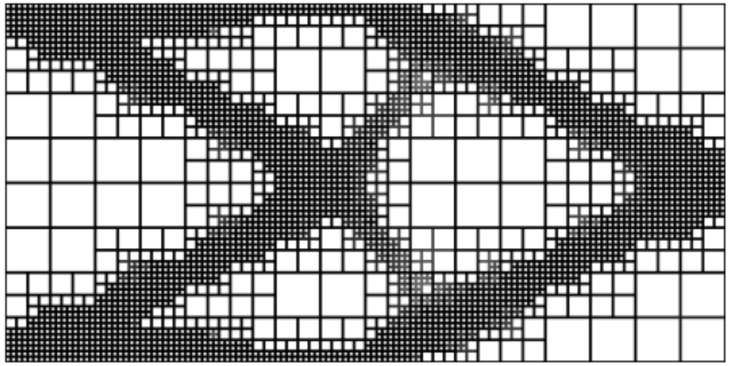
(c) It. 160, c = 215.4, s = 0.362



(d) It. 220, c = 177.9, s = 0.151



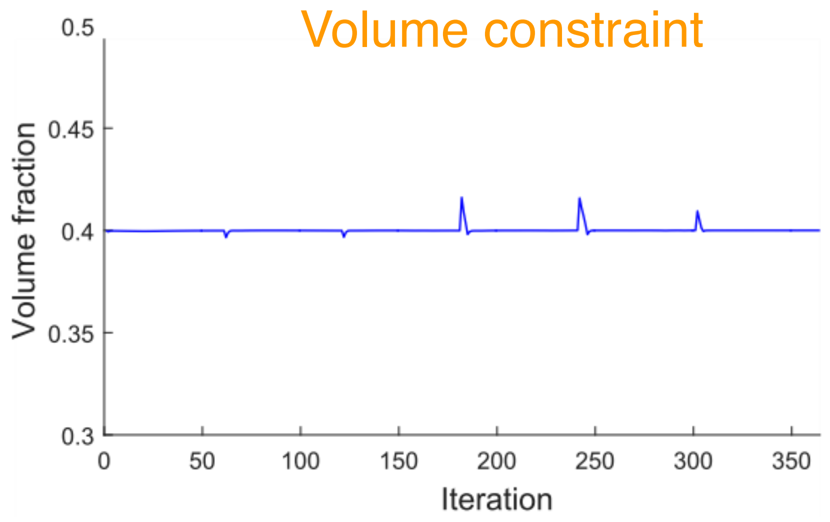
(e) It. 280, c = 169.2, s = 0.070

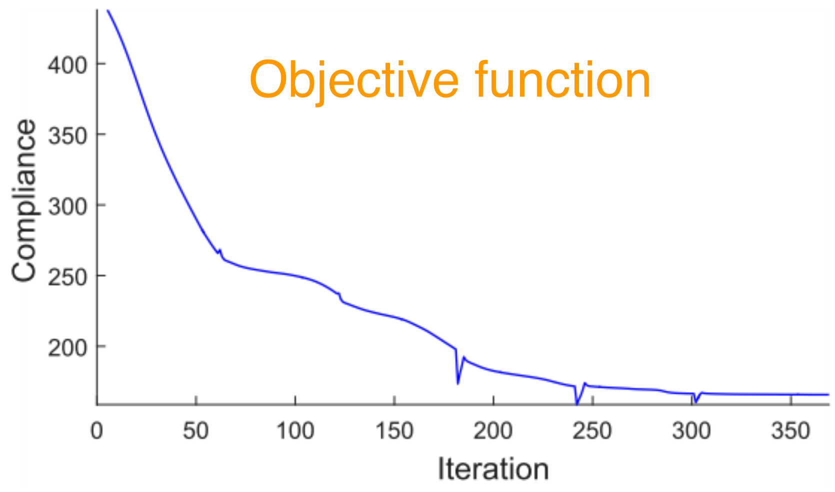


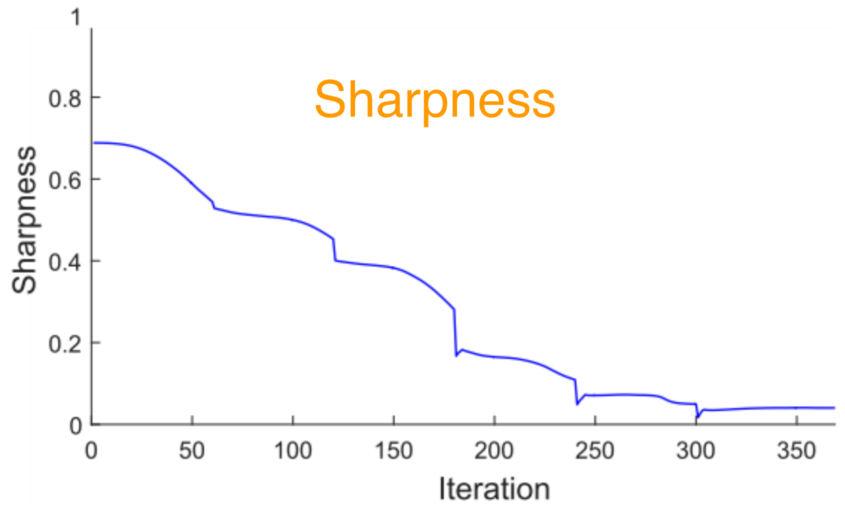
(f) It. 360, c = 165.7, s = 0.040

Results: Convergence

- Objective $\min_{\boldsymbol{x}}: c = \boldsymbol{U}^T \boldsymbol{K}(\boldsymbol{\rho})\boldsymbol{U}$
- Volume $V(\boldsymbol{\rho}) = \sum_{\forall e} \rho_e v_e \leq V^*$
- Sharpness $s = \frac{4}{n} \sum_e (\rho_e(1 - \rho_e))$

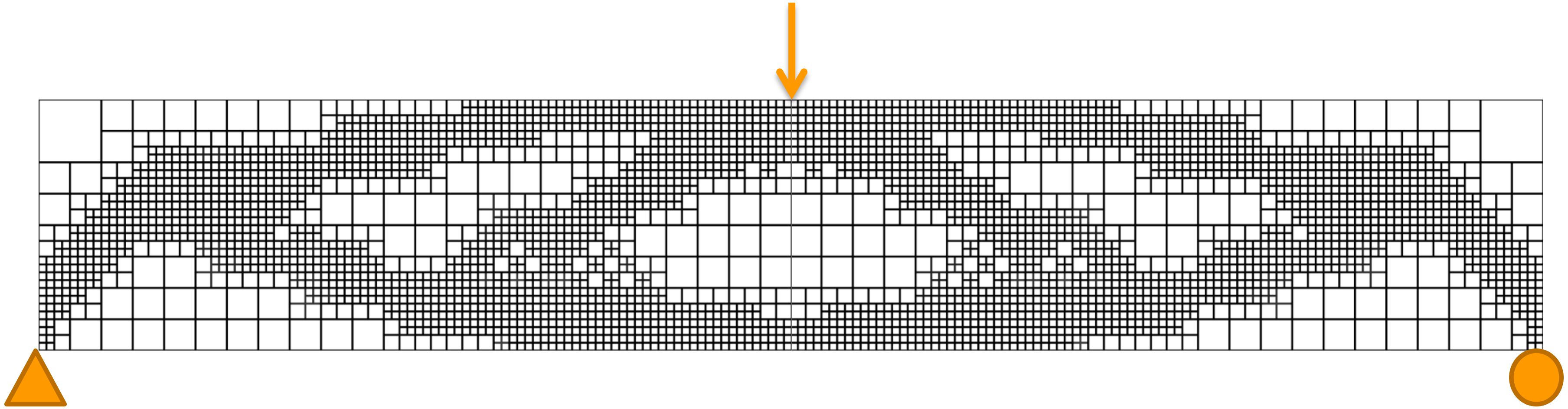


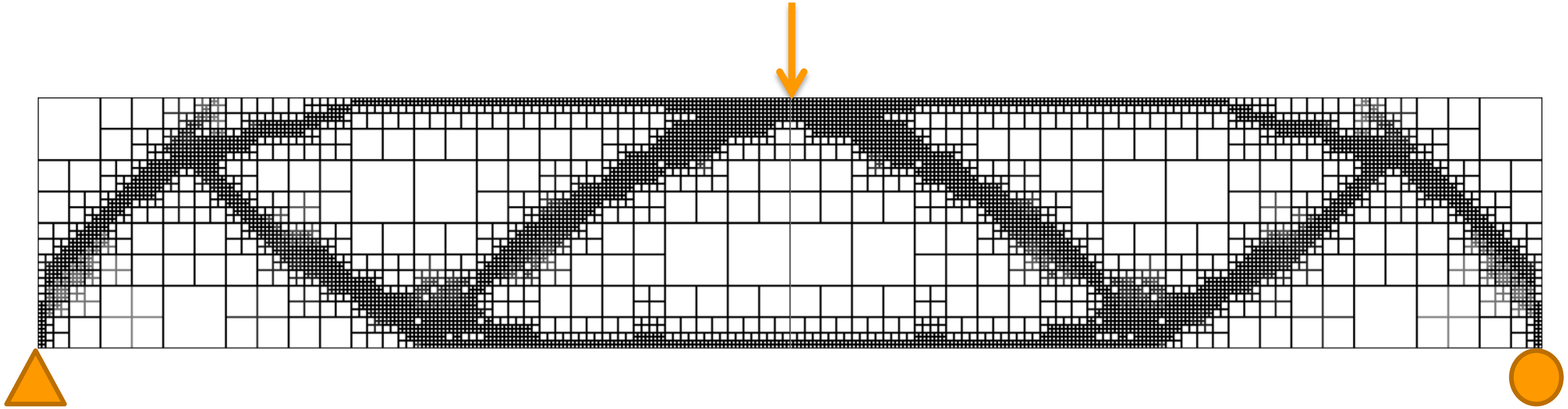




Results: Feature Size

- Control feature size by allowing different refinement levels

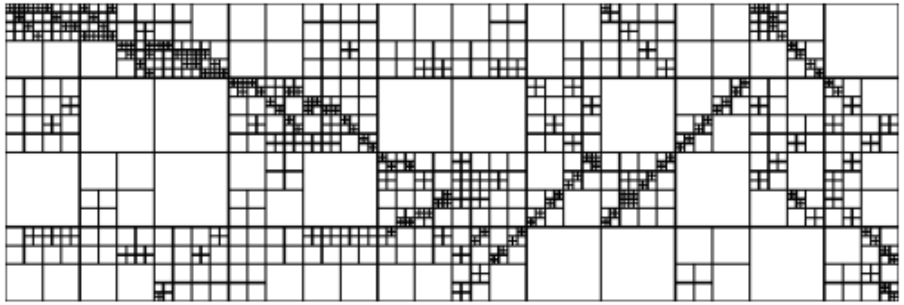




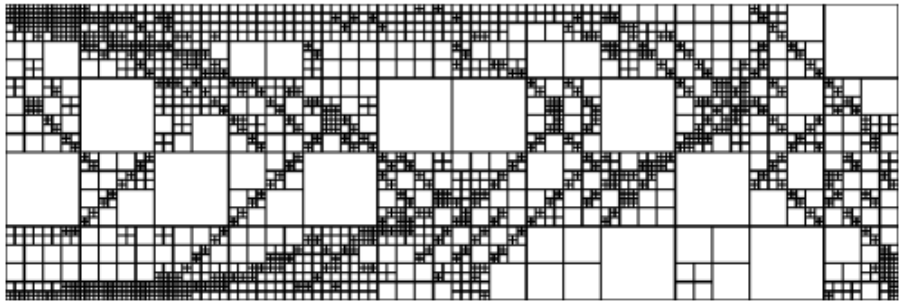
Results: Comparison

- Continuous optimization achieves more optimal solution than the heuristic approach

Greedy approach

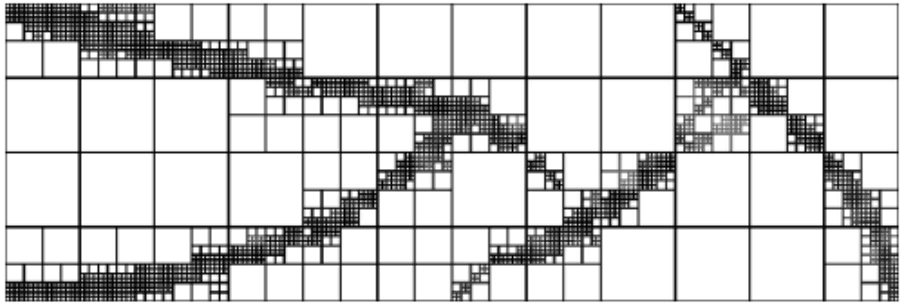


$v_f = 0.20 \quad c = 1919.9$

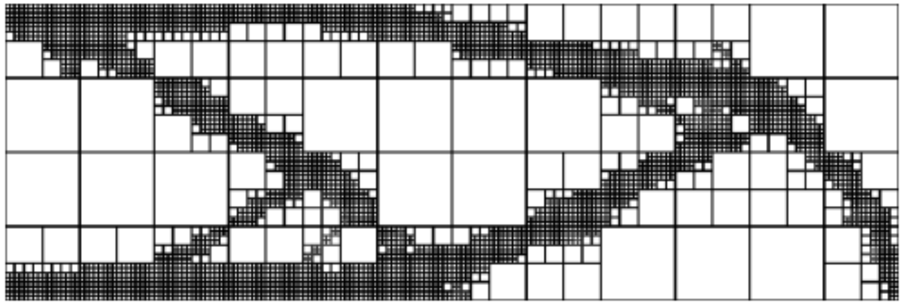


$v_f = 0.30 \quad c = 742.5$

Continuous optimization

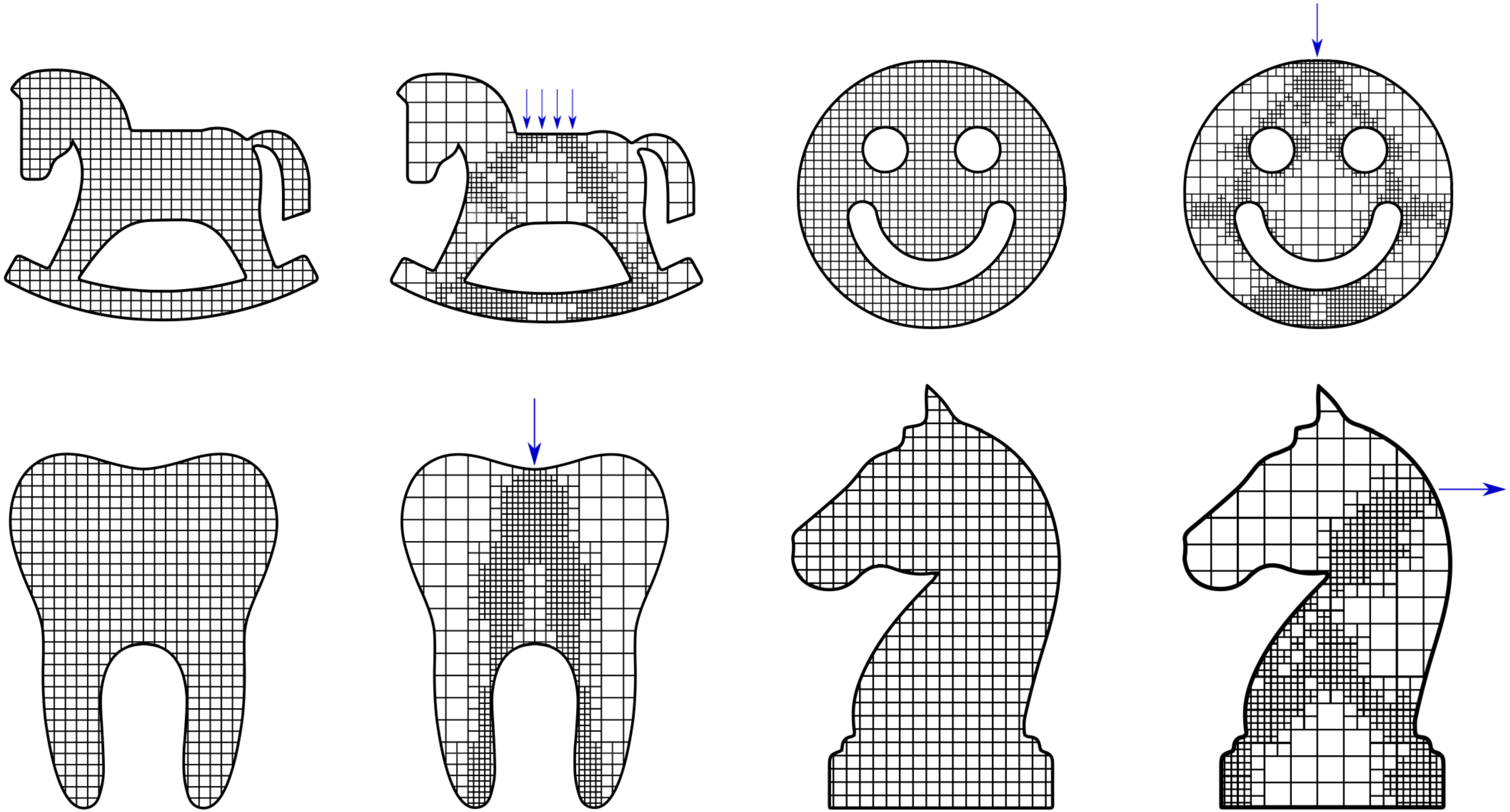


$v_f = 0.20 \quad c = 1266.7$

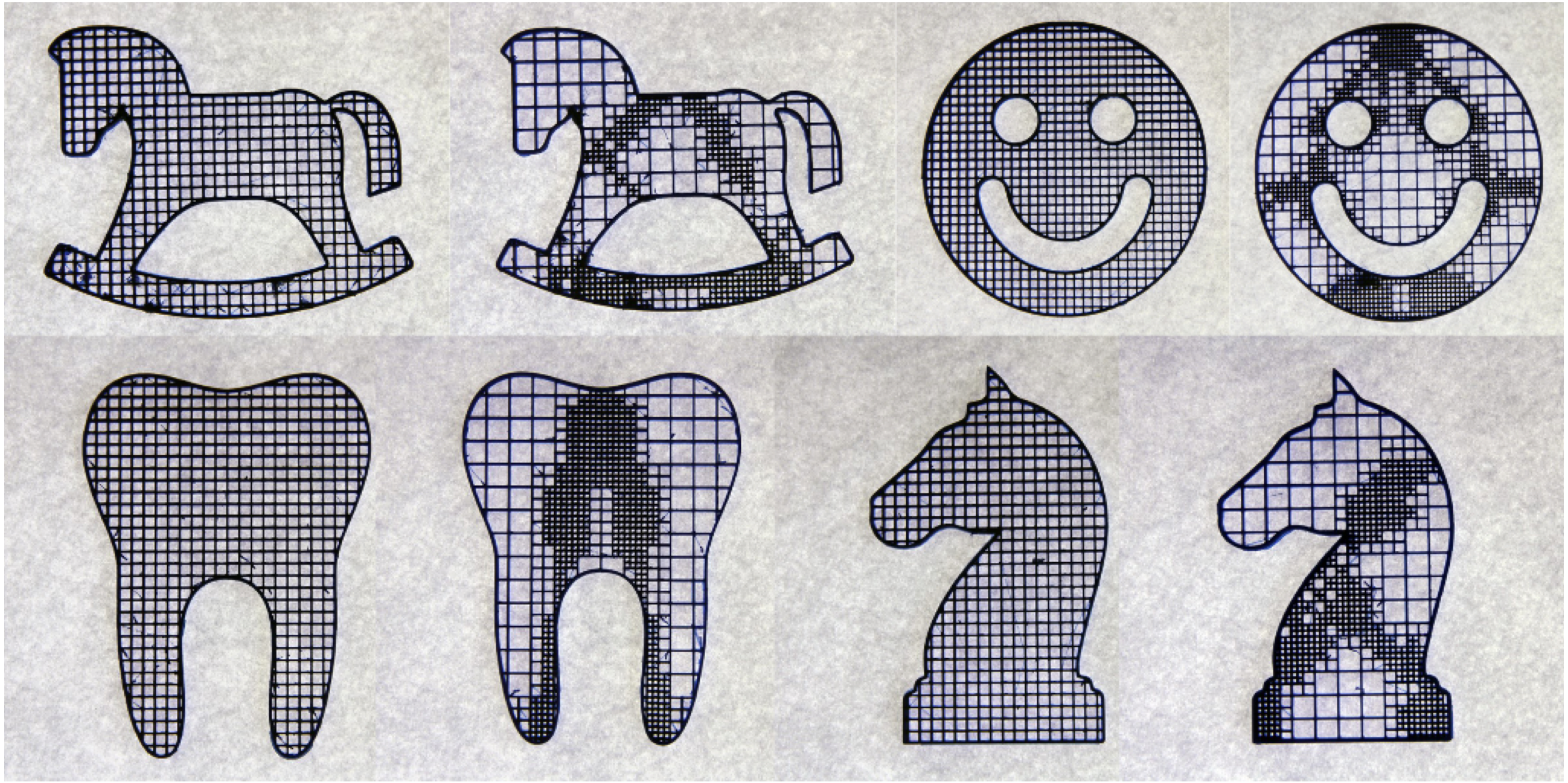


$v_f = 0.30 \quad c = 571.6$

Results: Fabrication

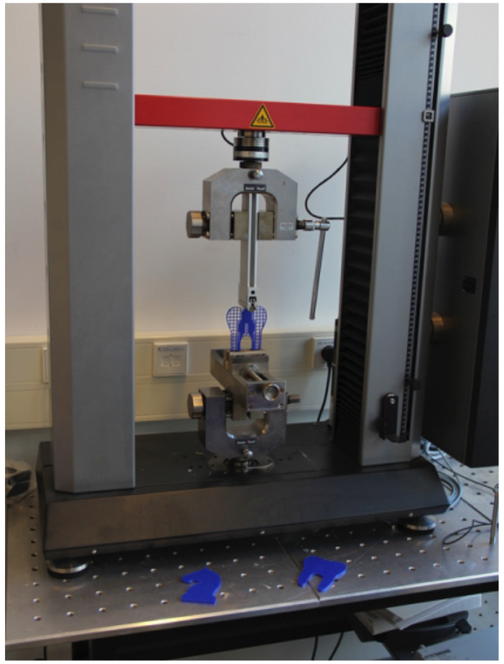


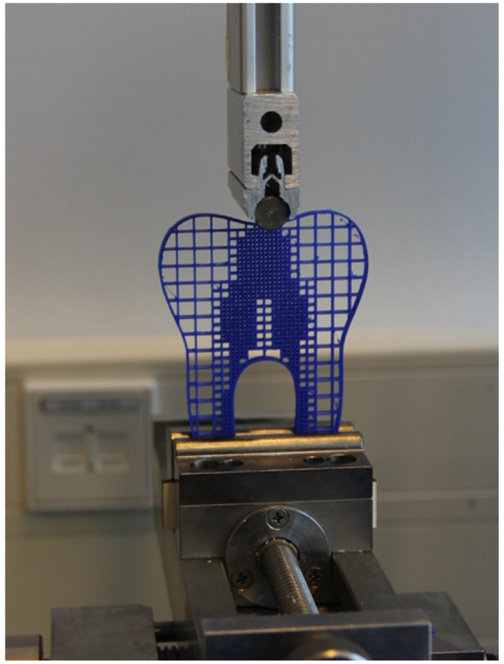
Results: Fabrication

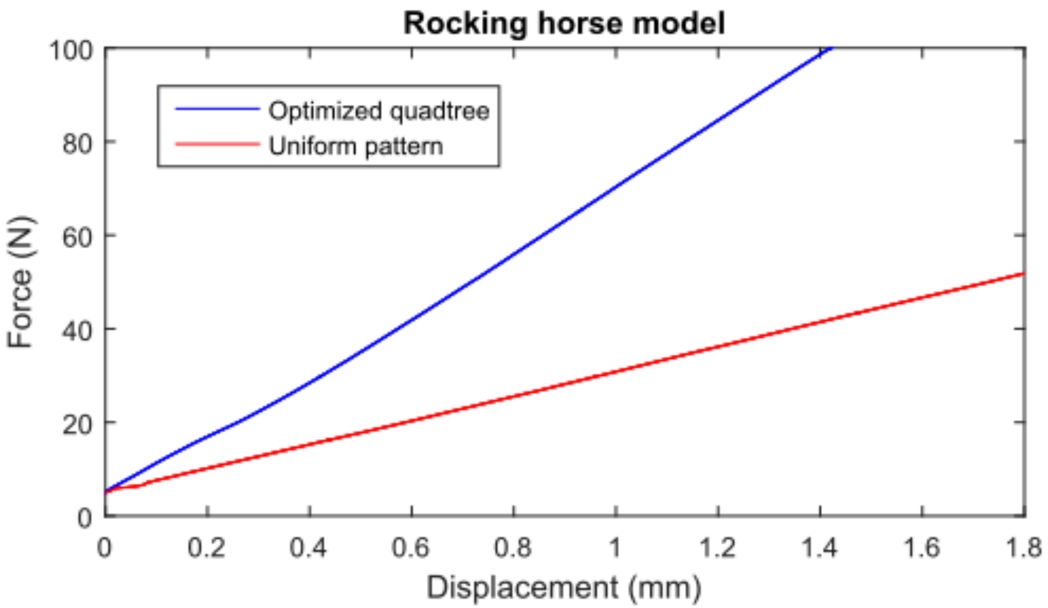


Results: Fabrication

- Adaptively refined infill is much stiffer than uniform infill

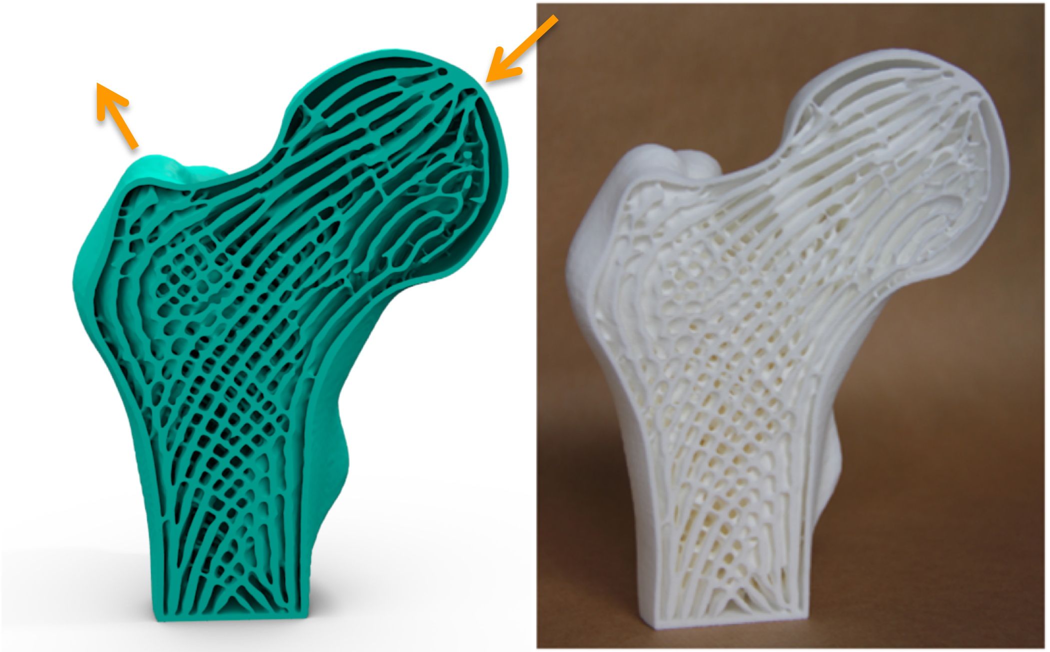




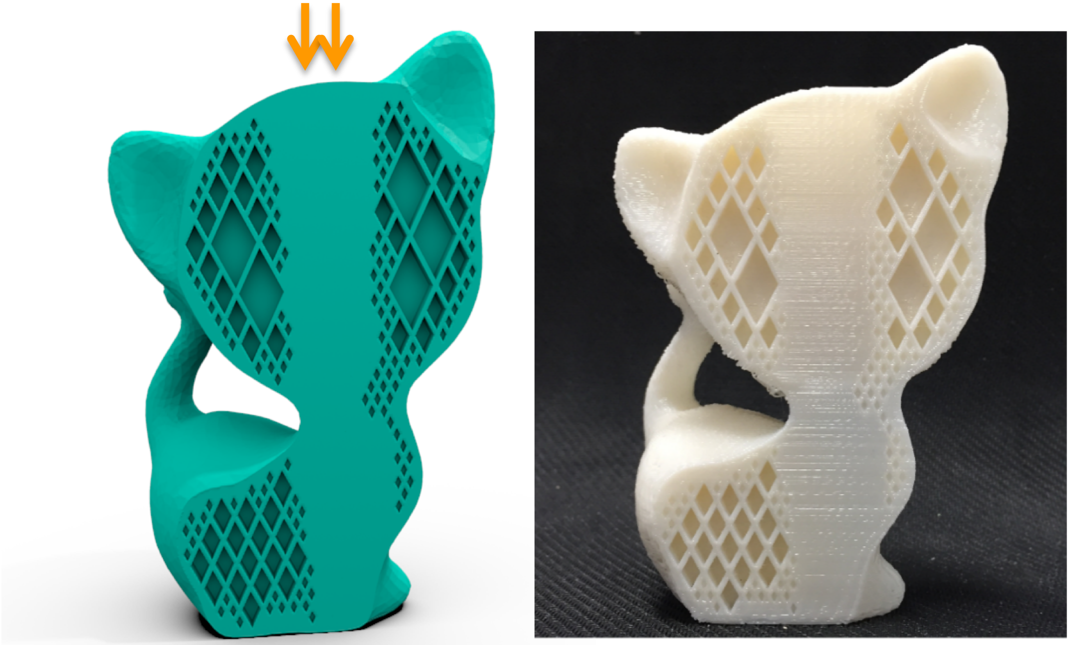


Outline

- Basics of Topology Optimization
- Topology Optimization for Additive Manufacturing
  - Geometric feature control by **density filters**
  - Geometric feature control by **alternative parameterizations**



Bone-inspired infill



Self-supporting infill

Topology Optimization

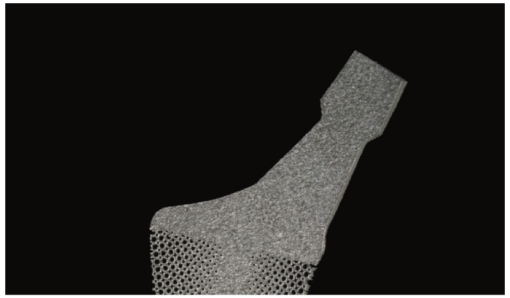
- Lightweight
- Free-form shape
- Customization
- Mechanically optimized
- …

Additive Manufacturing

- Customization
- Geometric complexity
- …

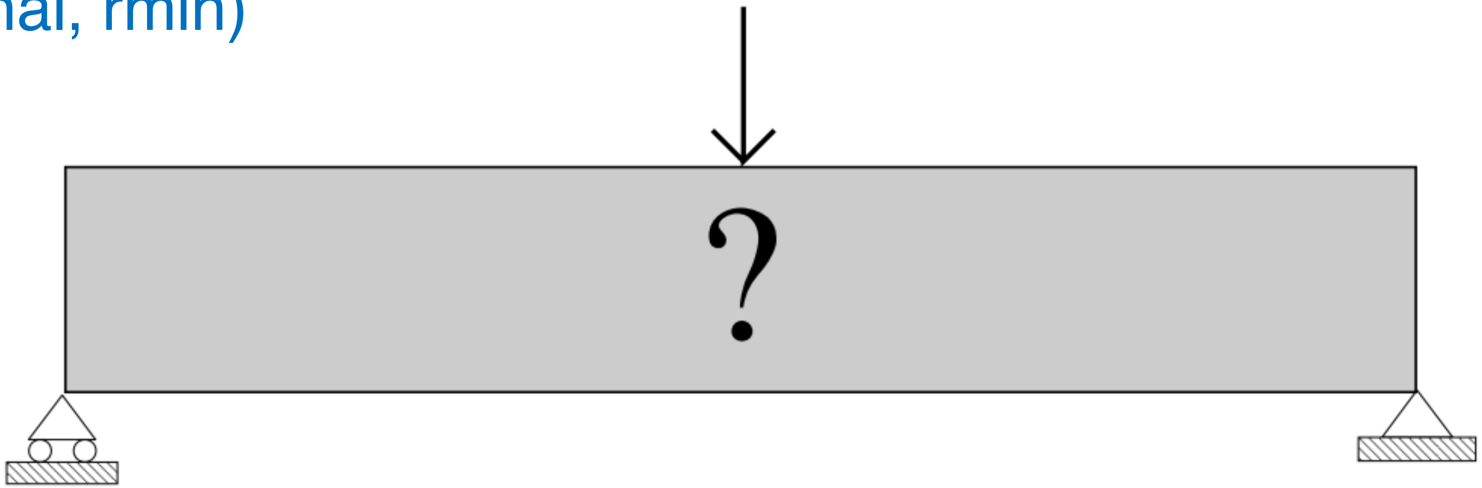
Research topics

- Multiscale structural optimization
  - Homogenization
- Integrating manufacturing constraints (and process simulation)
  - Overhang, process planning, toolpath, anisotropic behavior etc
- Novel applications -> unconventional design objectives
  - Soft robotics, implants, 4d printing, shape morphing etc
- Optimality, convergence, stress constraint etc
- Geometric perspective in topology optimization
  - Alternative parameterizations for elasticity analysis
  - Alternative design parameterizations
  - Meshing for designing lattice structures

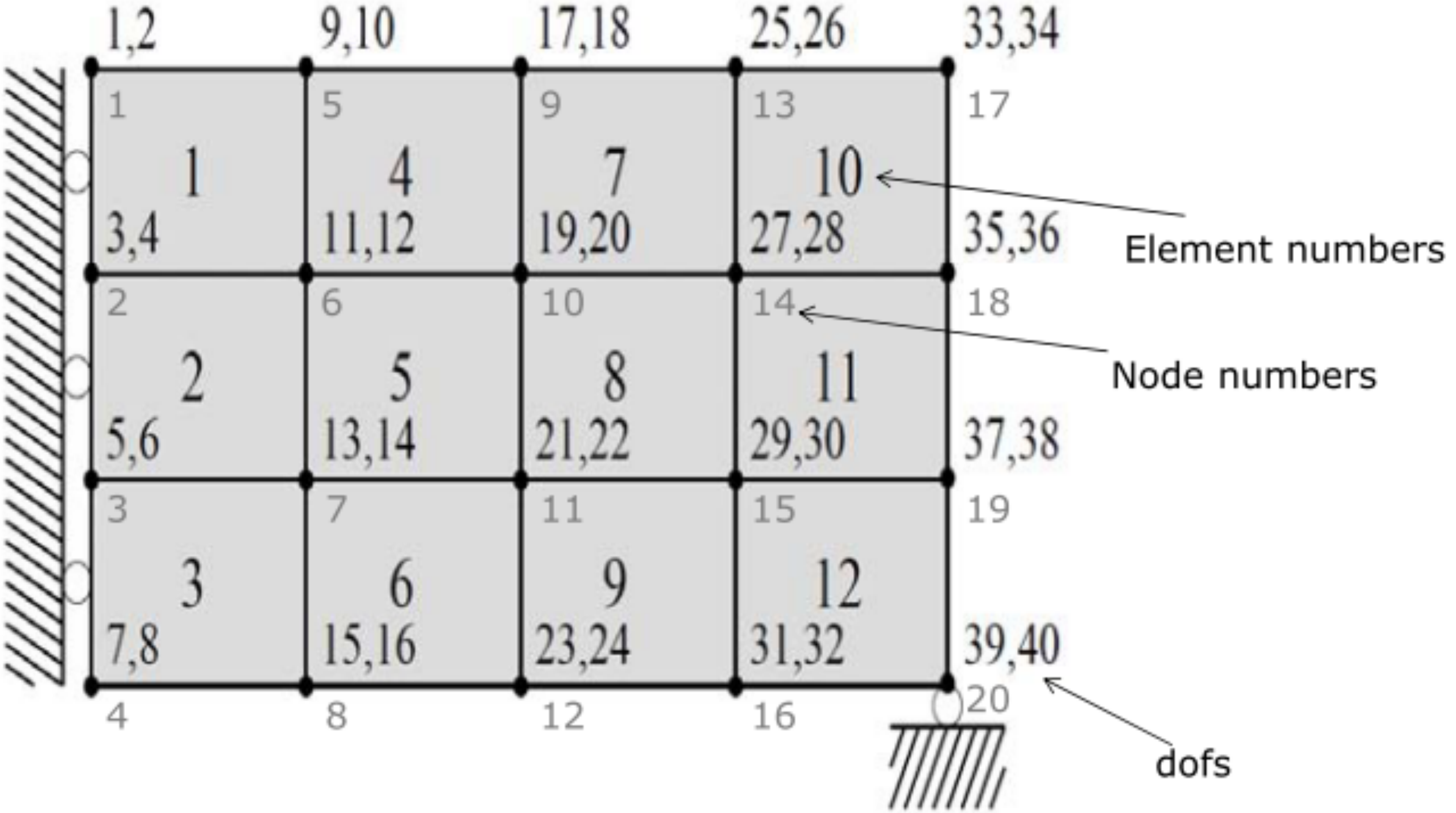


Matlab code

- Download top.m or top88.m
  - www.topopt.dtu.dk -> Choose Applets and Software
  - Select “Matlab Program” (top.m) or “New Matlab Program” (top88.m)
- Start Matlab, and run the default MMB-example
- top88(nelx, nely, volfrac, penal, rmin, ft)
  - top88(80, 40, 0.3, 3, 1.6, 1)
- top(nelx, nely, volfrac, penal, rmin)
  - top(80, 40, 0.3, 3, 1.6)

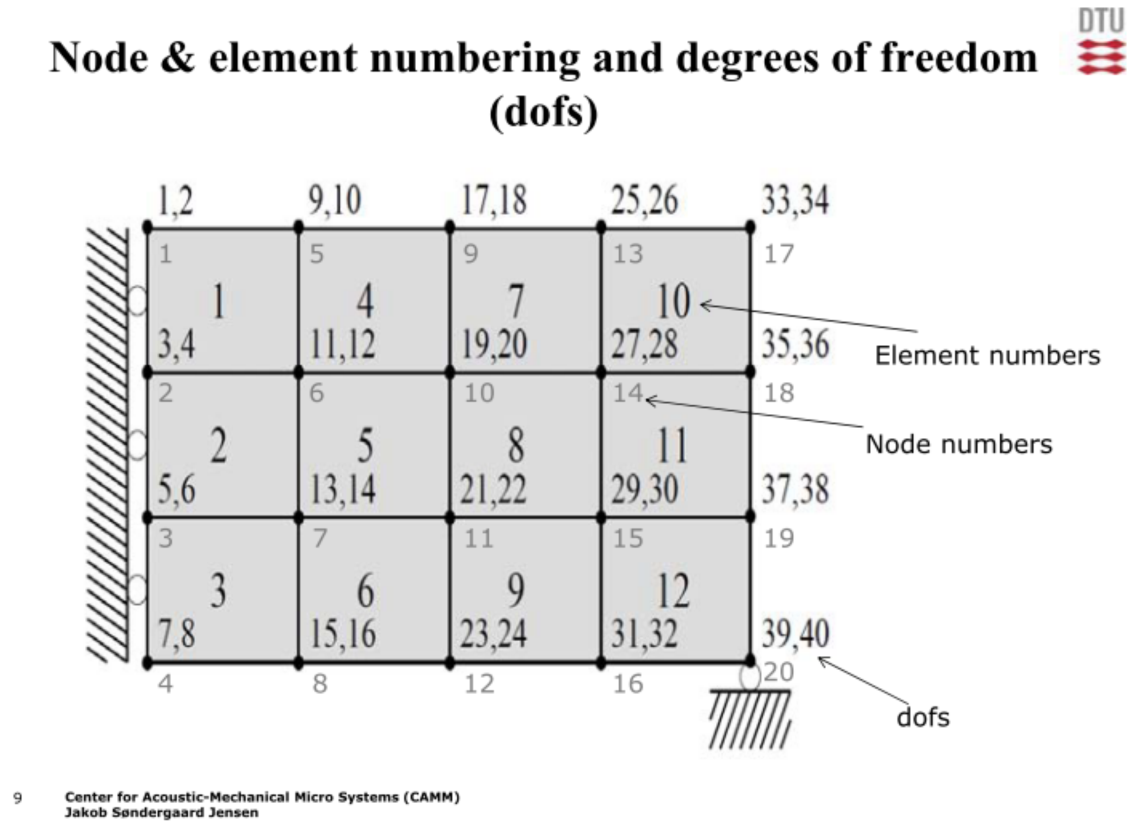


Node & element numbering and degrees of freedom (dofs)

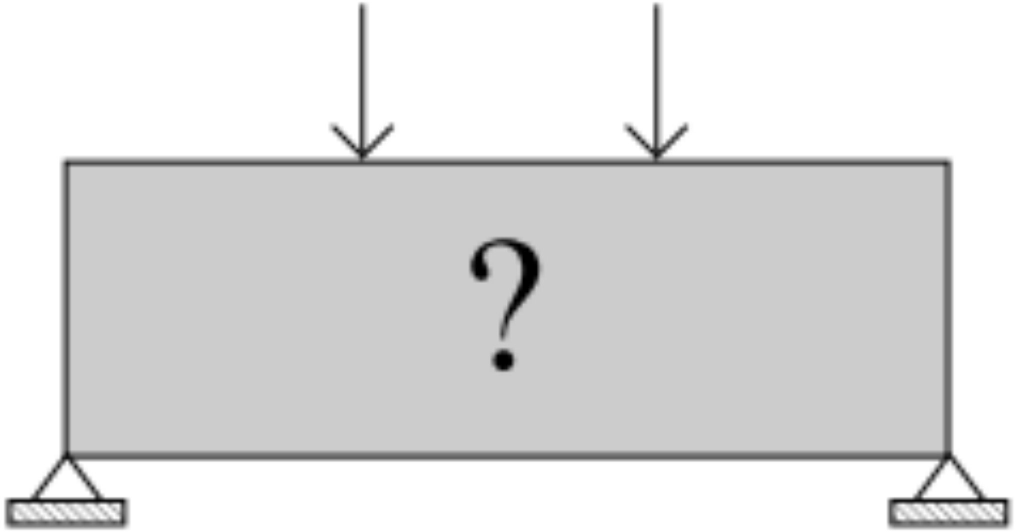


Center for Acoustic-Mechanical Micro Systems (CAMM)
Jakob Søndergaard Jensen

- F = sparse(2, 1, -1, 2*(nely+1)*(nelx+1), 1);
- fixeddofs = union([1:2:2*(nely+1)], [2*(nelx+1)*(nely+1)]);



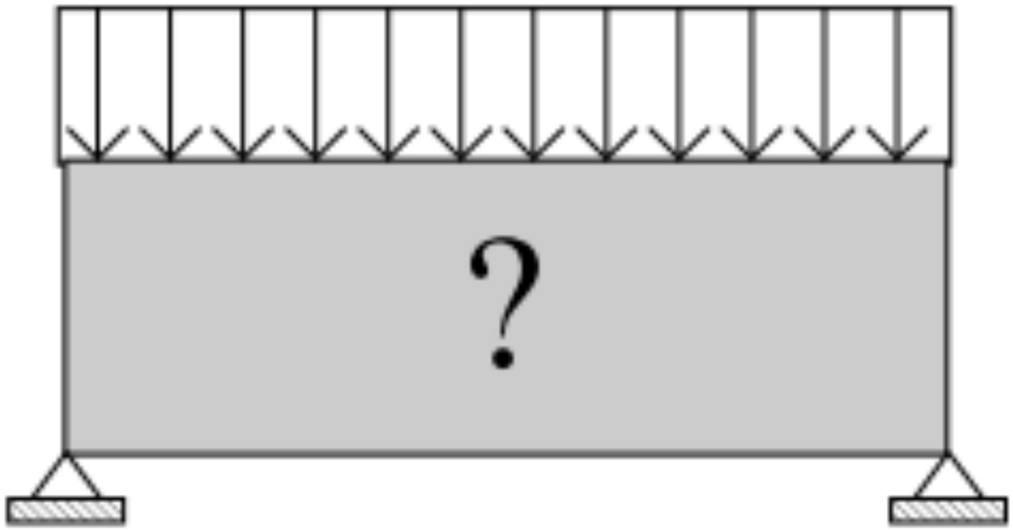
- iForce1 = (nely+1)*2*nelx/3+2;
- iForce2 = (nely+1)*2*nelx*2/3+2;
- iF = [iForce1 iForce2];
- jF = [1 1];
- vF = [-1 -1];
- F = sparse(iF, jF, vF, 2*(nely+1)*(nelx+1), 1);



- iFix1 = [(2*(nely+1)-2):2*(nely+1)];
- iFix2 = [(2*(nelx+1)*(nely+1)-2):2*(nelx+1)*(nely+1)];
- fixeddofs = union(iFix1, iFix2);

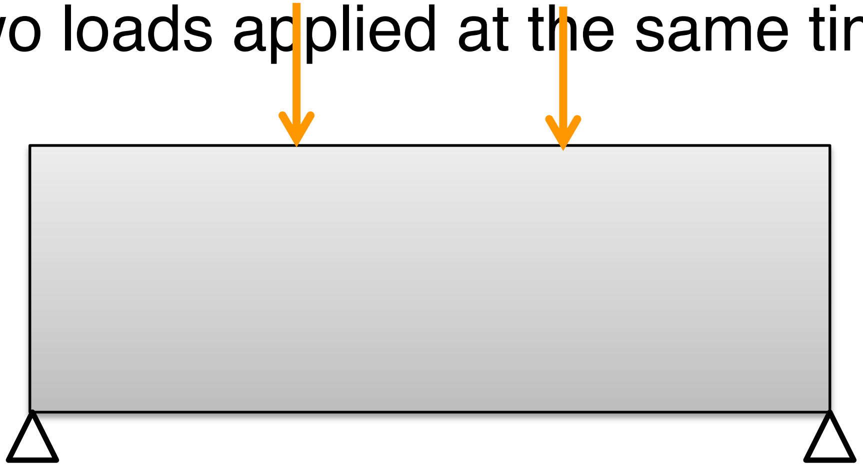
Exercise 1: Set distributed loads

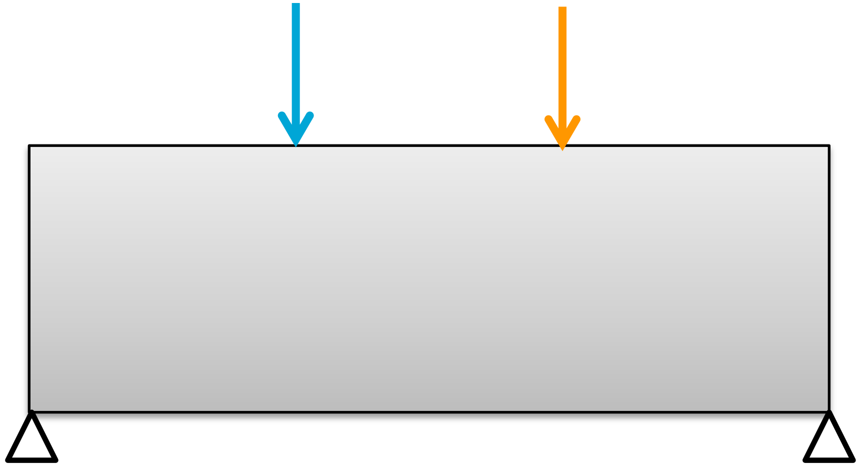
- iF = -----
- jF = -----
- vF = -----
- F = sparse(iF, jF, vF, 2*(nely+1)*(nelx+1), 1);



Exercise 2: Set multiple load cases

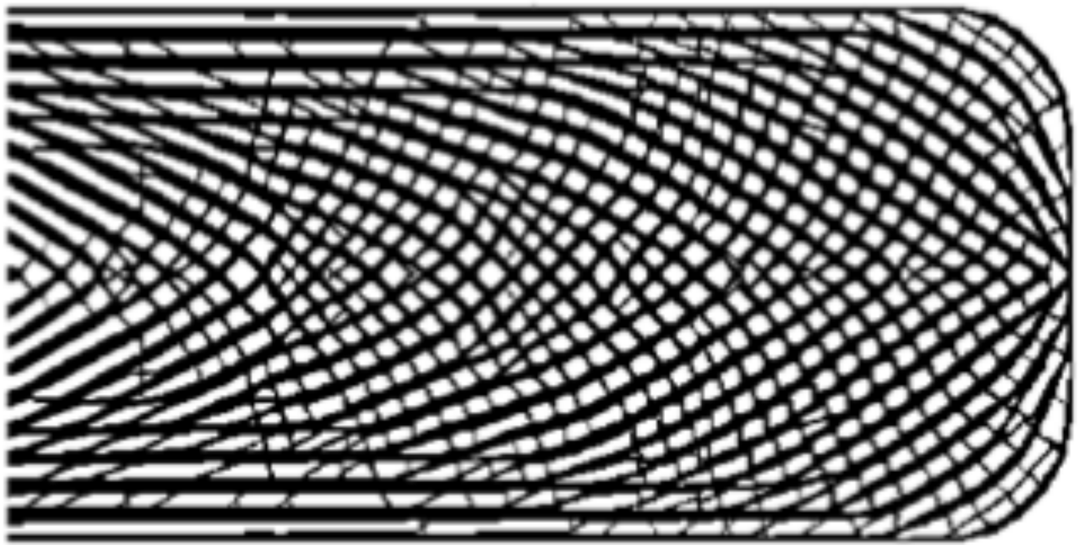
- Set two loads which are applied alternatively
- Compare to the situation where the two loads applied at the same time
- Min: $c = \frac{1}{2}U_1^T K U_1 + \frac{1}{2}U_2^T K U_2$
- s.t. : $KU_1 = F_1$
- $KU_2 = F_2$
- $\rho_i \in [0,1], \forall i$
- $\sum_i \rho_i \leq V_0$

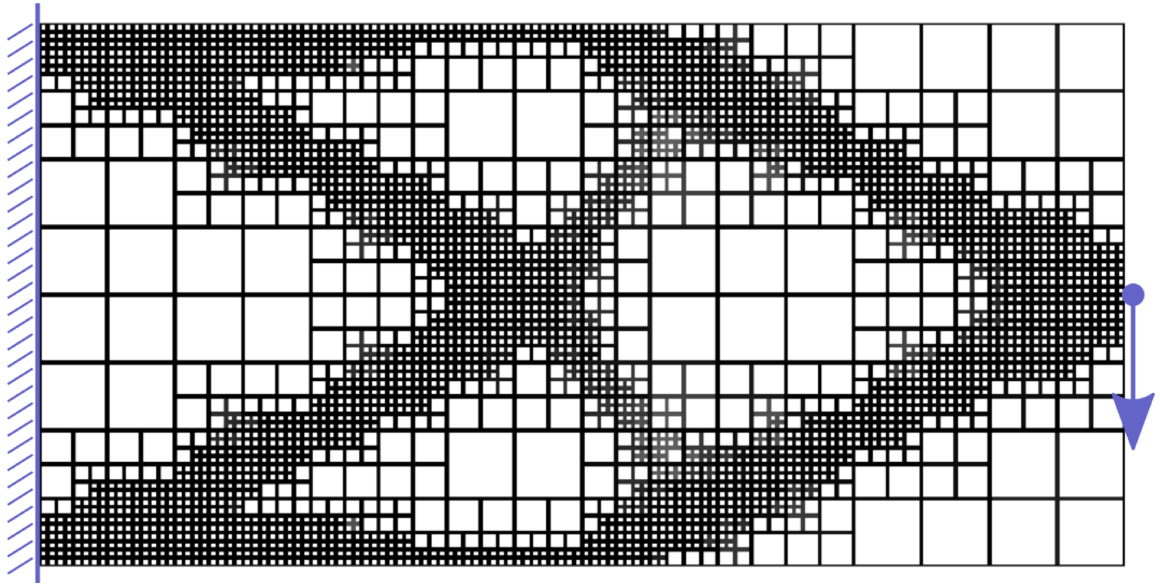




Exercise 3: Integrate a different solver for infill optimization

- Download infill.m or topQuadtreeOC.m
- Replace the MMA solver by, e.g IPOPT





Thank you for your attention!

Jun Wu

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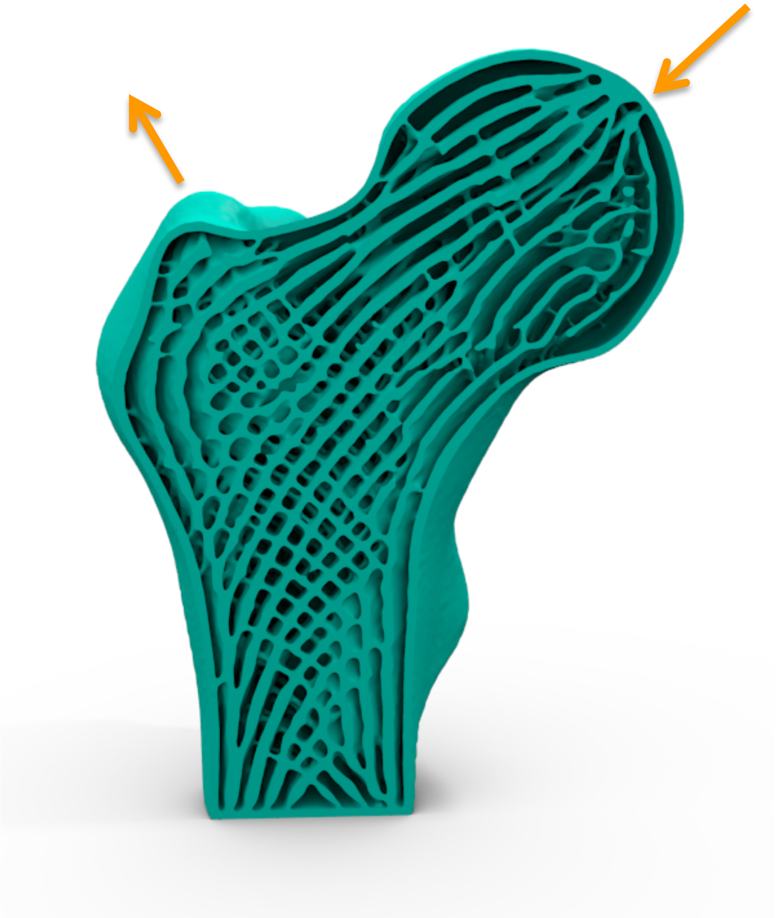
j.wu-1@tudelft.nl



TU Delft