SGP Graduate School
Fabrication

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Niloy Mitra

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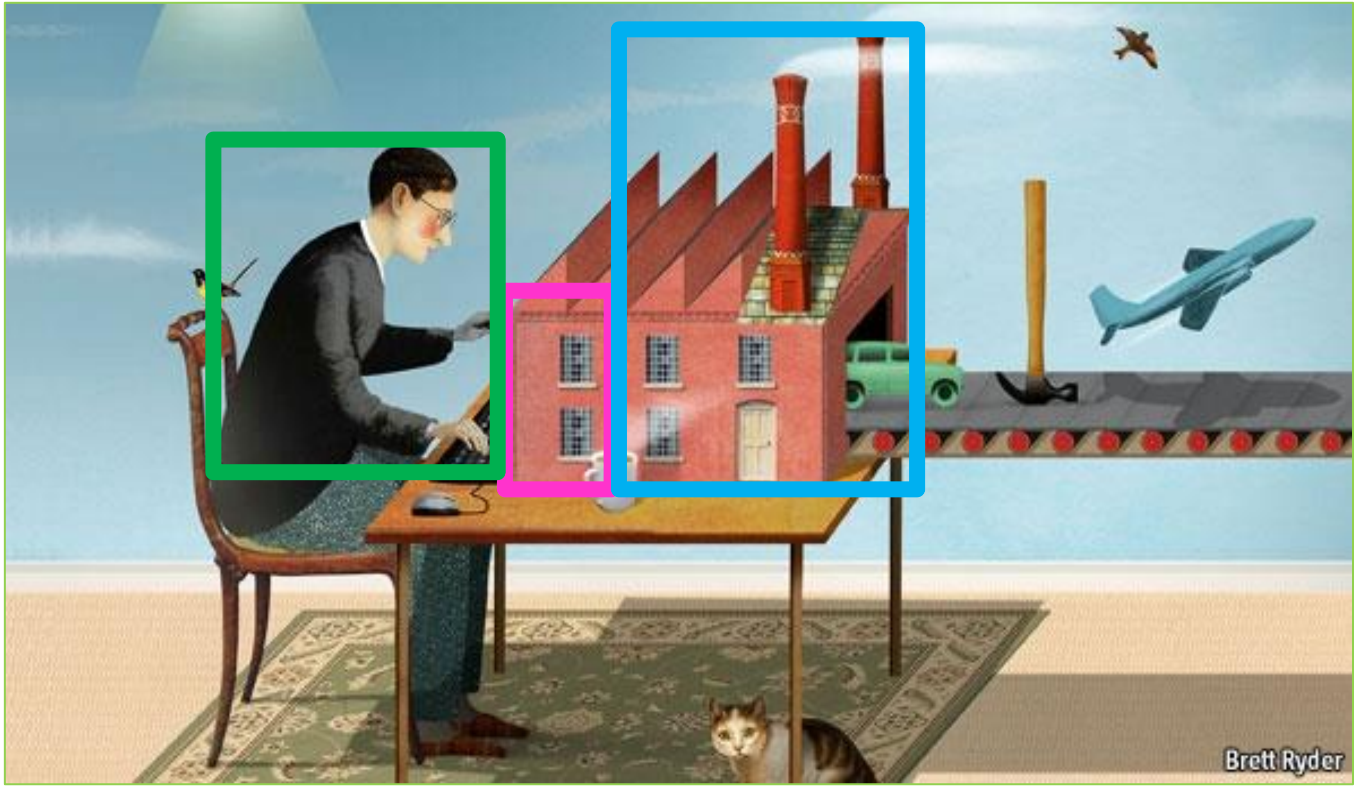
Institute of Science and Technology Austria
(IST Austria)

About This Course

- Introduction and general challenges
- Computational tools and design tools for
  - Deformable Shapes
  - Foldable Shapes
- Latest research
- Advanced manufacturing
- Inspiration

Source: The Economist (Cover)

About This Course



Source: The Economist (Cover)

Other 3D Printing Courses at Siggraph/ Siggaph Asia

- Siggraph Asia 2014
  - **3D printing oriented design: geometry and optimization** http://staff.ustc.edu.cn/~lgliu/Courses/SigAsia_2014_course_3Dprinting/index.html
- Siggraph 2015
  - **Modeling and Toolpath Generation for Consumer-Level 3D Printing**
  - http://webloria.loria.fr/~slefebvr/sig15fdm/
- Siggraph 2016
  - **Computational Tools for 3D Printing**
  - http://computational-fabrication.com/2016/
- Eurographics 2017
  - **Topology Optimization for Computational Fabrication**
  - https://topopt.weblog.tudelft.nl/

Course Schedule

16:00 - 16:15, Welcome & Introduction **Bernd**

16:15 - 16:45, Inverse Design – Deformable Shapes **Bernd**

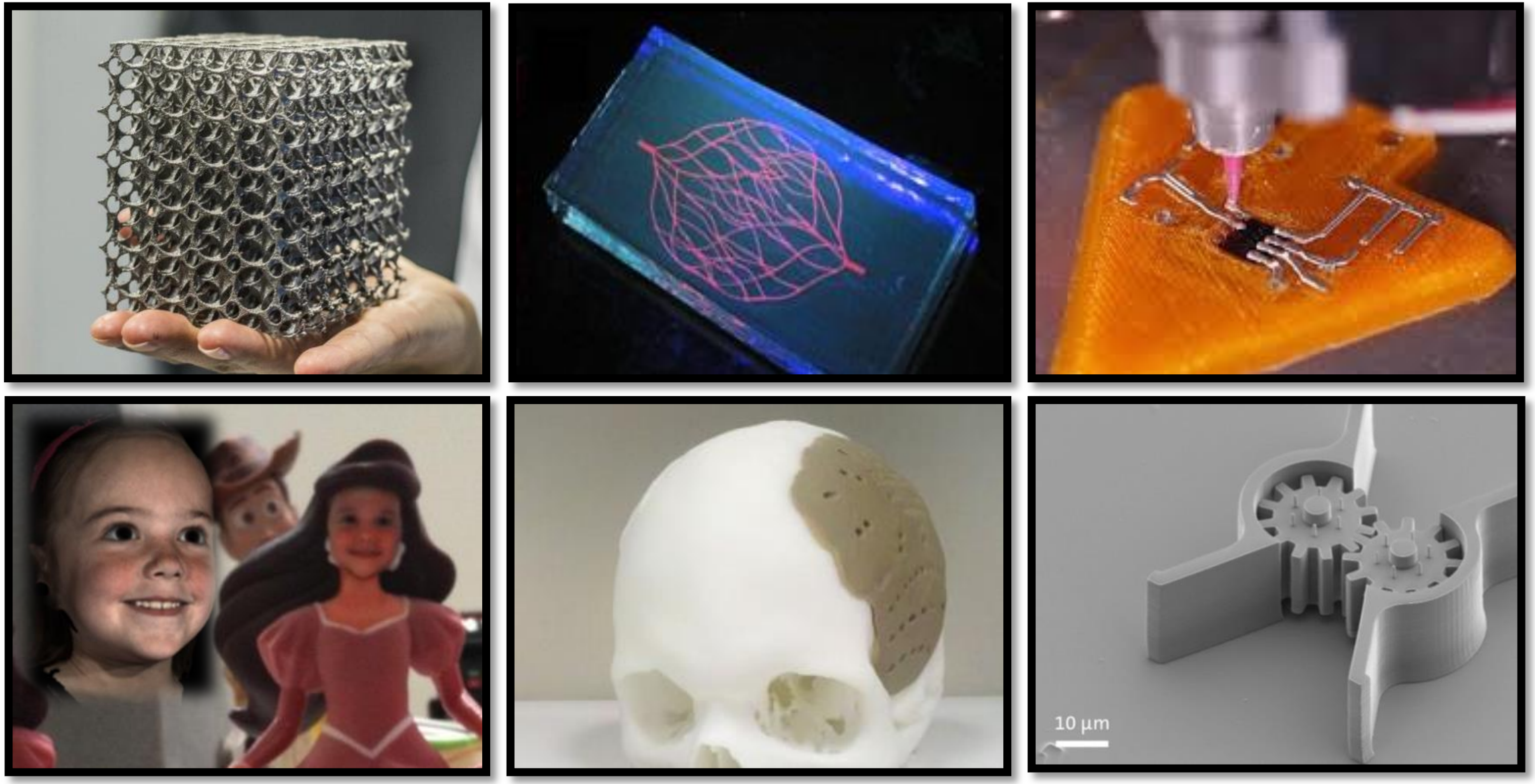
16:45 - 17:15, Inverse Design – Foldable Shapes **Niloy**

17:15 - 17:25, Advanced Manufacturing and Open Challenges, **Bernd**

17:25 – 17:30 Q&A

Introduction

Computational Fabrication



Engineering / Customized Products

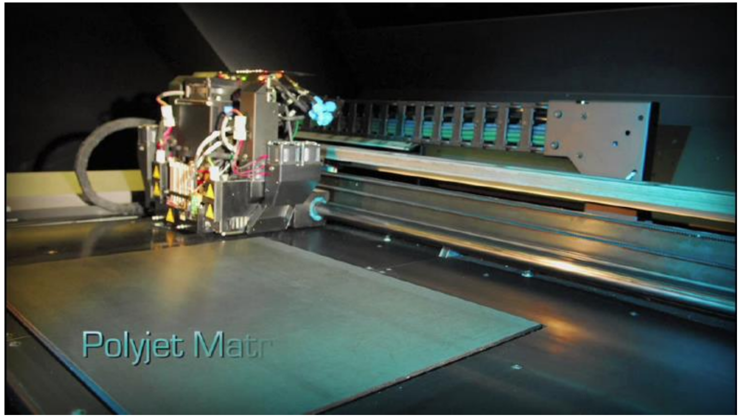
Bioprinting / Medicine

Robotics / Nanofabrication

[The Economist]

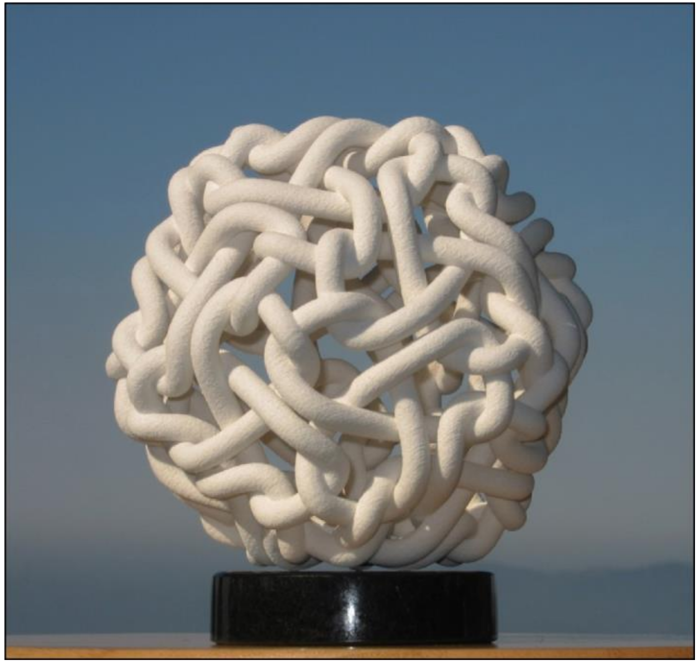
«Game Changer» 3D Printer

3D Printers



[Stratasys]

Complexity (almost) for Free



Arabic Icosahedron
(Carlo H. Séquin)

Benefits of Additive Manufacturing

- Very flexible
- Rapid fabrication
- Excellent for customization
- Complexity for free
- AM has minimal material waste

Limitations

- Limited part sizes
- Limited fabrication speed
- Limited materials
- Poor surface finish
- Inconsistent part quality
- High cost (machine, material, pre- and postprocessing)

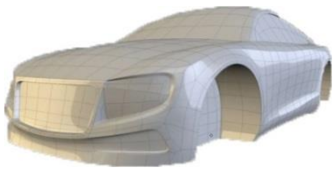
Computational Fabrication

Physical World
Information
Ideas

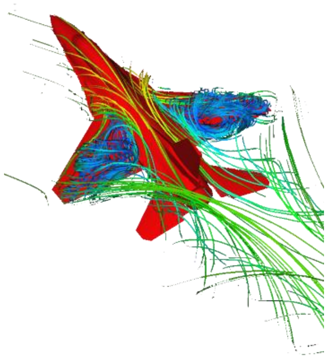
Virtual World

Output

State of the Art

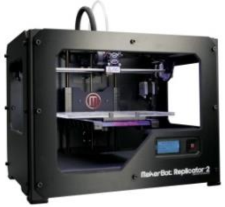


Shape modeling

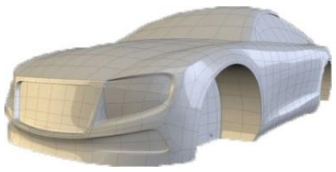


Simulation

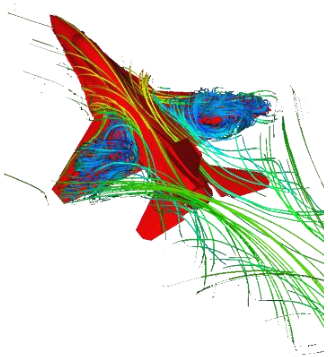
Triangle Surface Mesh



State of the Art

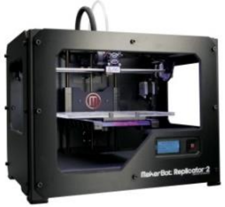


Shape modeling



Simulation

Triangle Surface Mesh



Trillions of voxels

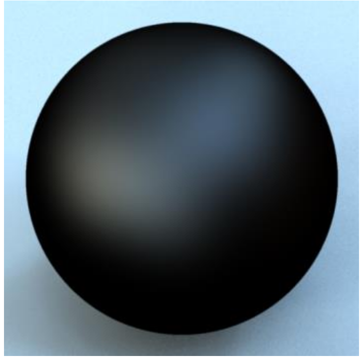
Finite Element method too slow

Prescriptive, no semantics

Direct Specification

- Decompose into regions
- Assign one material for each region

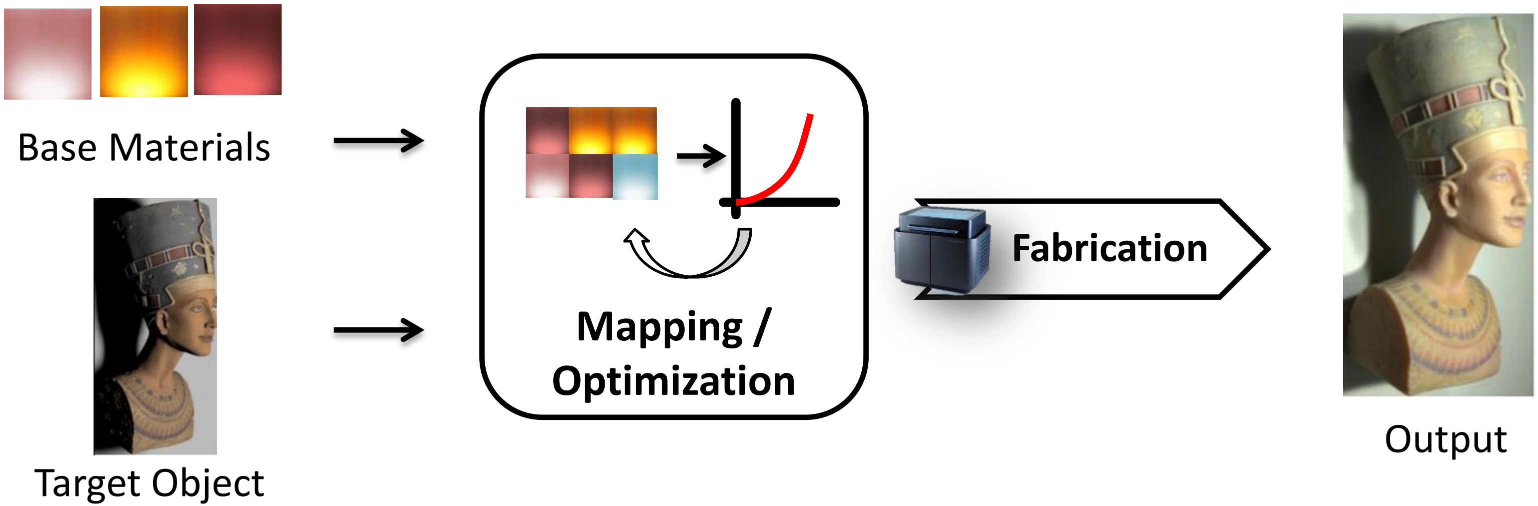
Functional Specification



Appearance Properties

Texture

From Functional 2 Direct Specification



Questions?

Inverse Design of Deformable Shapes

Bernd Bickel

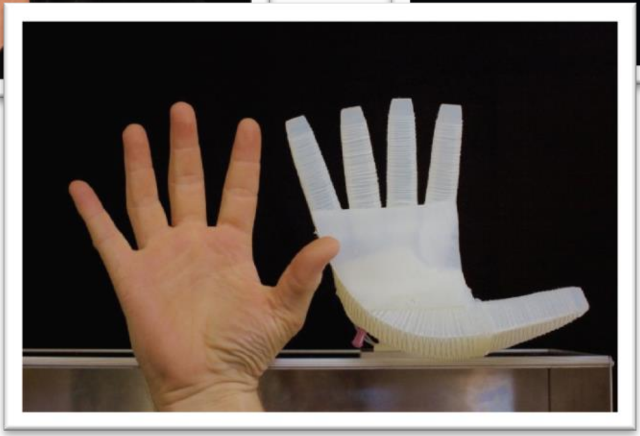
IST AUSTRIA
Institute of Science and Technology

UCL

Deformable Objects

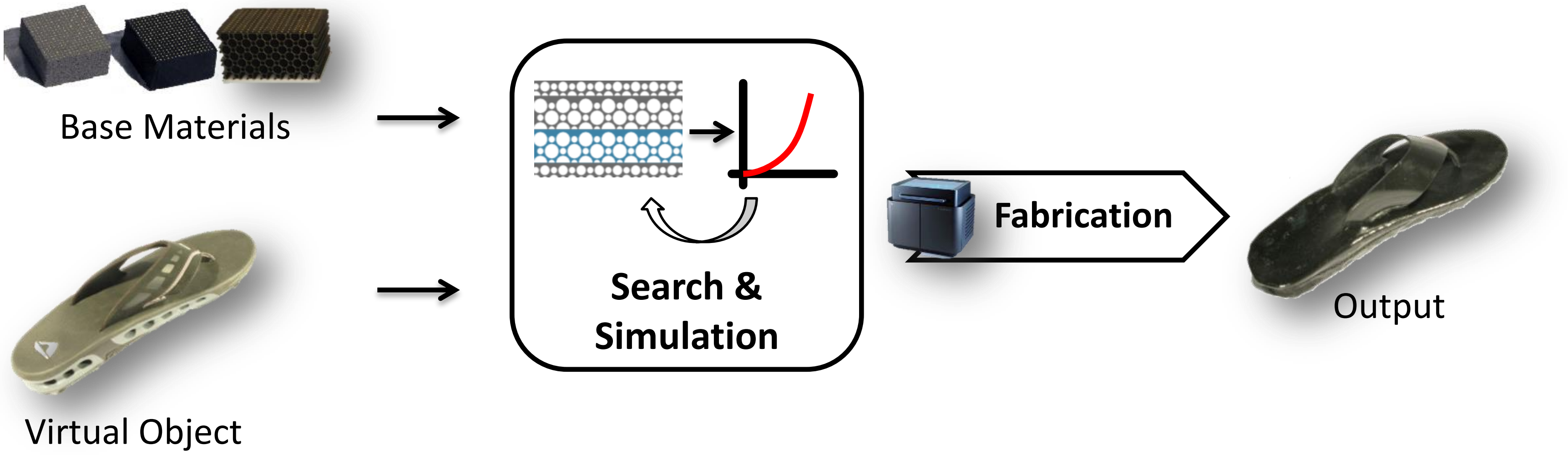
[shapeways.com]

[Festo]

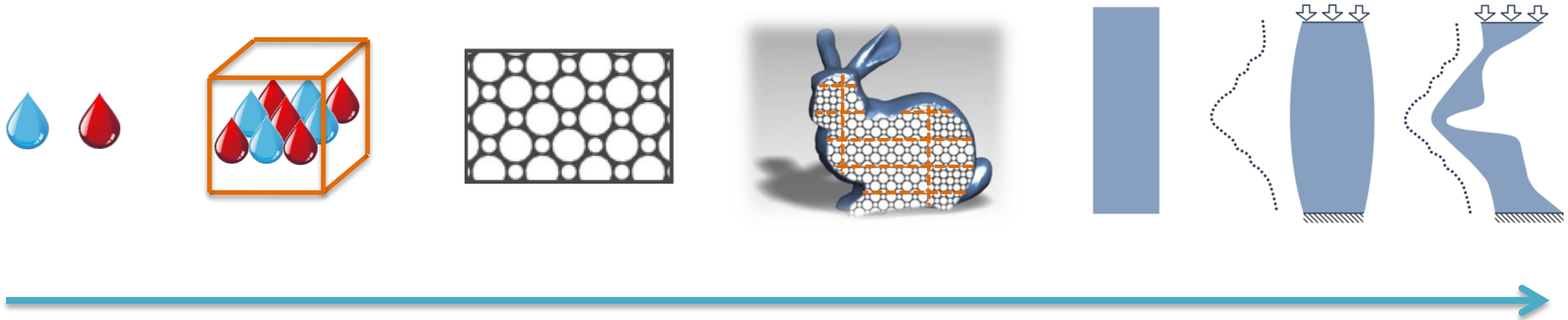


[RBO Hand, TU Berlin]

From Functional 2 Direct Specification



Manufacturing Deformable Materials



intrinsic

extrinsic

[Hiller and Lipson 2009]

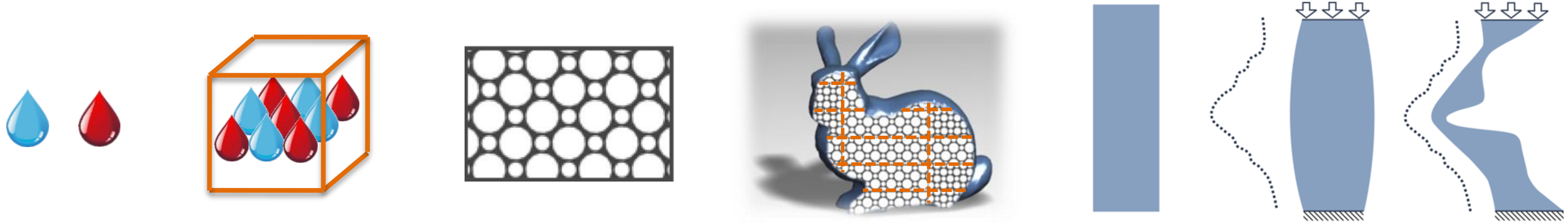
[Bickel et al. 2010]
[Skouras et al. 2013]

[Bickel et al. 2012]
[Chen et al. 2014]

[Chen et al. 2013]

[Schumacher et al. 2015]
[Panetta et al. 2015]
[Martínez et al. 2016] [Konakovic et al. 2016]

4D Printing

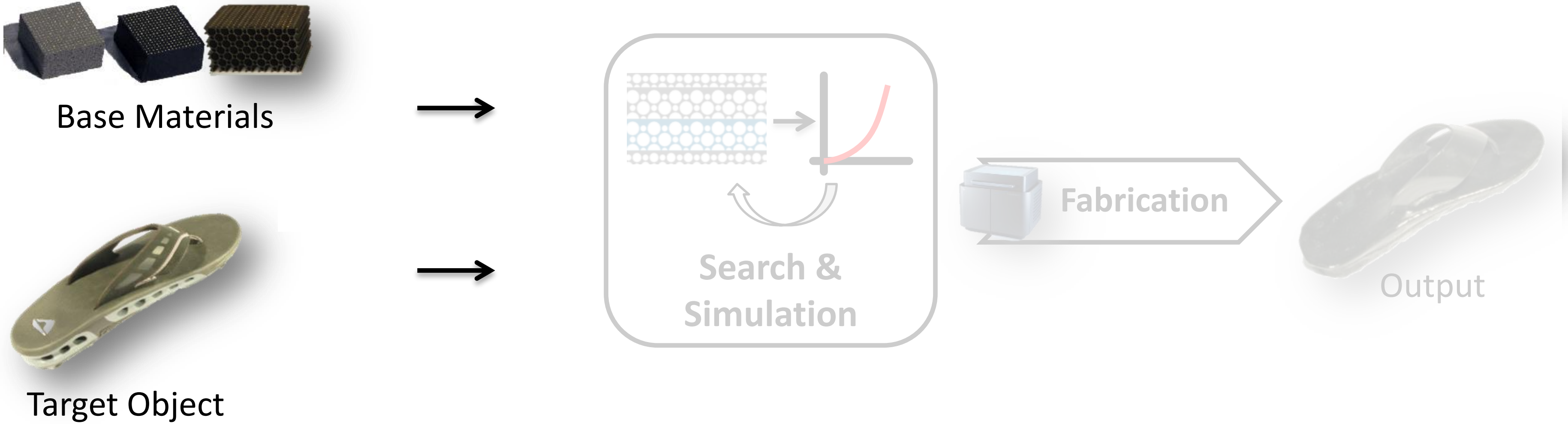


intrinsic — extrinsic

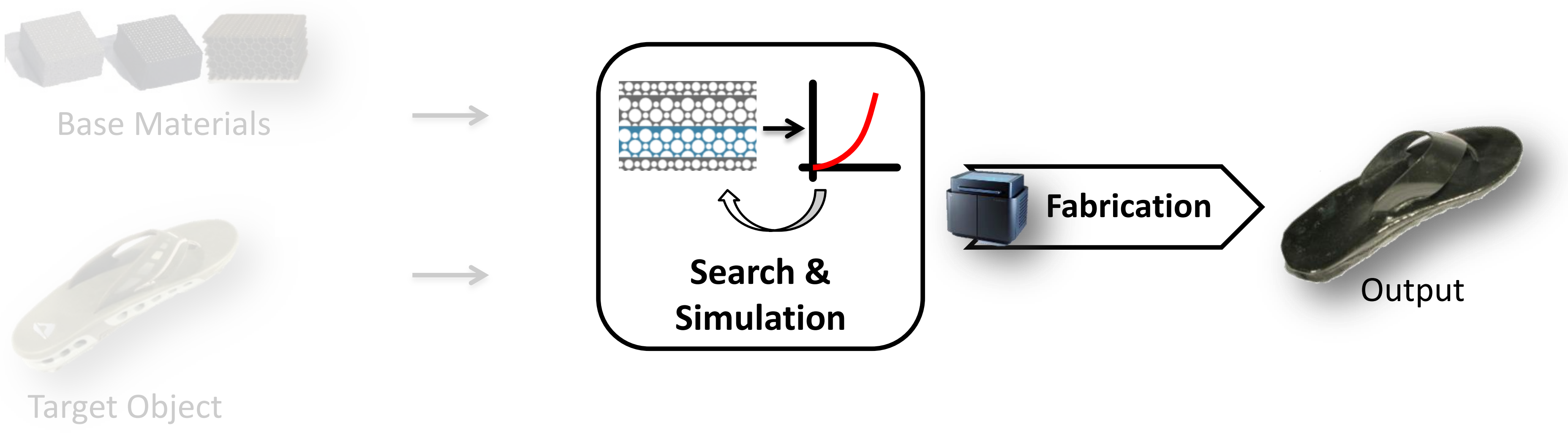
Pre-programmed behavior

The 4th dimension is the set of behavioral rules that are pre-programmed into a 3D-printed shape. Based on any number of stimuli, the object can be set to respond differently. These responses can take a near limitless number of forms including (but definitely not limited to) changes in color, temperature, shape, movement, ...

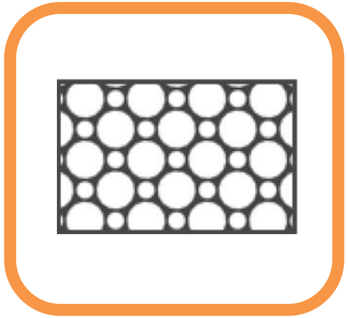
From Functional 2 Direct Specification

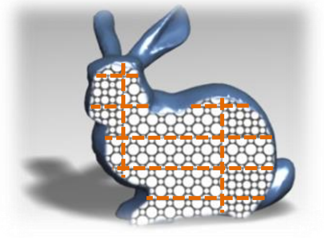


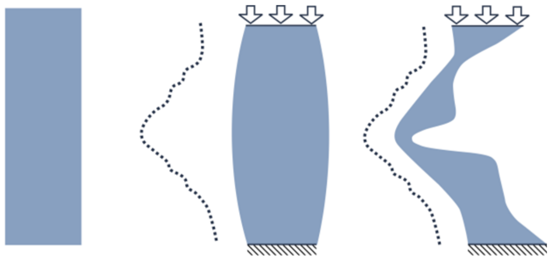
From Functional 2 Direct Specification



Manufacturing Deformable Materials







intrinsic

extrinsic

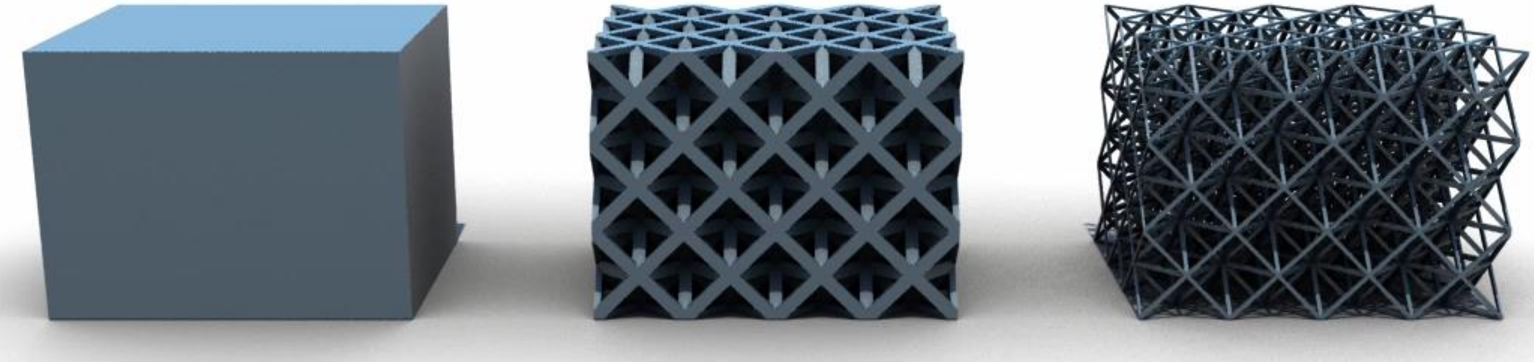
[Hiller and Lipson 2009]

[Bickel et al. 2010]
[Skouras et al. 2013]

[Bickel et al. 2012]
[Chen et al. 2014]

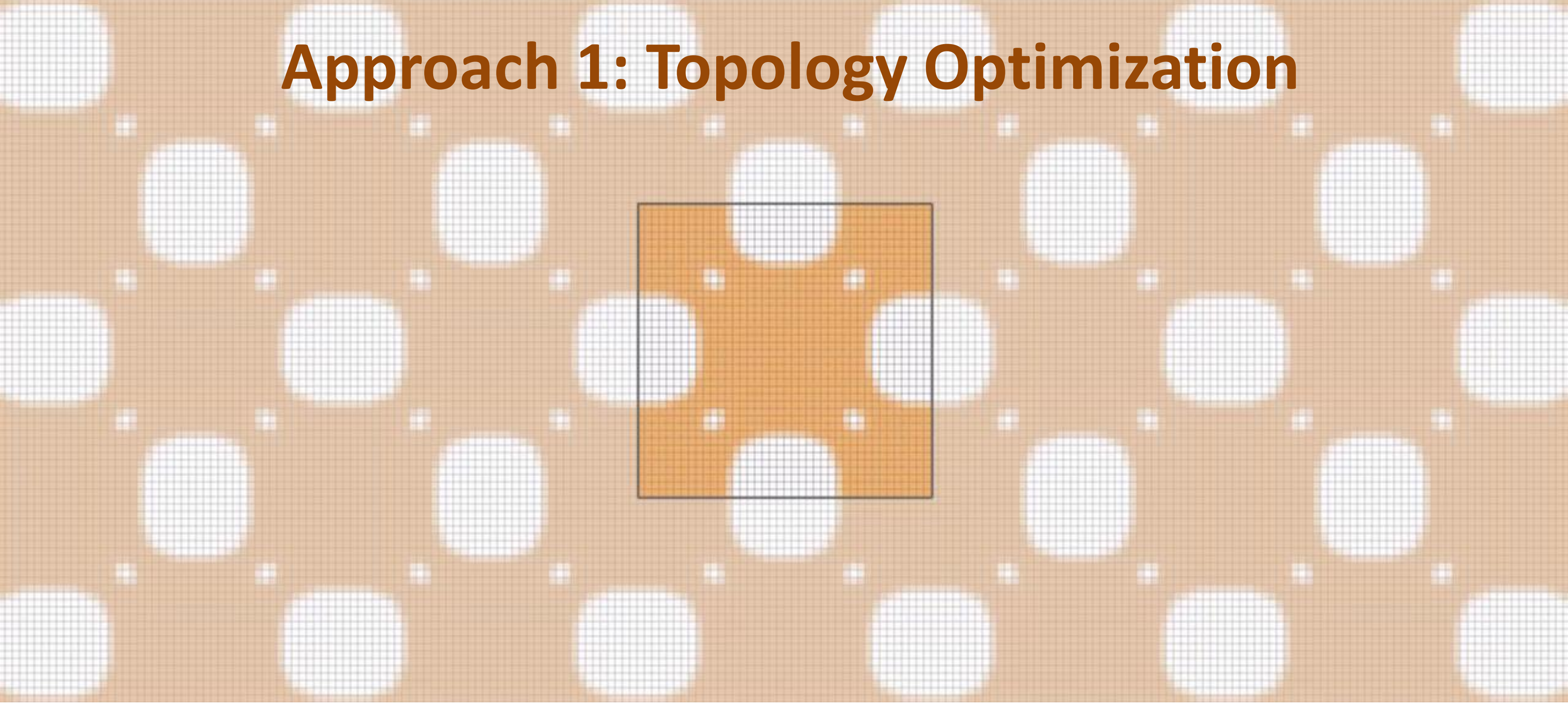
[Schumacher et al. 2015]
[Panetta et al. 2015]
[Martínez et al. 2016]

[Chen et al. 2013]



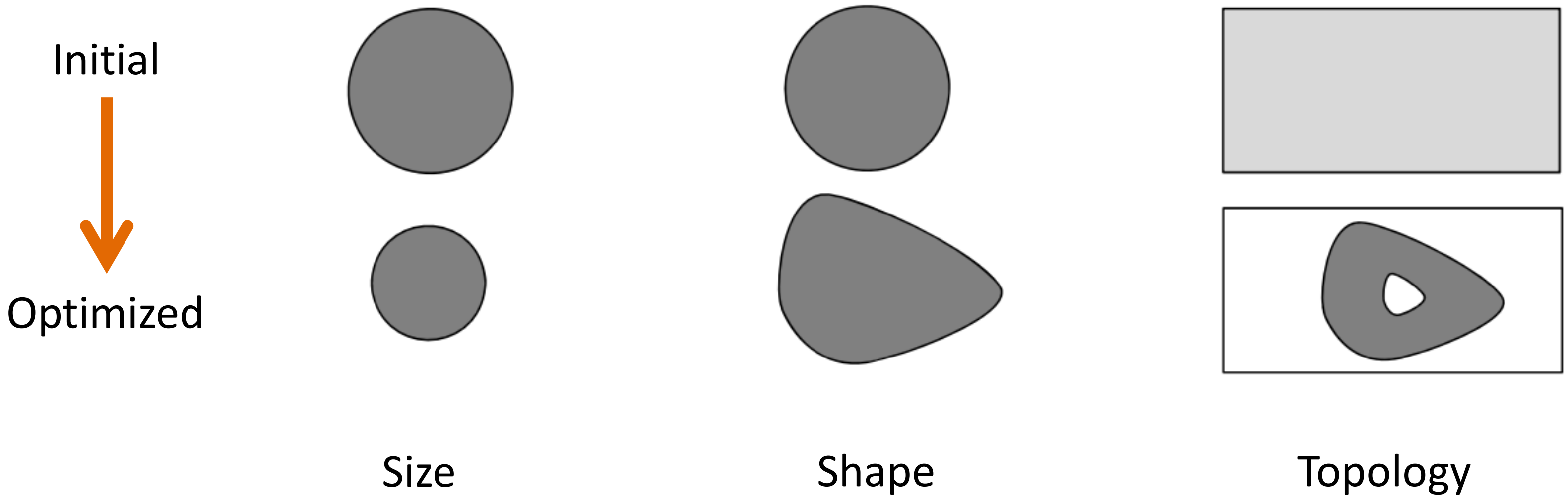
Goal

Approach 1: Topology Optimization



[Sigmund 2009]
[Schumacher et al. 2015]

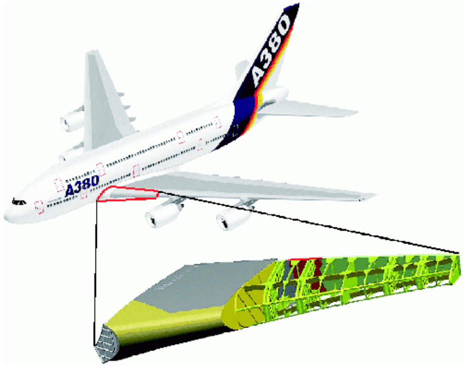
Classes of structural optimization methods

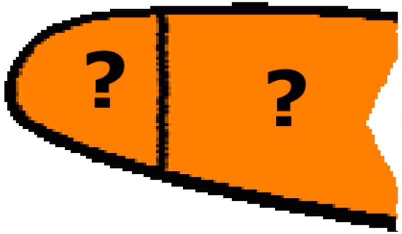


[courtesy Aage 2017]

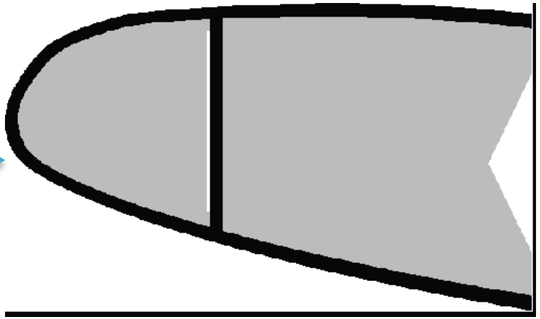
Generating Optimal Topologies

[Bendsøe and Kikuchi 1988]





Design domain

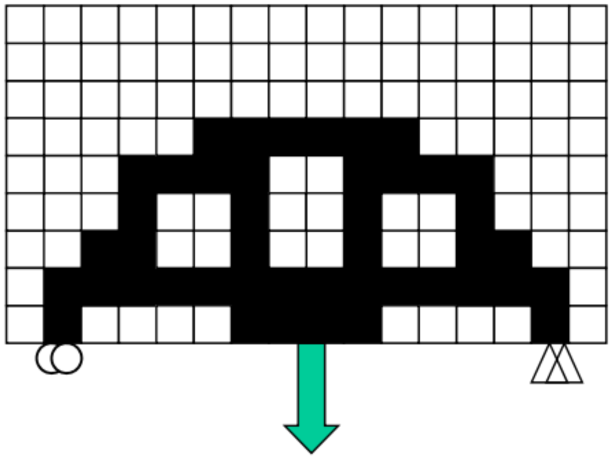


Optimal material redistribution

Interpretation

[courtesy Aage 2017]

Discrete Topopt Formulation



$$
\begin{aligned}
\min_{\boldsymbol{\rho}} : \quad & \Phi(\boldsymbol{\rho}, \mathbf{U}(\boldsymbol{\rho})) \\
s.t. : \quad & \sum_{e=1}^{N} v_e \rho_e = \mathbf{v}^T \boldsymbol{\rho} \leq V^* \\
: \quad & g_i(\boldsymbol{\rho}, \mathbf{U}(\boldsymbol{\rho})) \leq g_i^*, \quad i = 1, \ldots, M \\
: \quad & \rho_e = \begin{cases} 0 \text{ (void)} \\ 1 \text{ (material)} \end{cases}, \quad e = 1, \ldots, N \\
: \quad & \mathbf{K}(\boldsymbol{\rho})\mathbf{U} = \mathbf{F}
\end{aligned}
$$

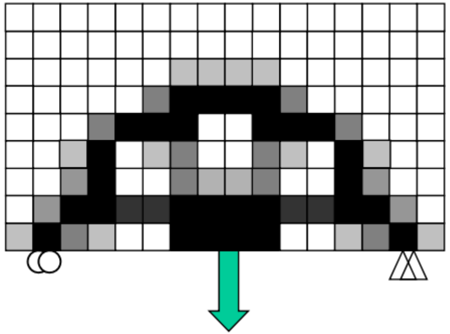
0/1 Integer problem

Huge number of combinations!

[courtesy Aage 2017]
[Sigmund 2015]

SIMP-approach
(Simplified Isotropic Material with Penalization)

[Bendsøe 1989, Zhou and Rozvany 1991, Mlejnek 1992]



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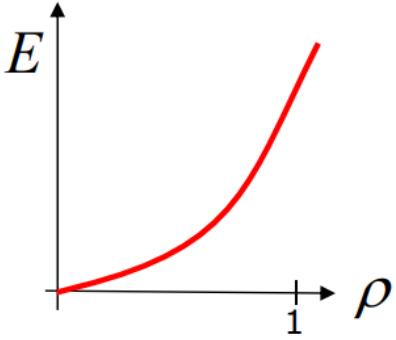
: \quad & g_i(\boldsymbol{\rho}, \mathbf{U}(\boldsymbol{\rho})) \leq g_i^*, \quad i = 1, ..., M \\
: \quad & \mathbf{0} < \boldsymbol{\rho}_{\min} \leq \boldsymbol{\rho} \leq \mathbf{1} \\
: \quad & \mathbf{K}(\boldsymbol{\rho})\mathbf{U} = \mathbf{F}
\end{aligned}
$$

Stiffness interpolation:



$$\boxed{E(\rho_e) = \rho_e^p E_0}$$

$$p \geq 1$$

[courtesy Aage 2017]

Sensitivity Analysis by Adjoint Method

- A general function and a general residual
  $\Phi = \Phi(\boldsymbol{\rho}, \boldsymbol{u}(\boldsymbol{\rho})), \boldsymbol{R}\big(\boldsymbol{\rho}, \boldsymbol{u}(\boldsymbol{\rho})\big) = 0$

- Step 1: differentiate using the chainrule
  $$\frac{d\Phi}{d\rho_e} = \frac{\partial \Phi}{\partial \rho_e} + \frac{\partial \Phi}{\partial \boldsymbol{u}} \boxed{\frac{d\boldsymbol{u}}{d\rho_e}} \qquad \frac{d\boldsymbol{R}}{d\rho_e} = \frac{\partial \boldsymbol{R}}{\partial \rho_e} + \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{u}} \boxed{\frac{d\boldsymbol{u}}{d\rho_e}} = 0$$

- Use the residual eqs:
  $$\frac{d\boldsymbol{u}}{d\rho_e} = -\left(\frac{\partial \boldsymbol{R}}{\partial \boldsymbol{u}}\right)^{-1} \frac{\partial \boldsymbol{R}}{\partial \rho_e}$$

[Slide from Aage 2017]

Sensitivity Analysis by Adjoint Method

- Step 2: Insert trouble term into derivative

$$\frac{d\Phi}{d\rho_e} = \frac{\partial\Phi}{\partial\rho_e} + \underbrace{\frac{\partial\Phi}{\partial\boldsymbol{u}}\left(-\frac{\partial\boldsymbol{R}}{\partial\boldsymbol{u}}\right)^{-1}}_{\lambda^T}\frac{\partial\boldsymbol{R}}{\partial\rho_e}$$

- Step 3: Adjoint problem

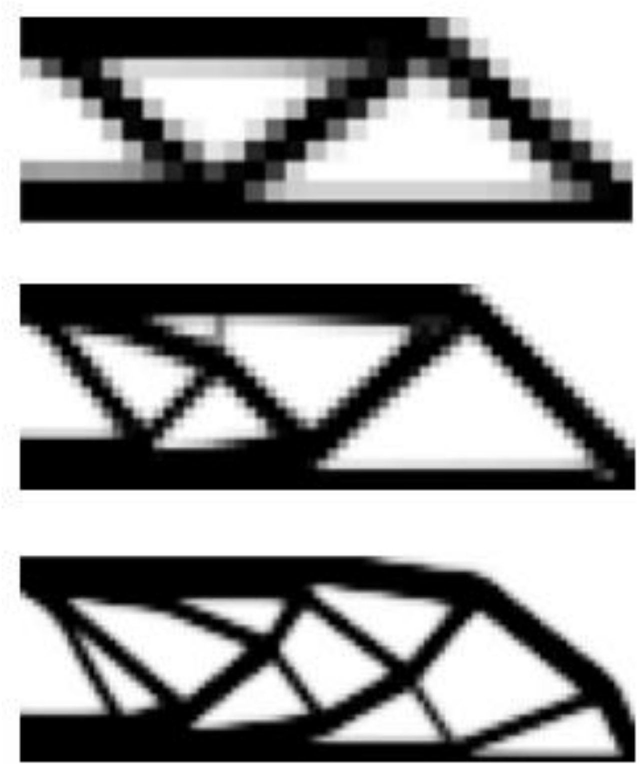
$$\lambda^T = -\frac{\partial\Phi}{\partial\boldsymbol{u}}\left(\frac{\partial\boldsymbol{R}}{\partial\boldsymbol{u}}\right)^{-1} \Rightarrow \frac{\partial\boldsymbol{R}^T}{\partial\boldsymbol{u}}\boldsymbol{\lambda} = -\frac{\partial\Phi}{\partial\boldsymbol{u}}$$

- Final sensitivity

$$\frac{d\Phi}{d\rho_e} = \frac{\partial\Phi}{\partial\rho_e} + \boldsymbol{\lambda}^T\frac{\partial\boldsymbol{R}}{\partial\rho_e}$$

[Slide from Aage 2017]

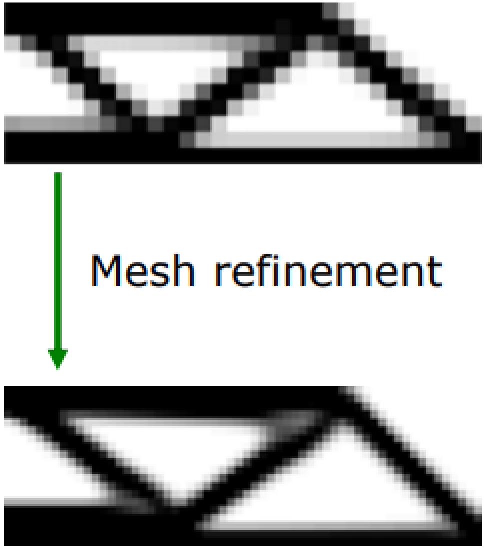
Mesh-dependence



[Slide from Aage 2017]

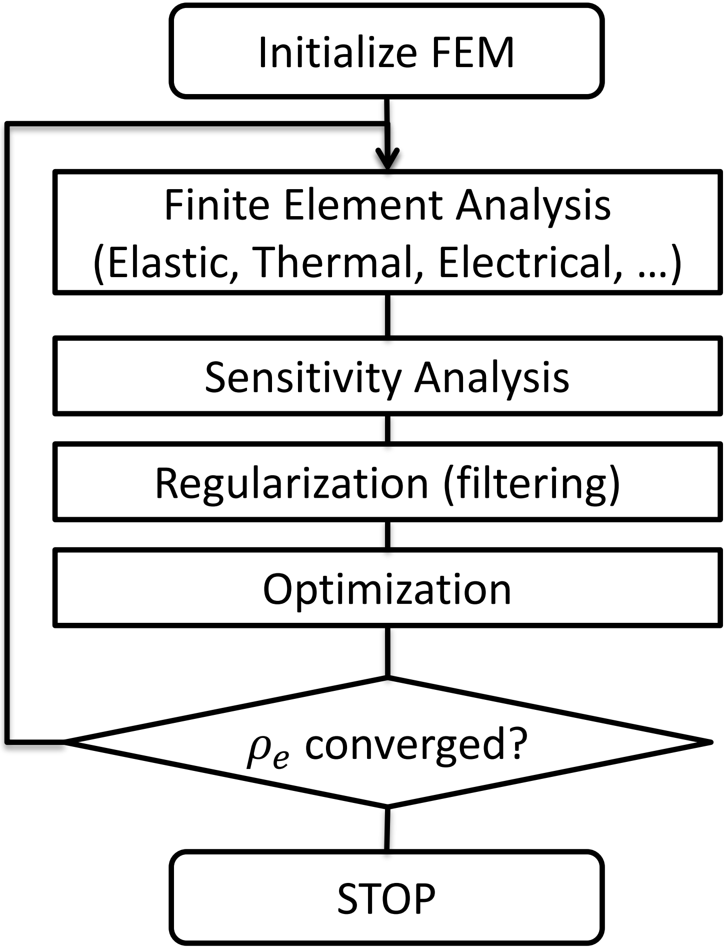
Regularization

Checkerboards

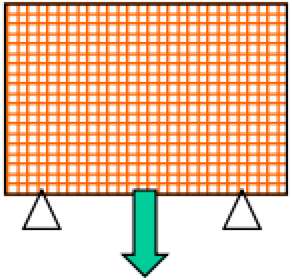


[Slide from Aage 2017]

TopOpt Scheme

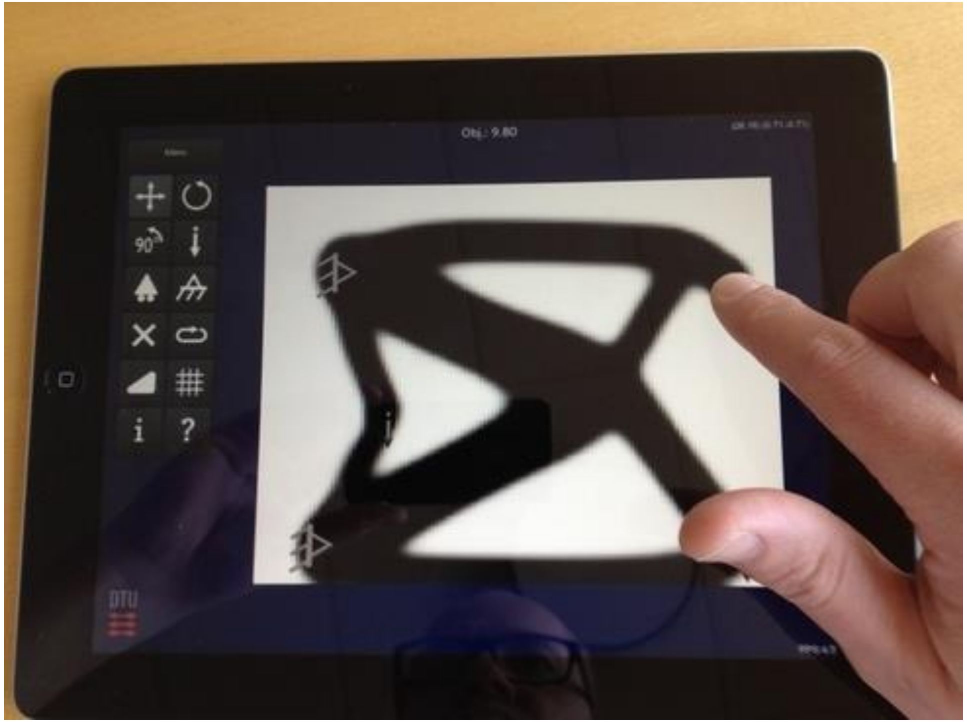


$$\boxed{\boldsymbol{KU} = \boldsymbol{F} \quad \text{or} \quad \boldsymbol{R}(\boldsymbol{U}) = \boldsymbol{0}}$$



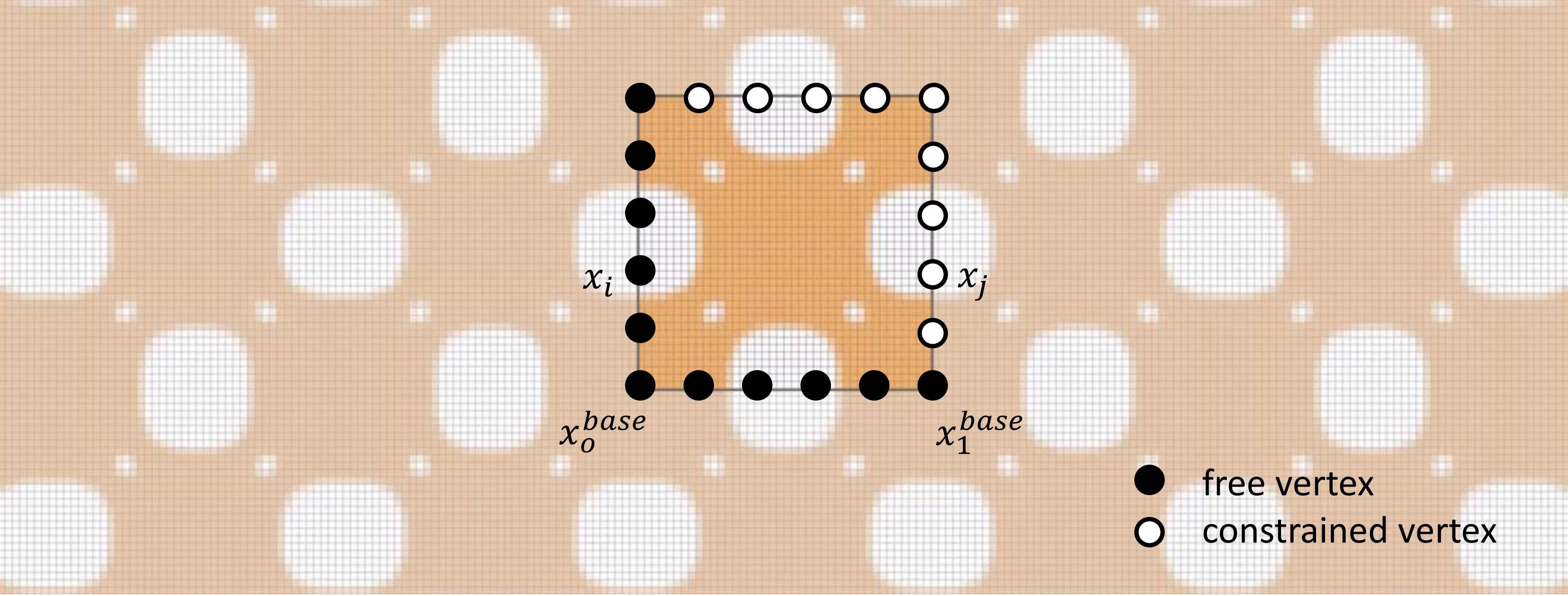
[Sigmund 2015]
[courtesy Aage 2017]

“TopOpt App” from DTU



http://www.topopt.dtu.dk

Approach 1: Topology Optimization

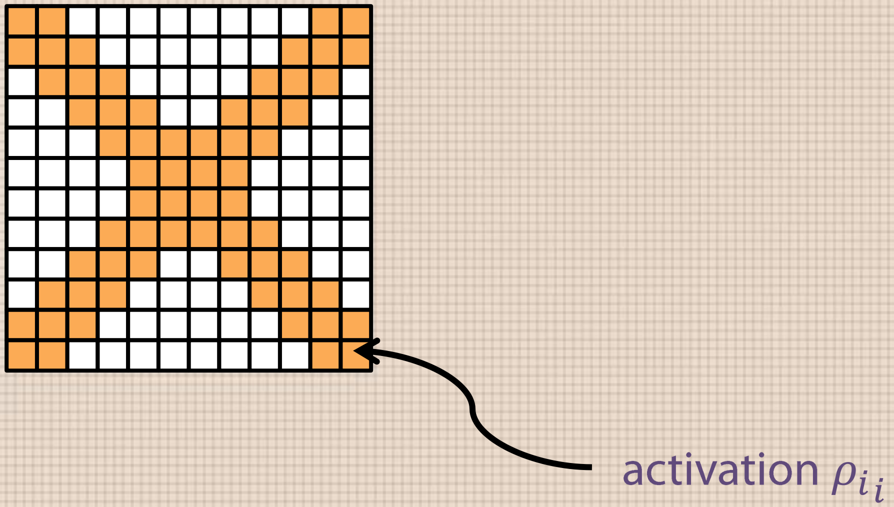


$$x_j = x_1{}^{base} + x_i - x_0^{base}$$

[Smit et al. 1998]

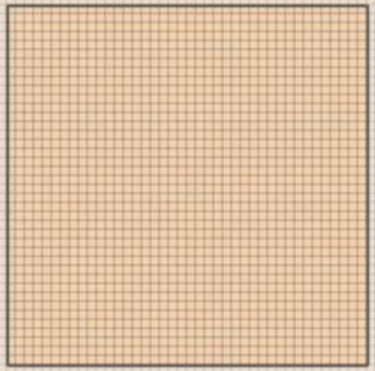
[Kharevych et al. 2009]

Approach 1: Topology Optimization



$$\boldsymbol{O(\alpha) = \left\| C_{goal} - \widetilde{C}(\rho_i) \right\|_F^2 + R}$$

Approach 1: Topology Optimization

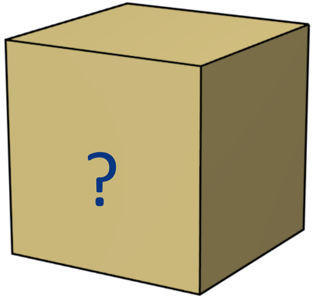


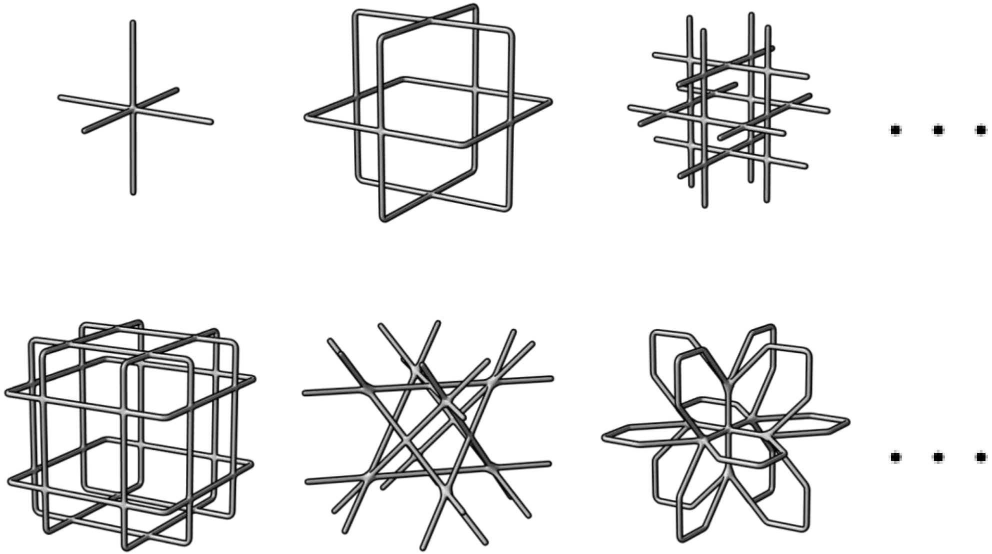
$$O(\alpha) = \left\| C_{goal} - \widetilde{C}(\rho_i) \right\|_F^2 + R$$

TopOpt – Family of Methods

- Density-based (what we have seen before)
- Implicit methods
- Topological derivatives
- Discrete approaches (Evolutionary methods)
- Combined shape and topology approaches

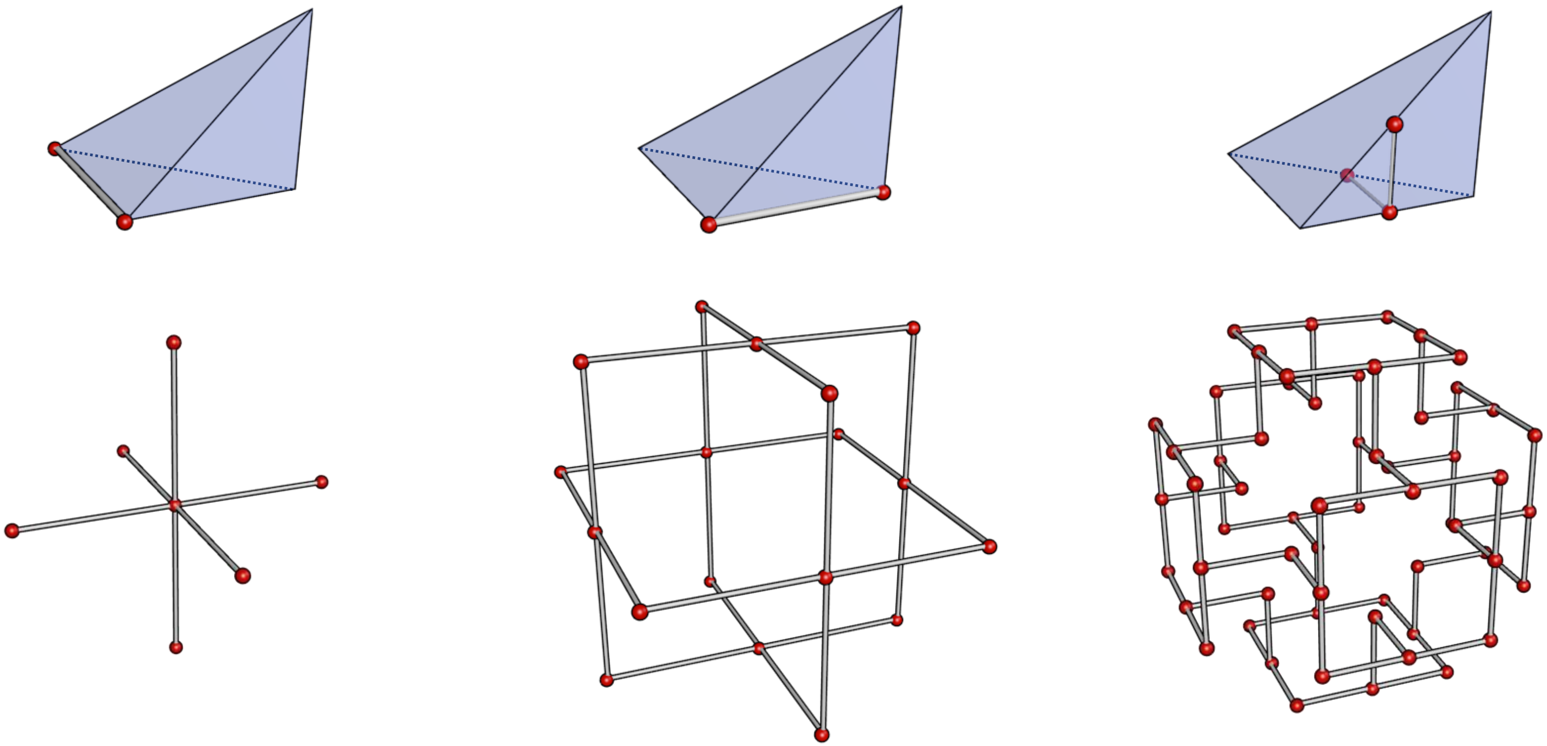
Approach 2: Systematic Topology Enumeration





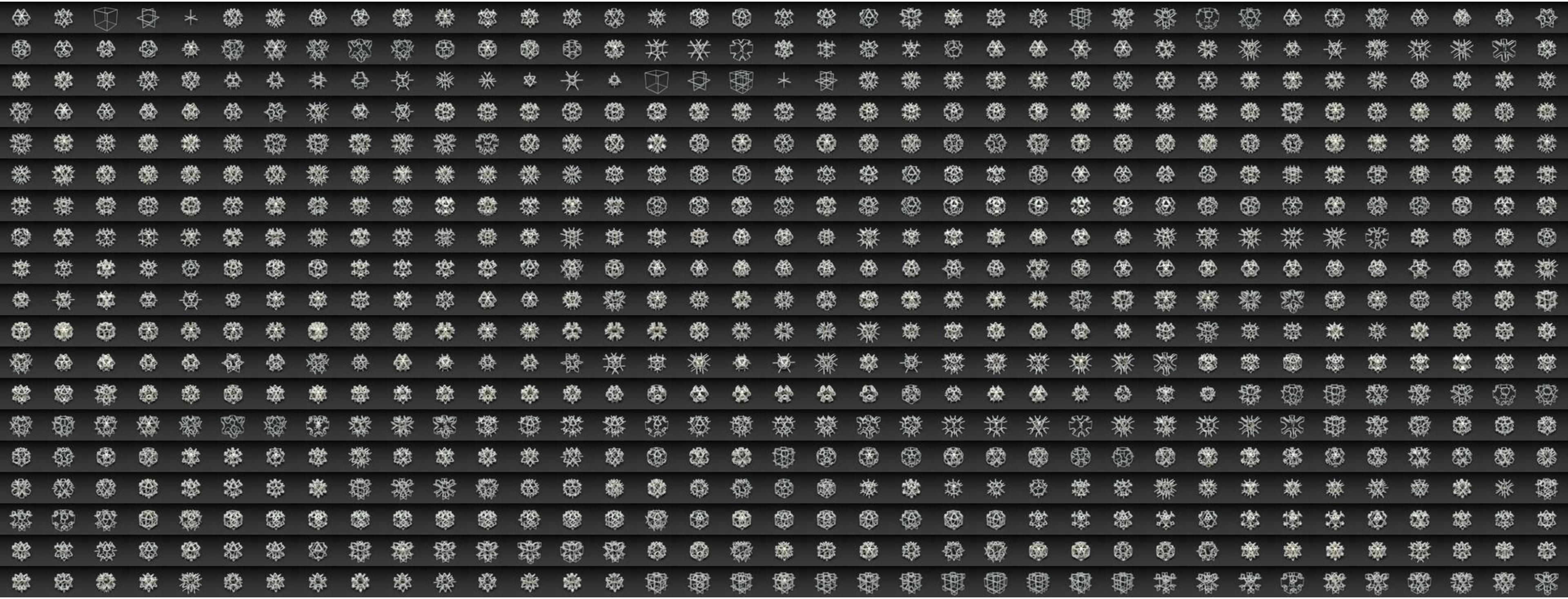
[Panetta et al. 2015]

Approach 2: Systematic Topology Enumeration



[Panetta et al. 2015]

Topology Sweep

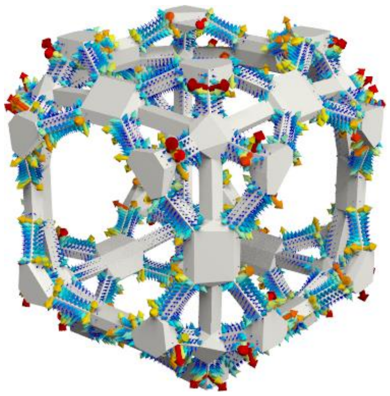


[Panetta et al. 2015]

Microstructure Shape Optimization

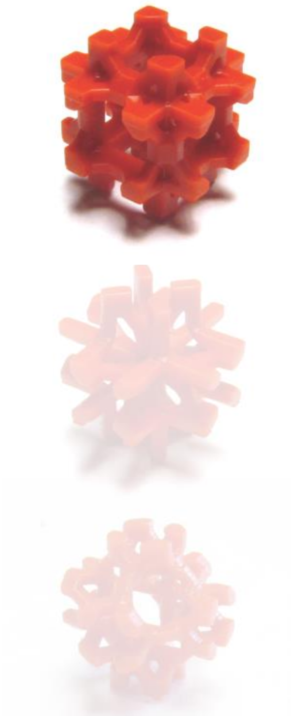
- **Thickness** and **offset** parameters continuously control microstructure's **shape**, ω
- Fit the microstructure to an elasticity tensor:

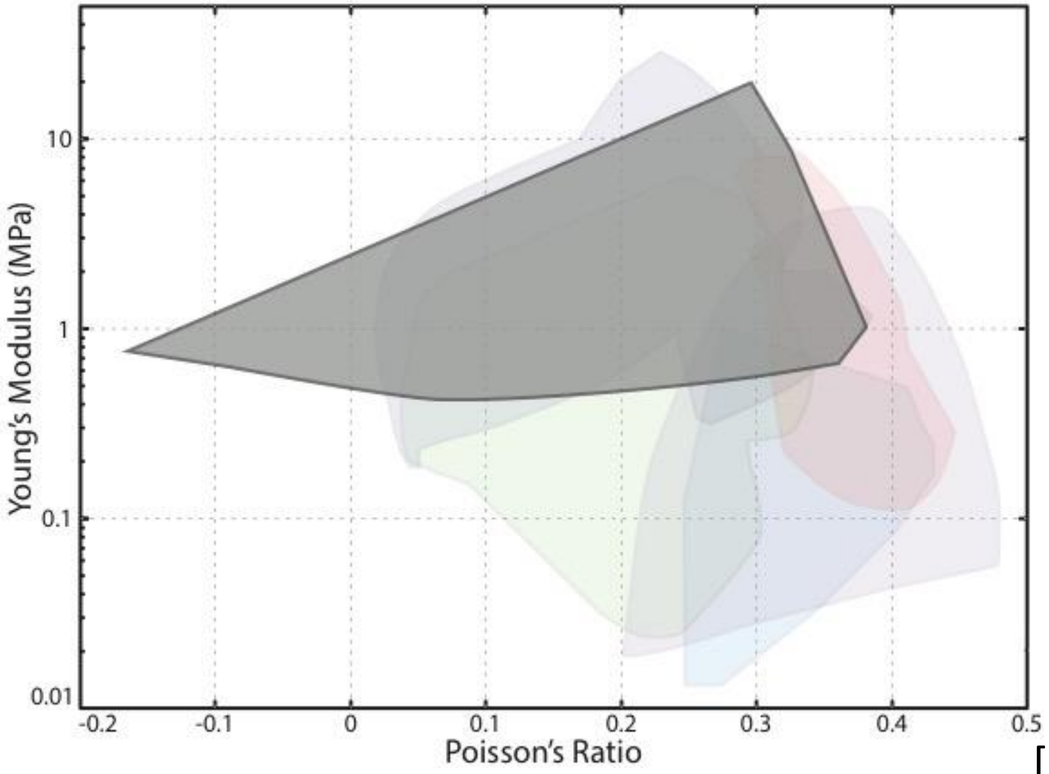
$$J(\omega) = \left|\left|C^H(\omega) - C^*\right|\right|^2$$



[Panetta et al. 2015]

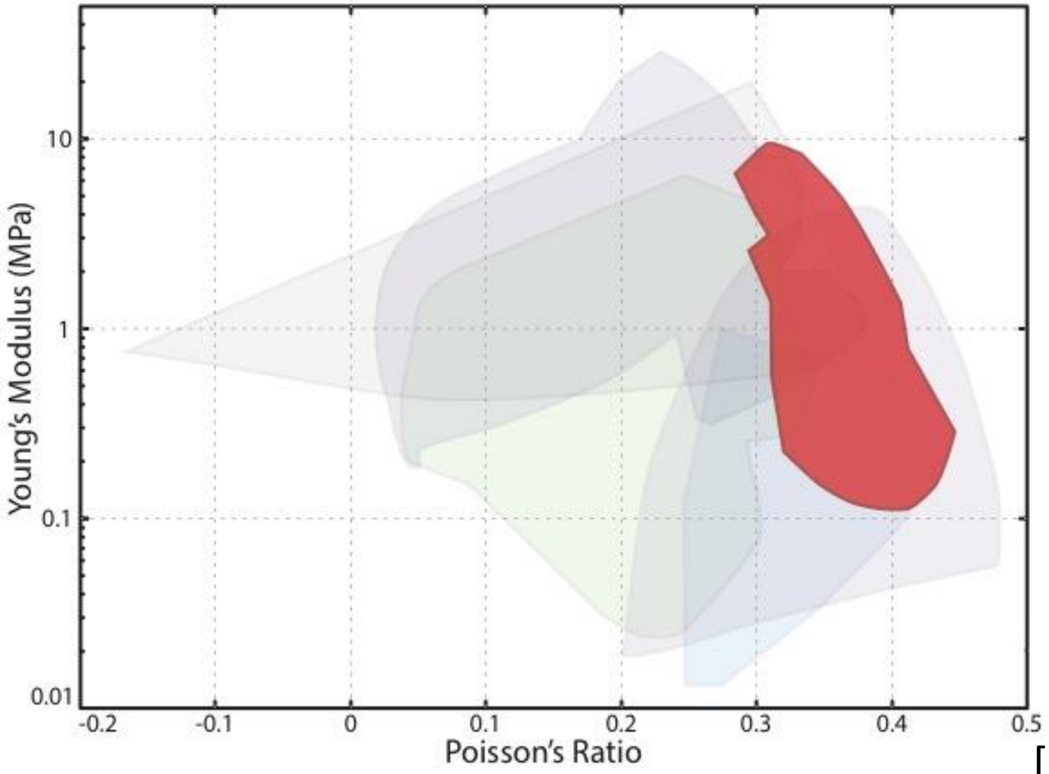
Shape Optimization Results





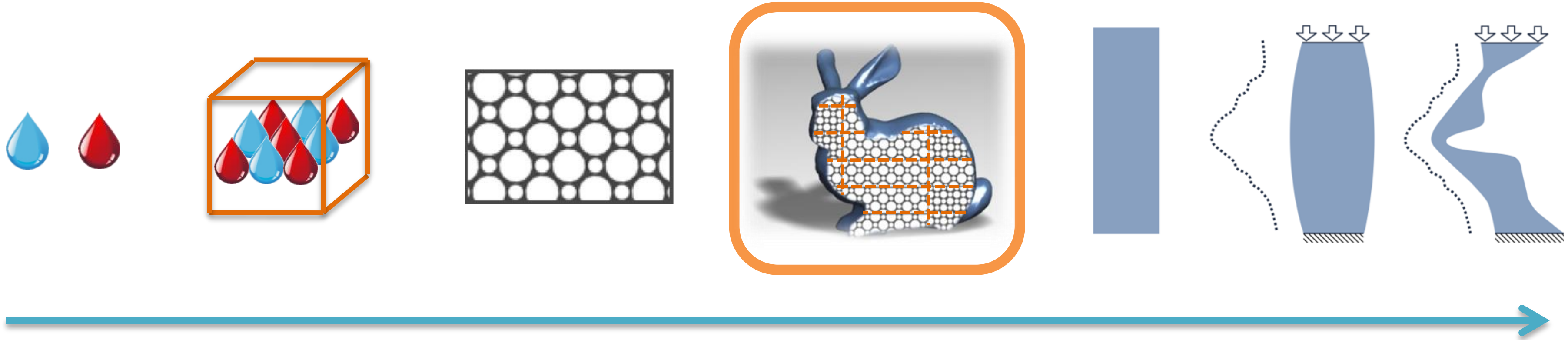
[Panetta et al. 2015]

Shape Optimization Results



[Panetta et al. 2015]

Manufacturing Deformable Materials



intrinsic — extrinsic

[Hiller and Lipson 2009]

[Bickel et al. 2010]
[Skouras et al. 2013]

[Bickel et al. 2012]
[Chen et al. 2014]

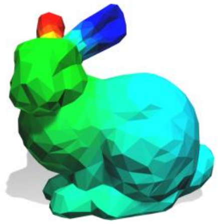
[Schumacher et al. 2015]
[Panetta et al. 2015]
[Martínez et al. 2016]

[Chen et al. 2013]

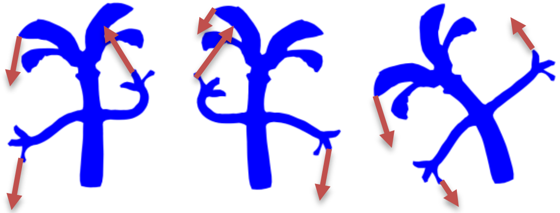
From Functional 2 Direct Specification

Input

- Shape with assigned Material Parameters

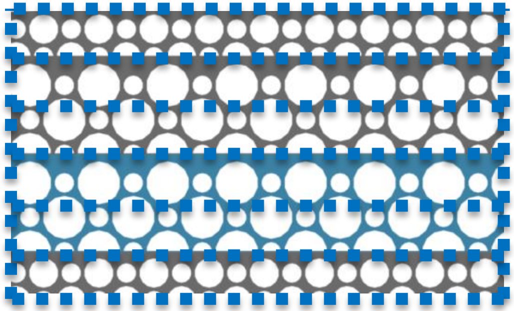


- Deformation Specification



Output

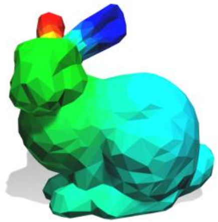
- Spatially-varying material structure



From Functional 2 Direct Specification

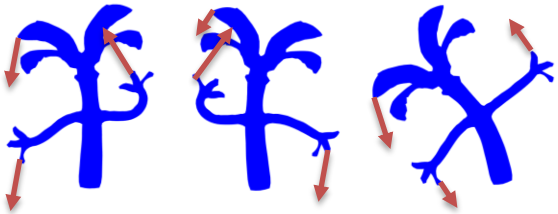
Input

- Shape with assigned Material Parameters



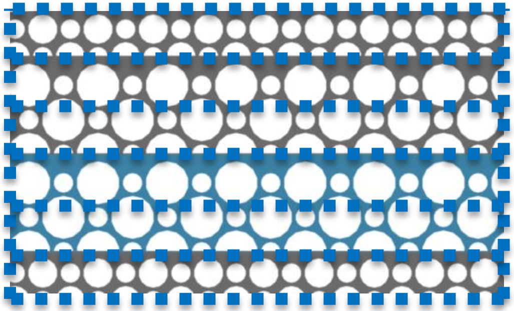
mapping

- Deformation Specification



Output

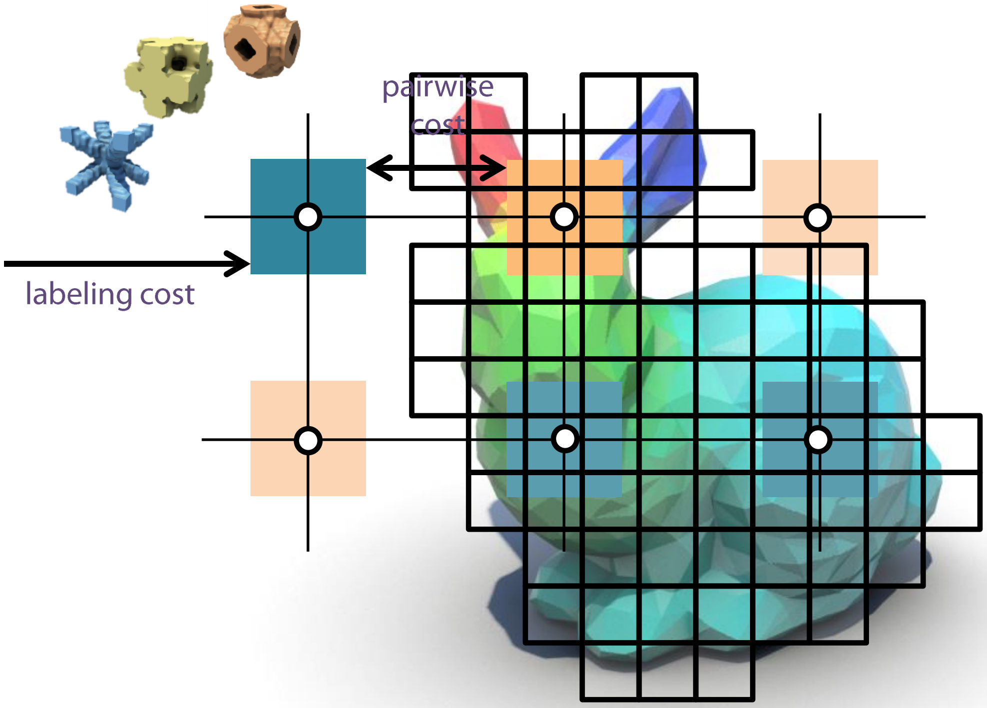
- Spatially-varying material structure



Challenge: Find optimal combinations

[Schumacher et al. 2015]

Synthesis



Message-Passing
ADMM

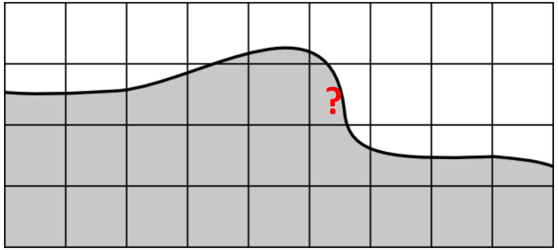
N. Derbinsky, J. Bento, V. Elser, J. S. Yedidia, *An improved three-weight message-passing algorithm* (arXiv:1305.1961, 2013)

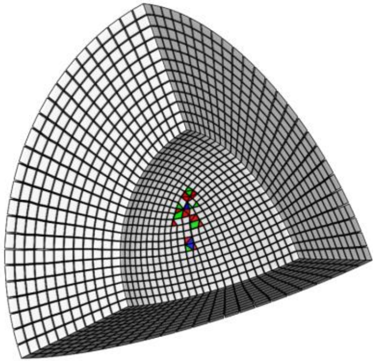
Challenge: Find optimal combinations

[Schumacher et al. 2015]

Periodic Tiling: Challenges

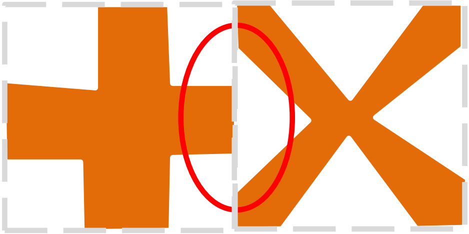
- Mapping? Possible, but difficult.

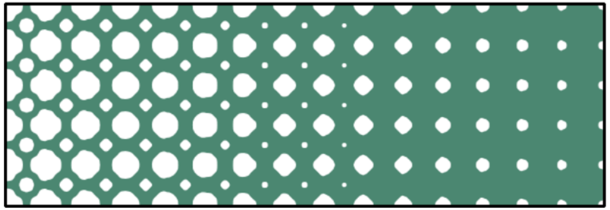




Hexahedral-dominant meshing
[Sokolov et al. 2015]

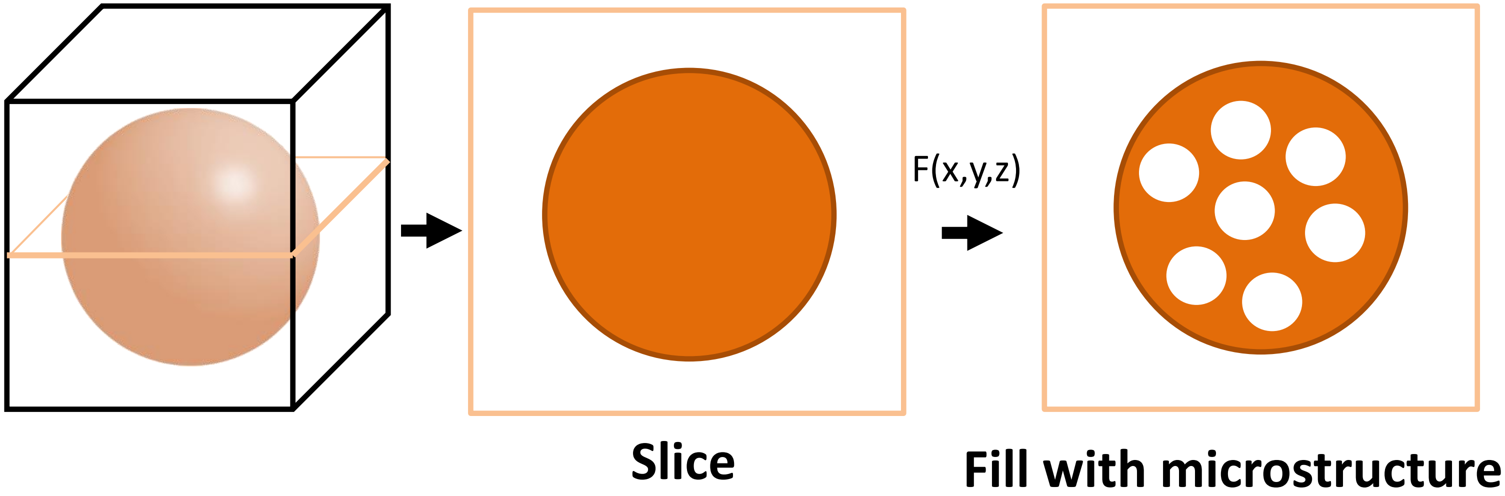
- Gradation? Possible, but transitions?



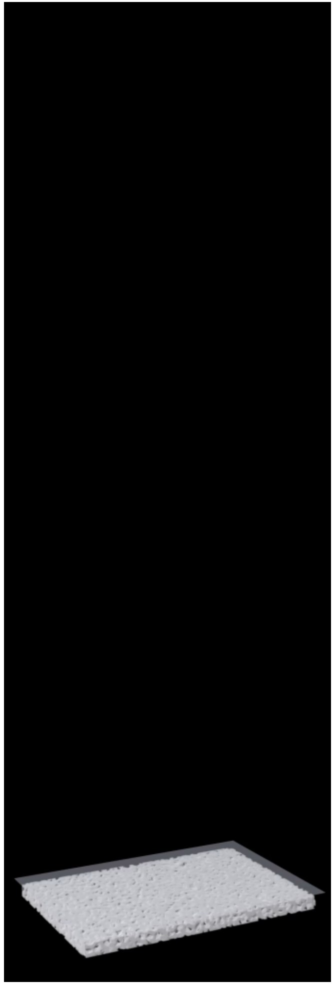


[Schumacher et al. 2015] – solves an optimization problem for finding compatible tilings

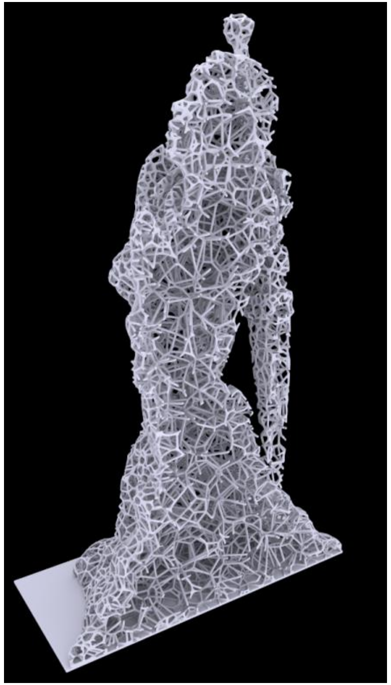
Procedural Synthesis for Fabrication



[Martínez et al. 2016]



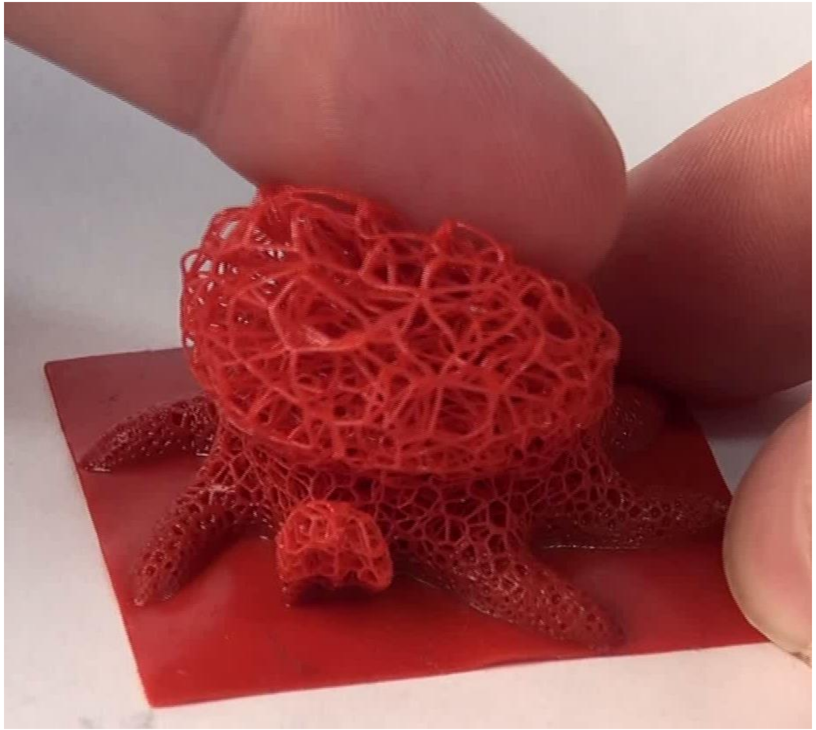
Procedural Synthesis for Fabrication

Printed with Autodesk Ember

[Martínez et al. 2016]

Procedural Synthesis for Fabrication: Result Cute Octopus



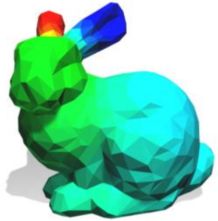
Printed with B9 Creator

[Martínez et al. 2016]

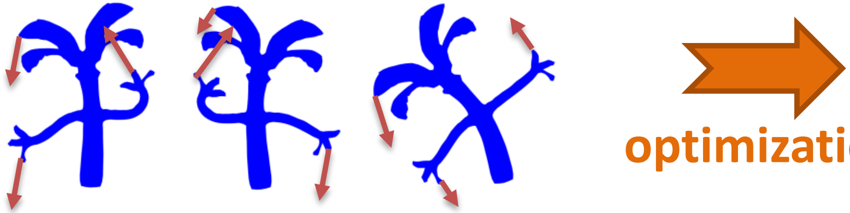
From Functional 2 Direct Specification

Input

- Shape with assigned Material Parameters



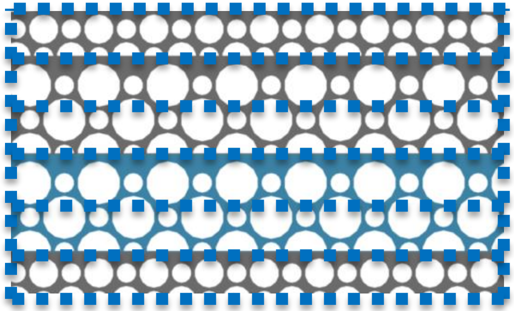
- Deformation Specification



optimization

Output

- Spatially-varying material structure



Problem Formulation

Design Parameters

$$E(\mathbf{x}, \mathbf{p}) = E_{match}(\mathbf{x}, \mathbf{x}_{target}) \quad \text{subject to} \quad \mathbf{f}^{total}(\mathbf{x}, \mathbf{p}) = 0$$

Problem Formulation

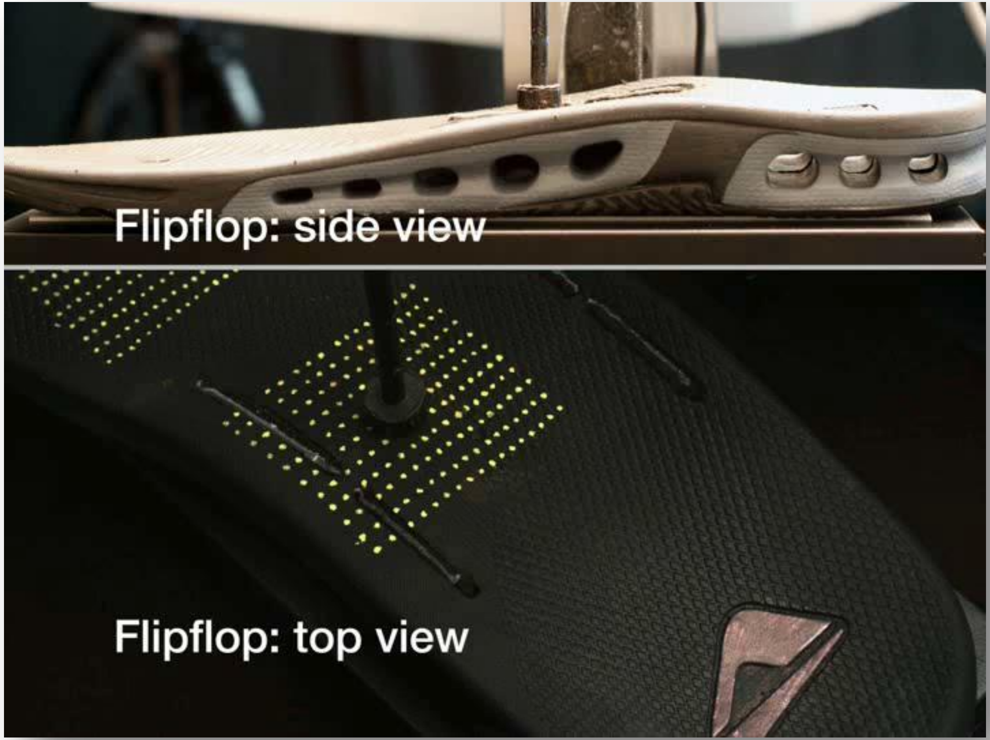
Design Parameters

$$E(\mathbf{x}, \mathbf{p}) = E_{match}(\mathbf{x}, \mathbf{x}_{target}) \quad \text{subject to} \quad \mathbf{f}^{total}(\mathbf{x}, \mathbf{p}) = 0$$

Optimization Strategies:

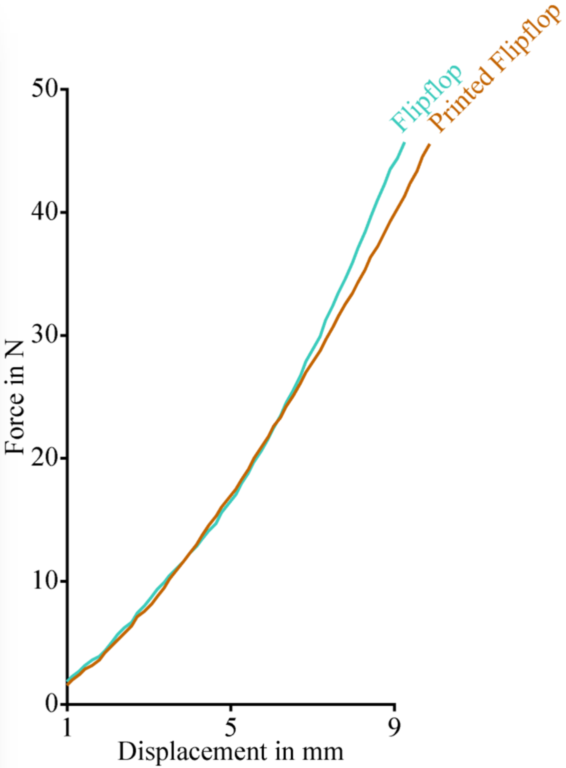
- Discrete ([Bickel et al. 2010])
- Continuous ([Skouras et al. 2013])

Result



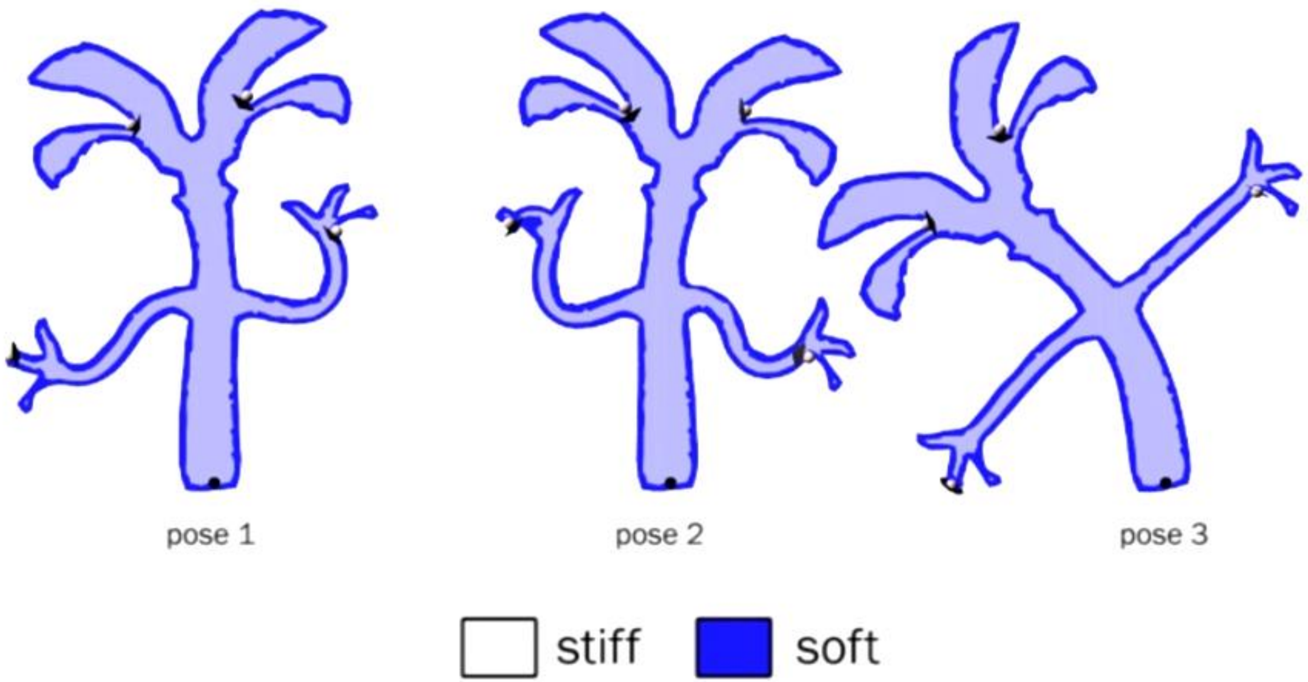
[Bickel et al. 2010]

Ready to Wear

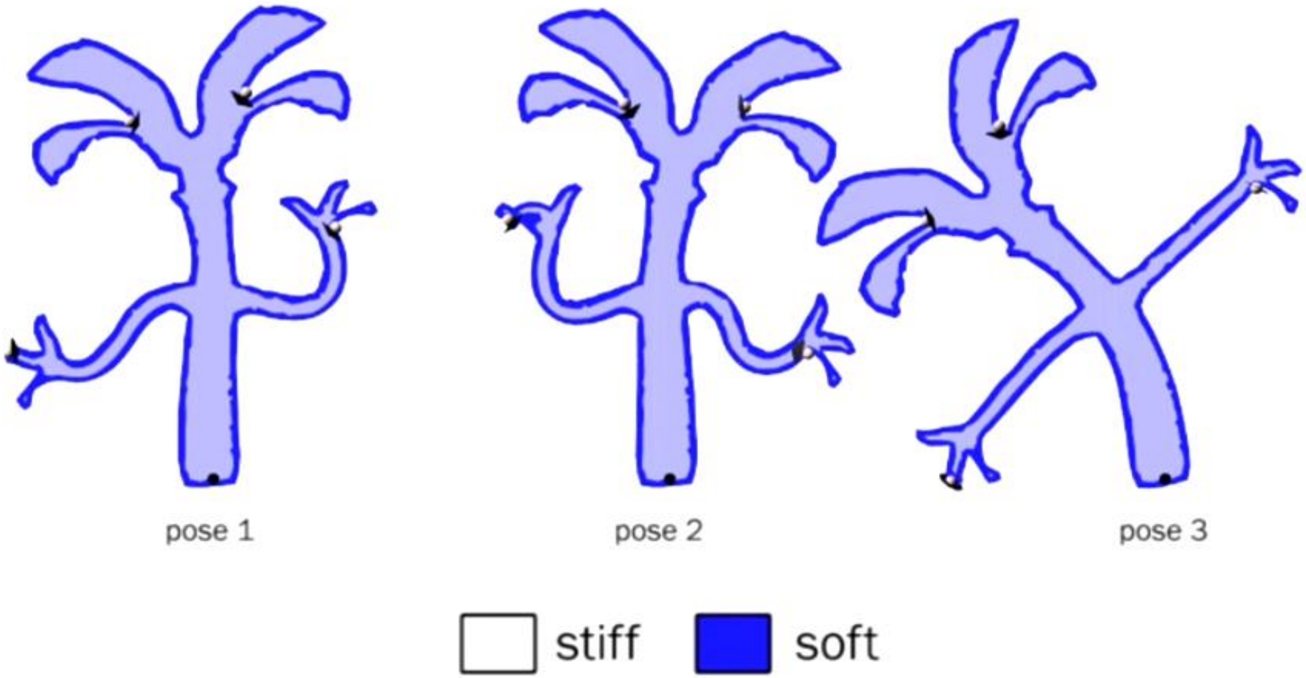


[Bickel et al. 2010]

Material Distribution Optimization

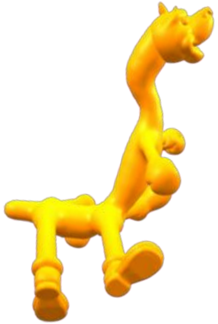


Material Distribution Optimization



Results

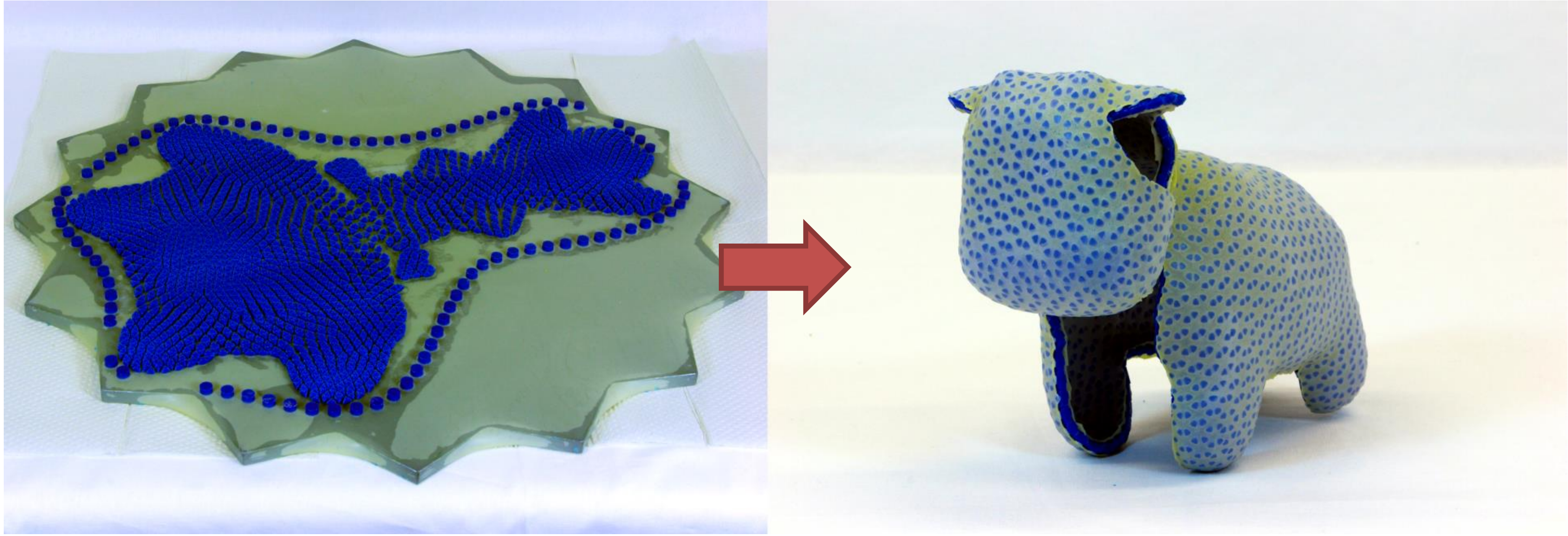
Rest Pose



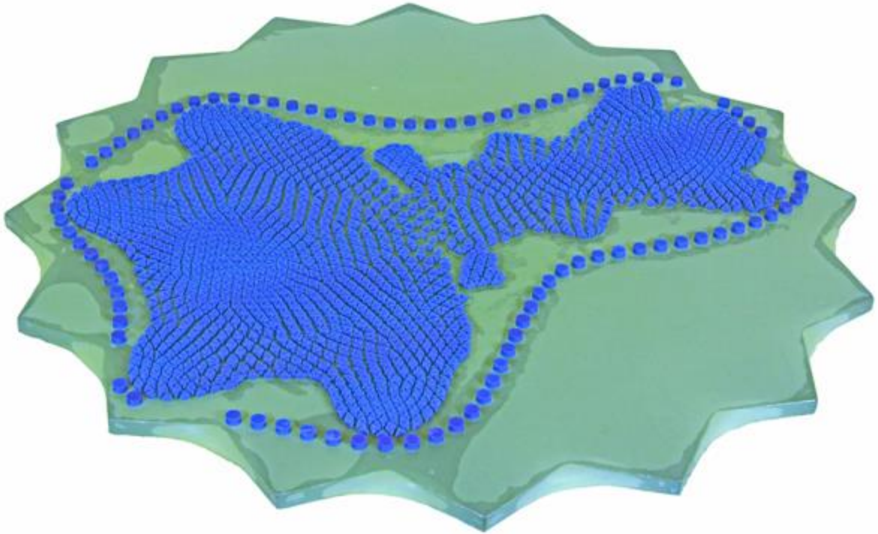
Target Pose

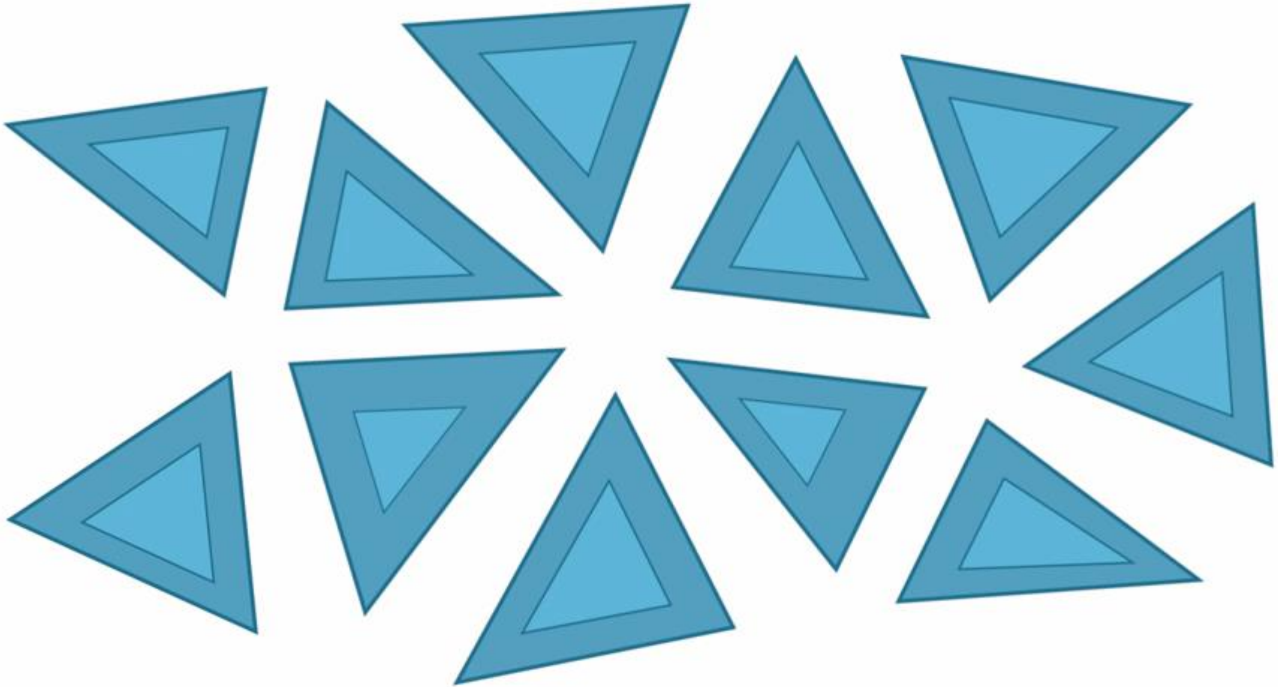
Stiff Soft

CurveUps



CurveUps: Shaping Objects from Flat Plates with Tension-Actuated Curvature
R. Guseinov, E. Miguel, B. Bickel
ACM Transactions on Graphics (Proc. SIGGRAPH 2017)



Fabrication

Conclusion

- Summary
  - Control at various levels
  - Techniques could be combined
  - Interactive vs. Specification-Based Design

- Limitations / Future Work
  - Scaling
  - Non-linear material behavior can be very complex
  - Fabrication constraints often quite specifc
  - 3D printer
    - Durability of materials
    - Handling of materials

Thank you!

Acknowledgements

Niels Aage, Melina Skouras, Jeremie Dumas, Sylvain Lefebvre, and Julian Panetta for providing slides/image material.

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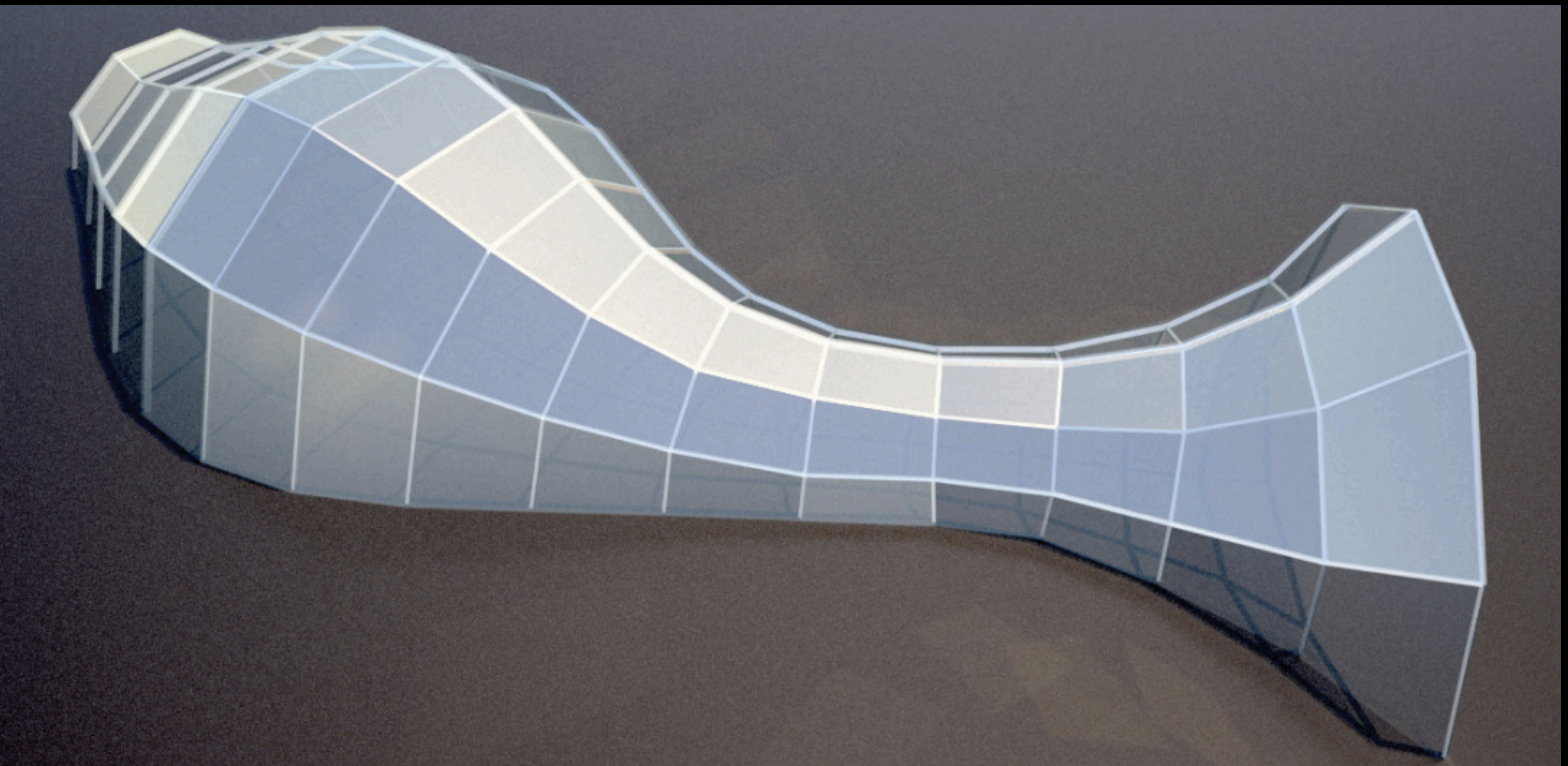
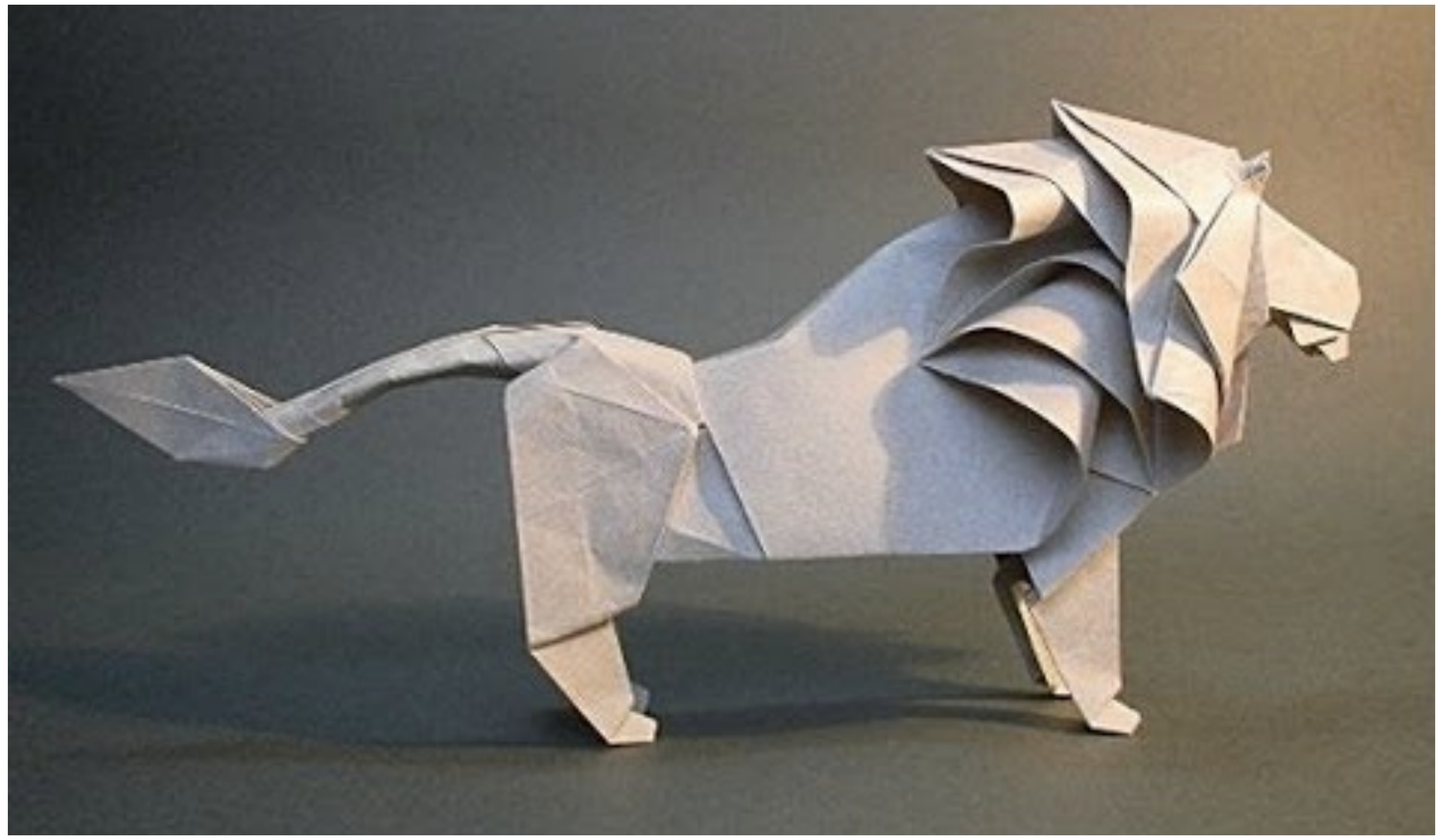
Martínez, J., Dumas, J., & Lefebvre, S. (2016). Procedural voronoi foams for additive manufacturing. ACM Transactions on Graphics (TOG), 35(4), 44.

*Simplifying Design of **Foldable** Shapes*

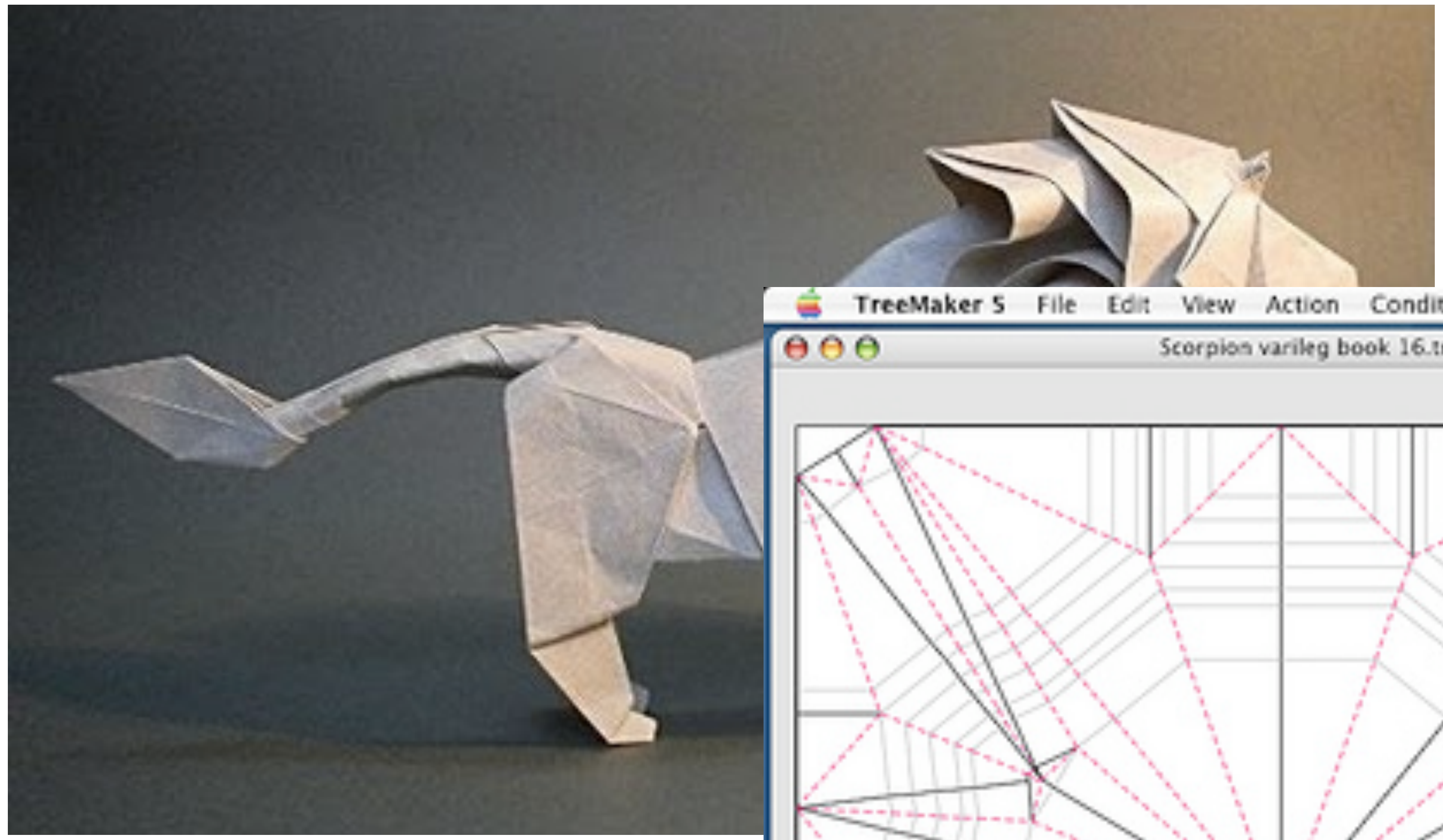
Niloy J. Mitra

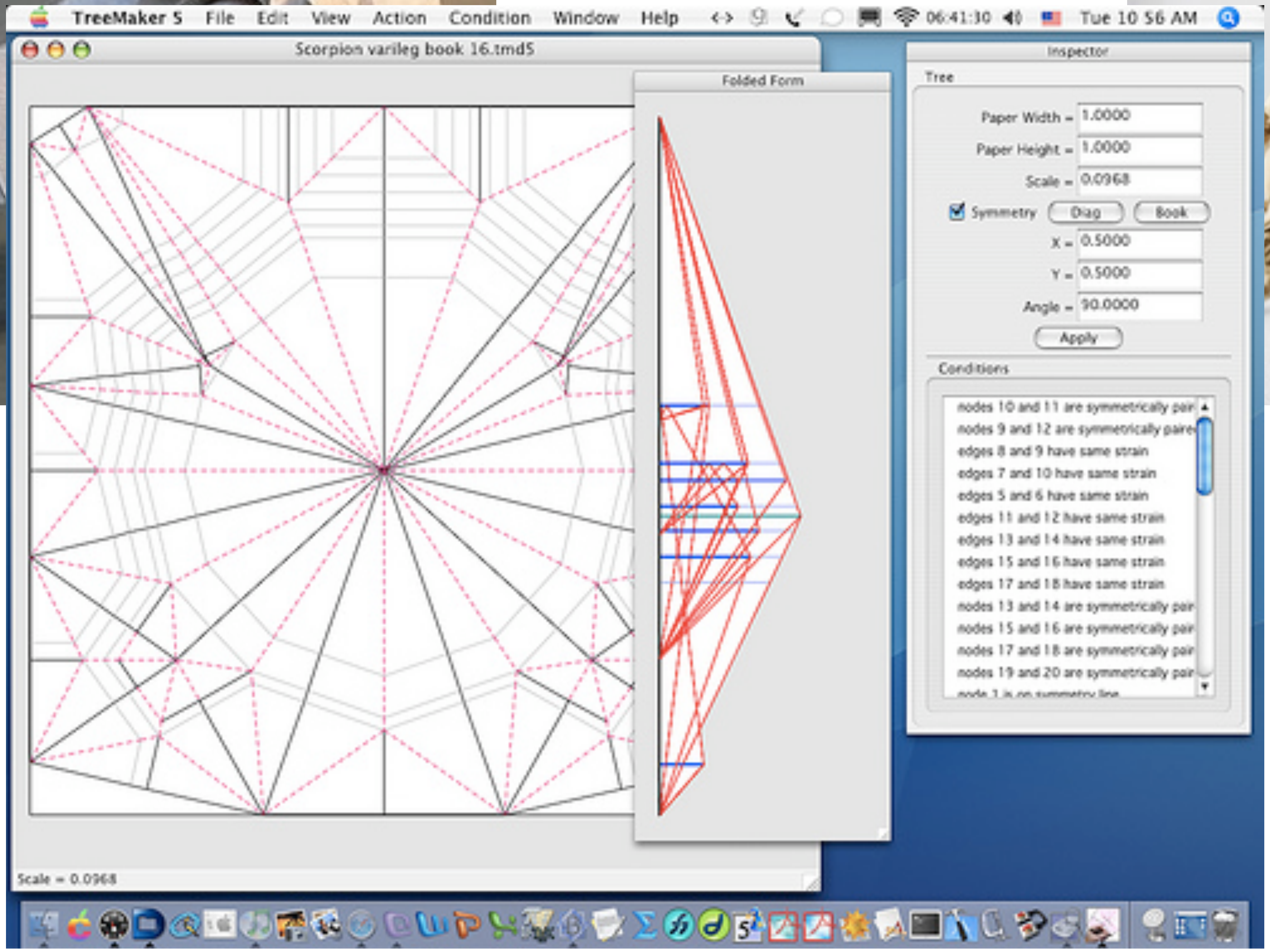


Origami



Origami





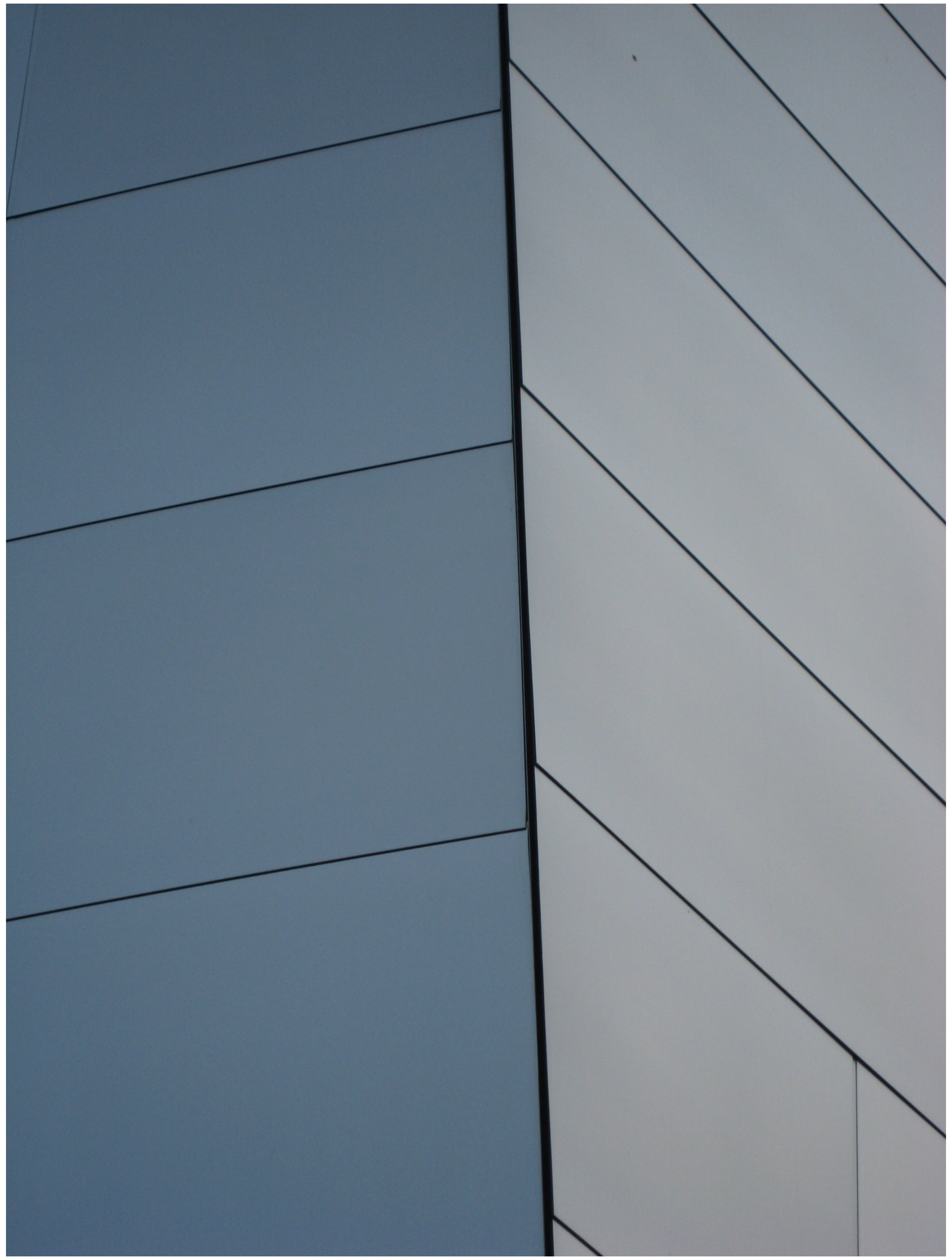
TreeMaker

Curved Folding

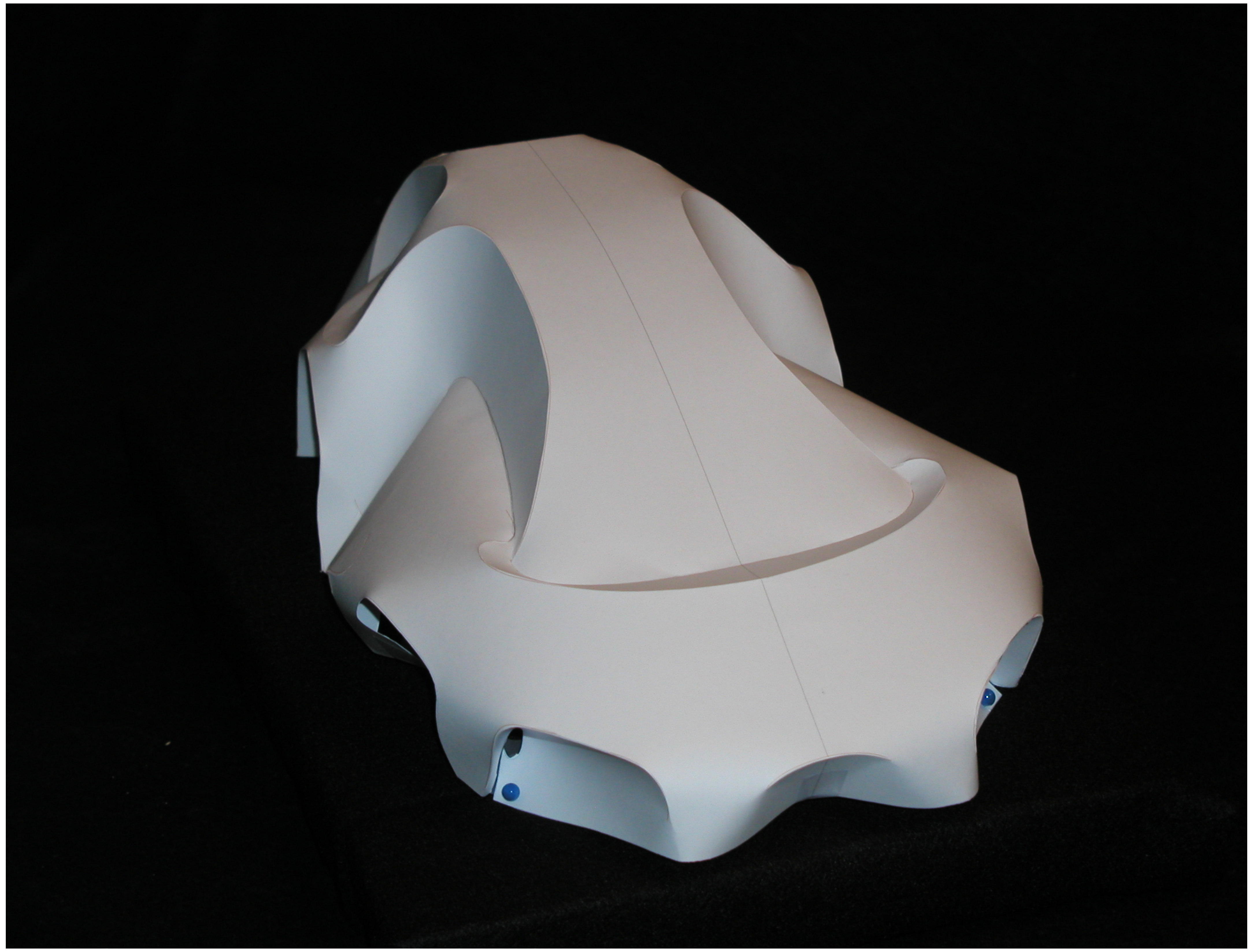
Disney Concert Hall, F. Gehry

[w/ Kilian, Sheffer, Pottmann, et al.]

Piecewise Developable Surface

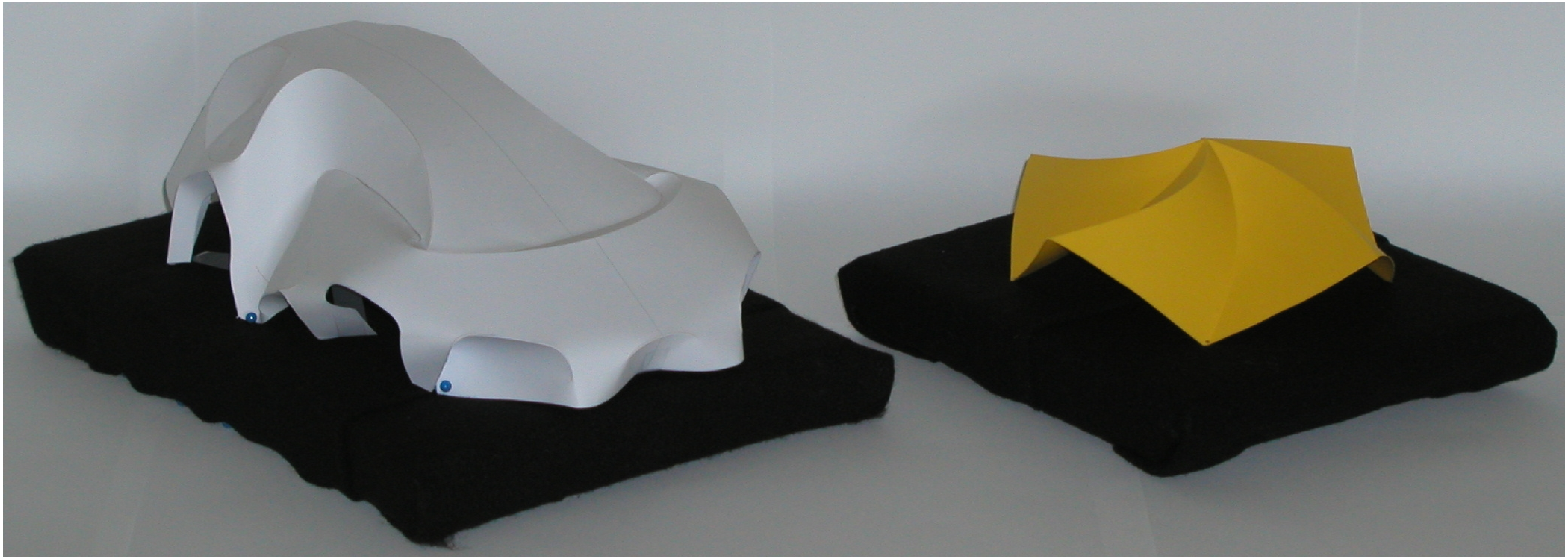


Inspiration

created by: David Huffman, Gregory Epps

How to Create One?



Developable Surface

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- vanishing Gaussian curvature

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- non-flat $\Rightarrow$ 1-parameter family of tangent planes

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- non-flat $\Rightarrow$ 1-parameter family of tangent planes
  - cones, cylinders, tangent surfaces

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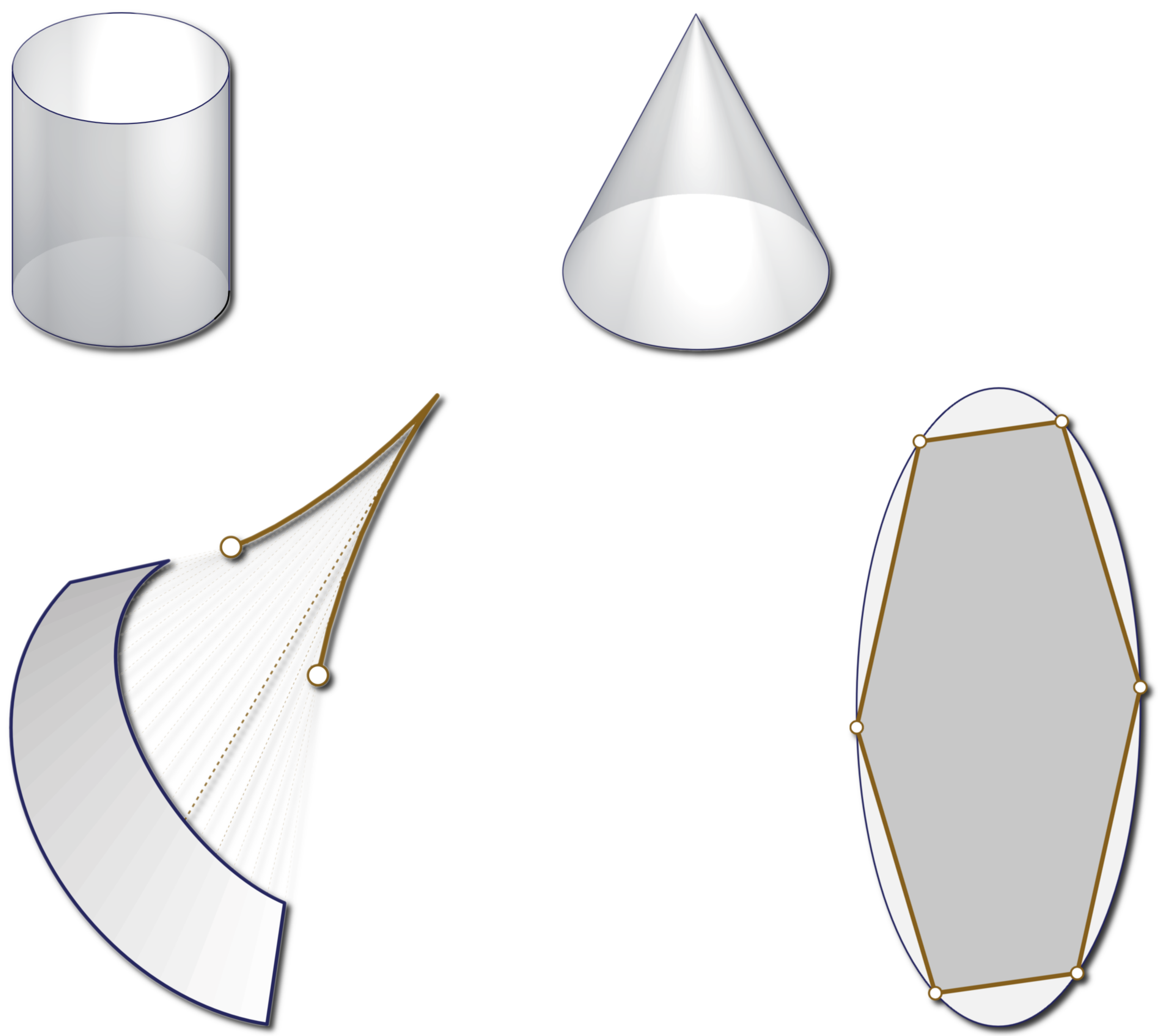
  - cones, cylinders, tangent surfaces
- $\Rightarrow$ ruled surface

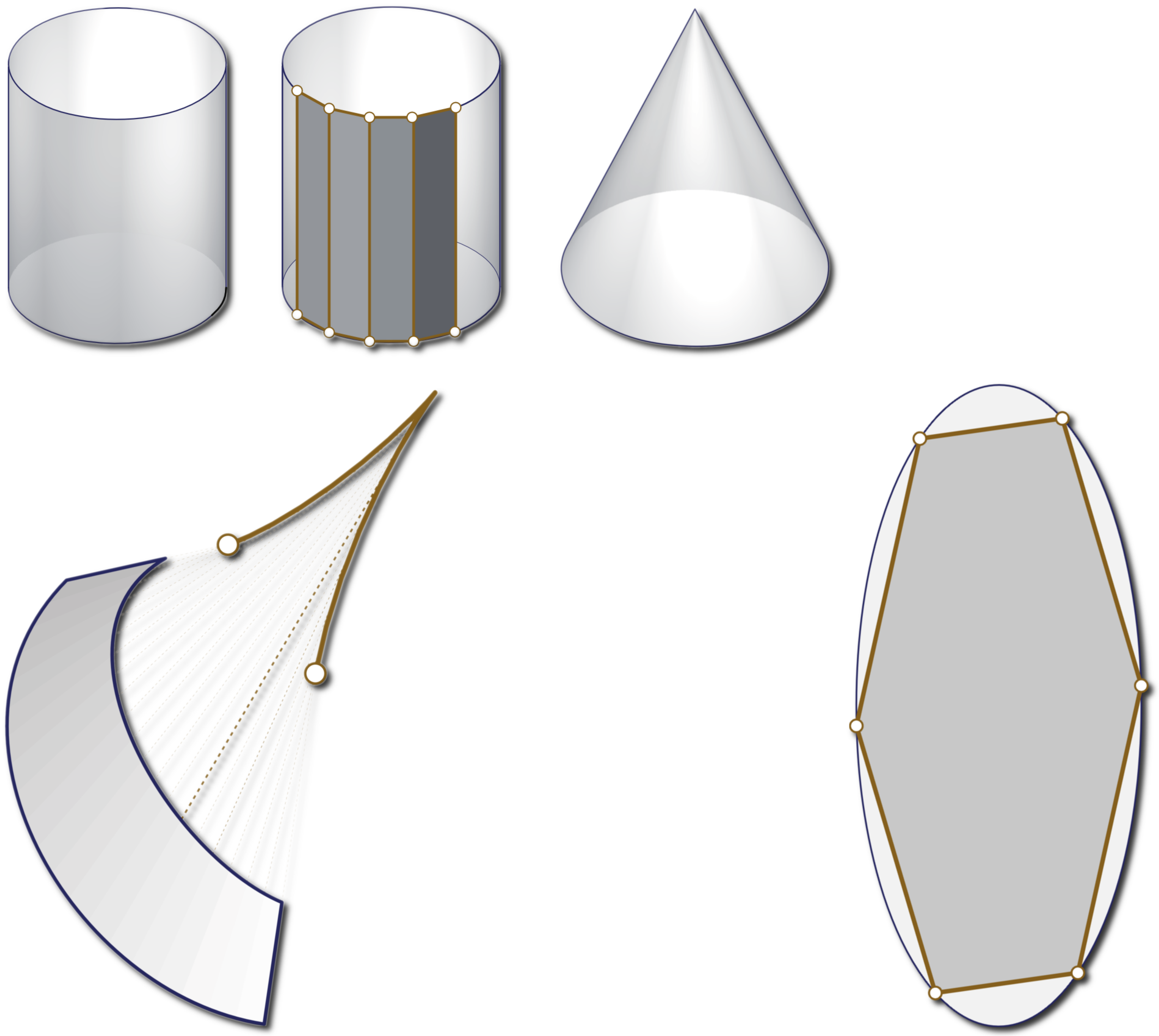
Developable Surface

- vanishing Gaussian curvature
- non-flat $\Rightarrow$ 1-parameter family of tangent planes
  - cones, cylinders, tangent surfaces
- $\Rightarrow$ ruled surface
  - same tangent plane along same generator (ruling)

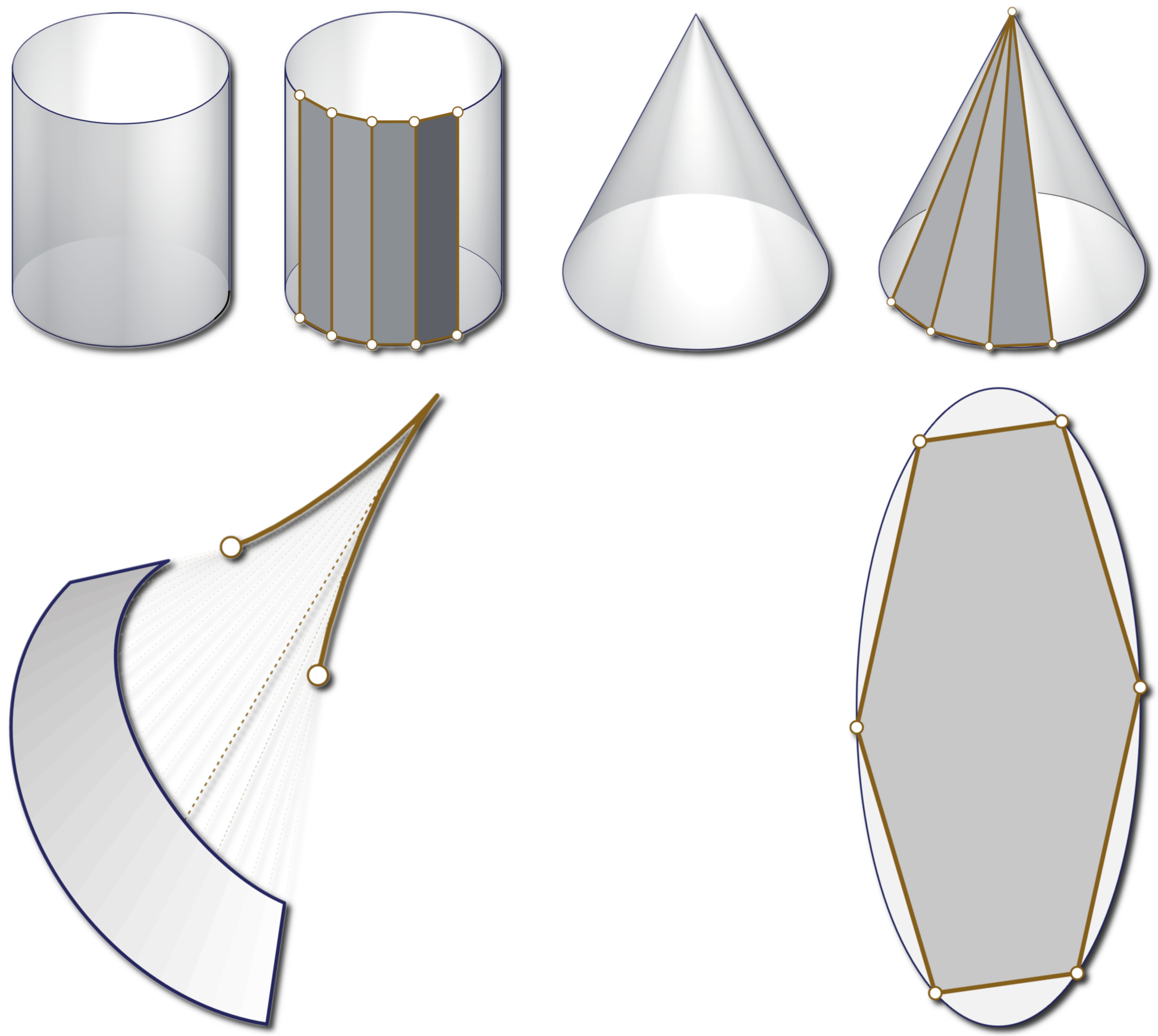
Developable Patches



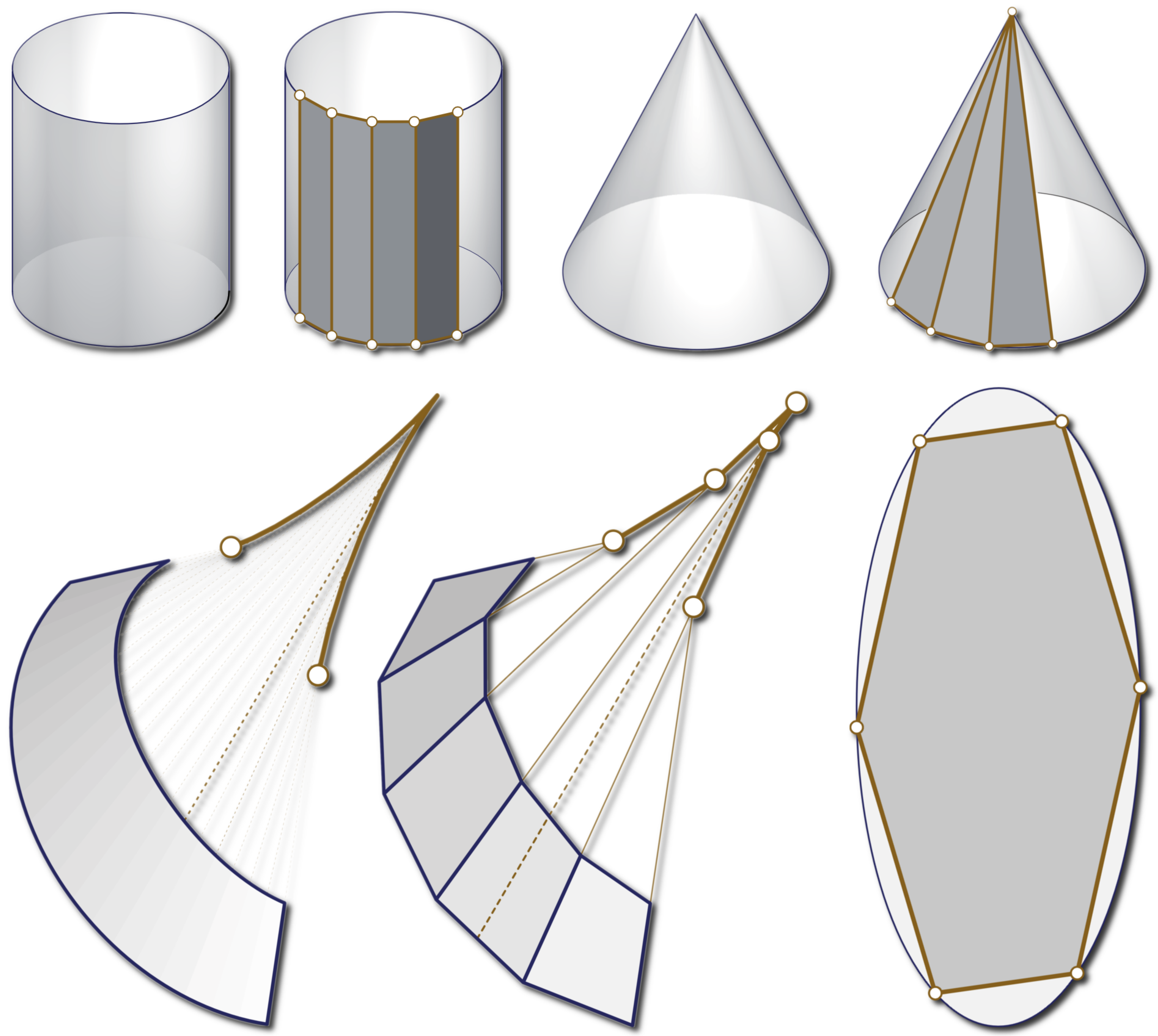
Developable Patches



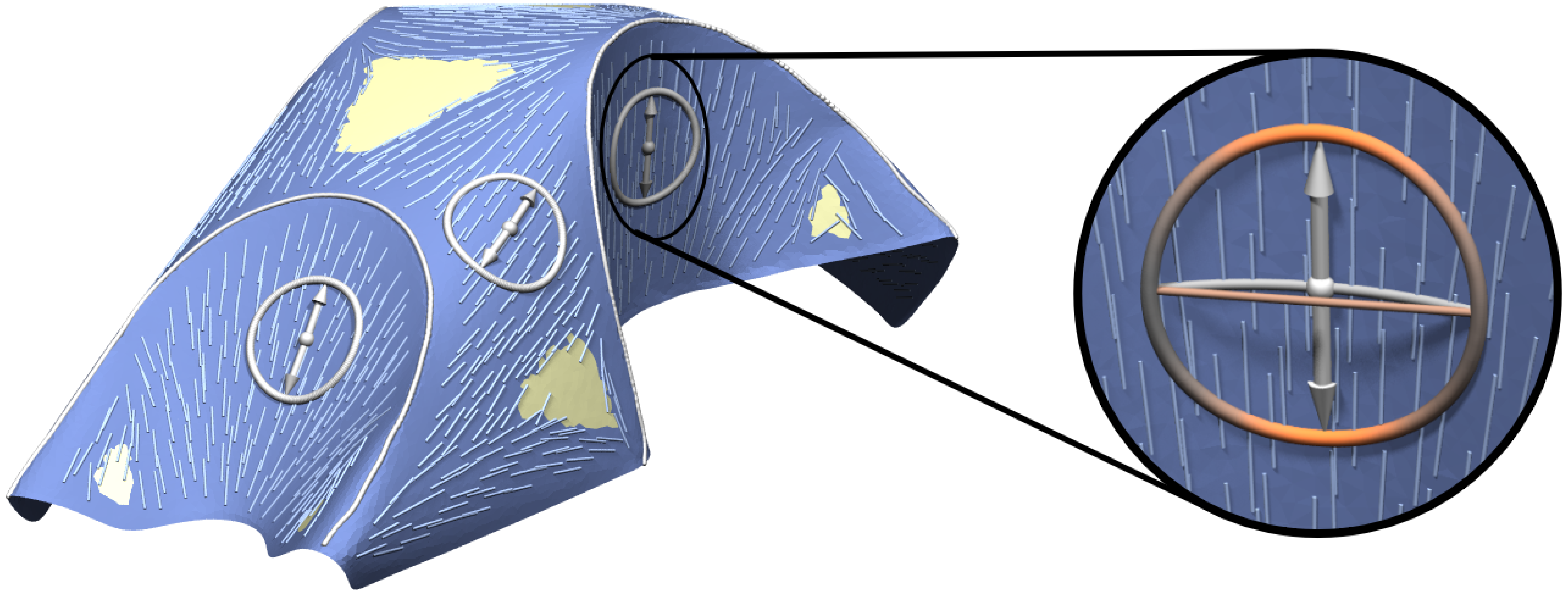
Developable Patches



Developable Patches



Step 1: Estimate Rulings



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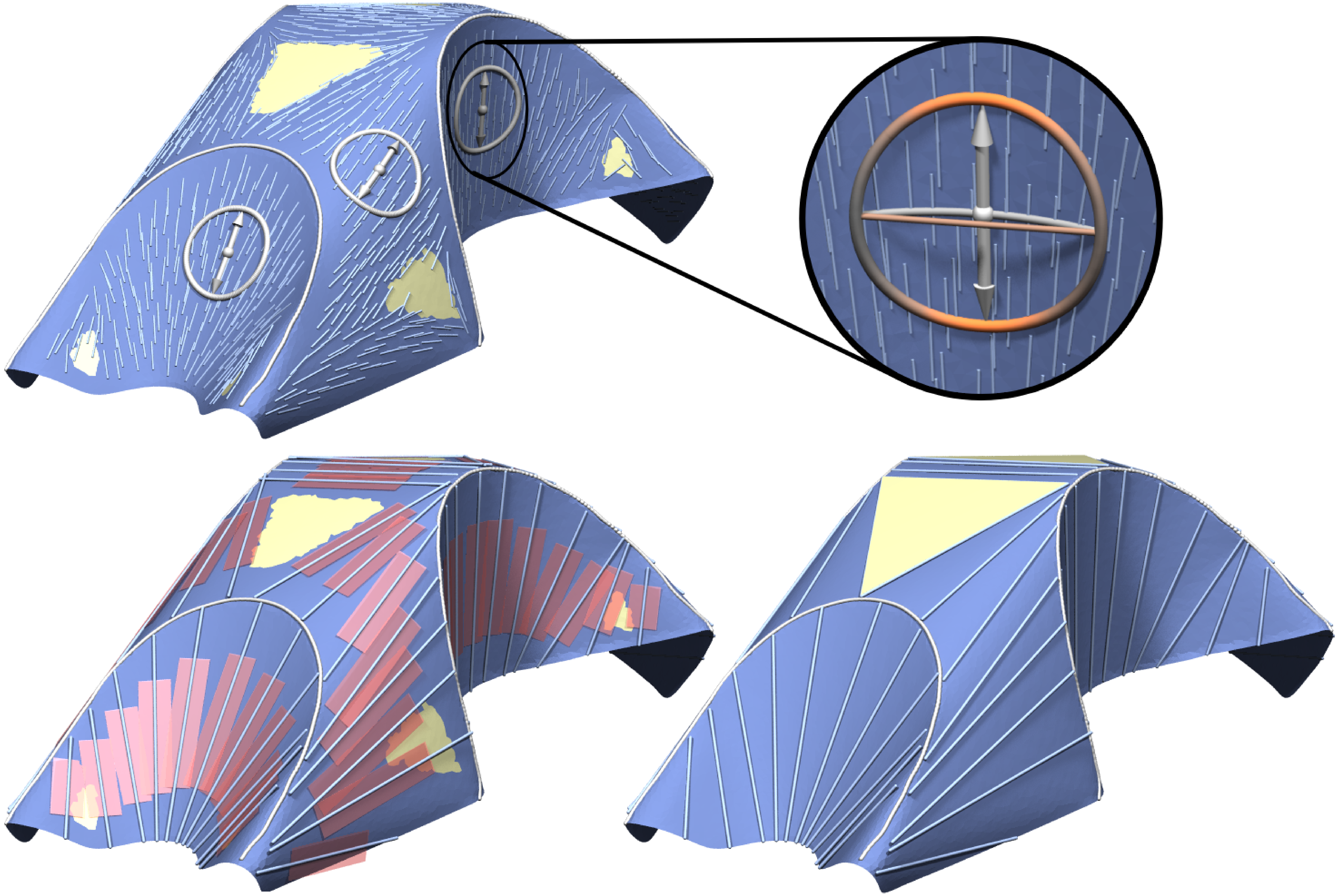
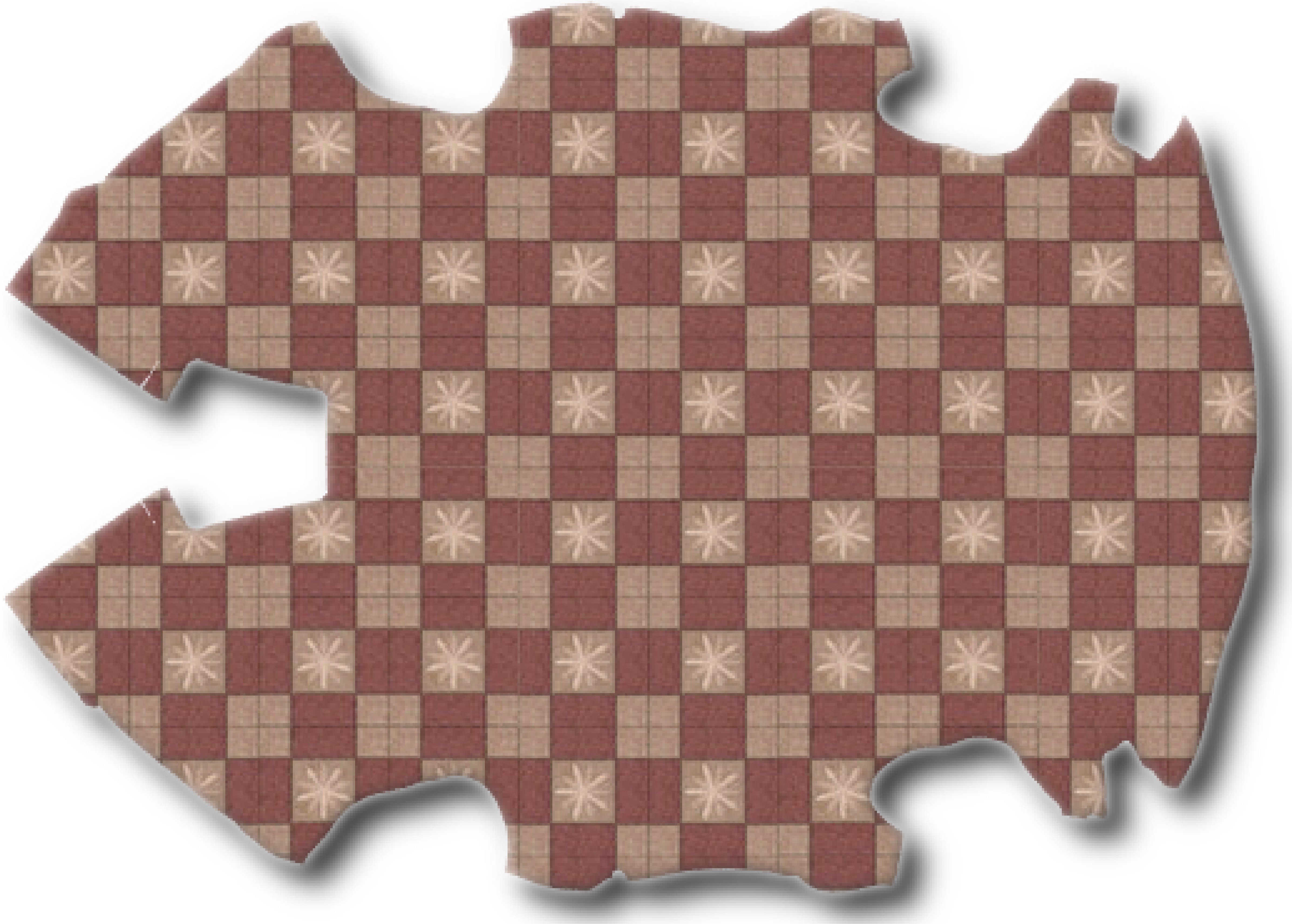
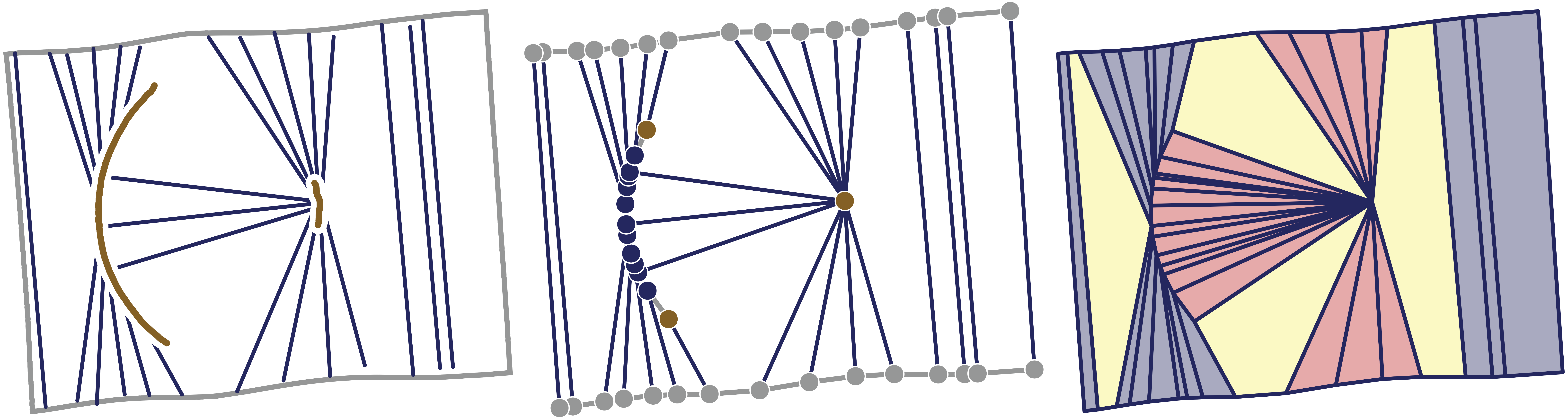


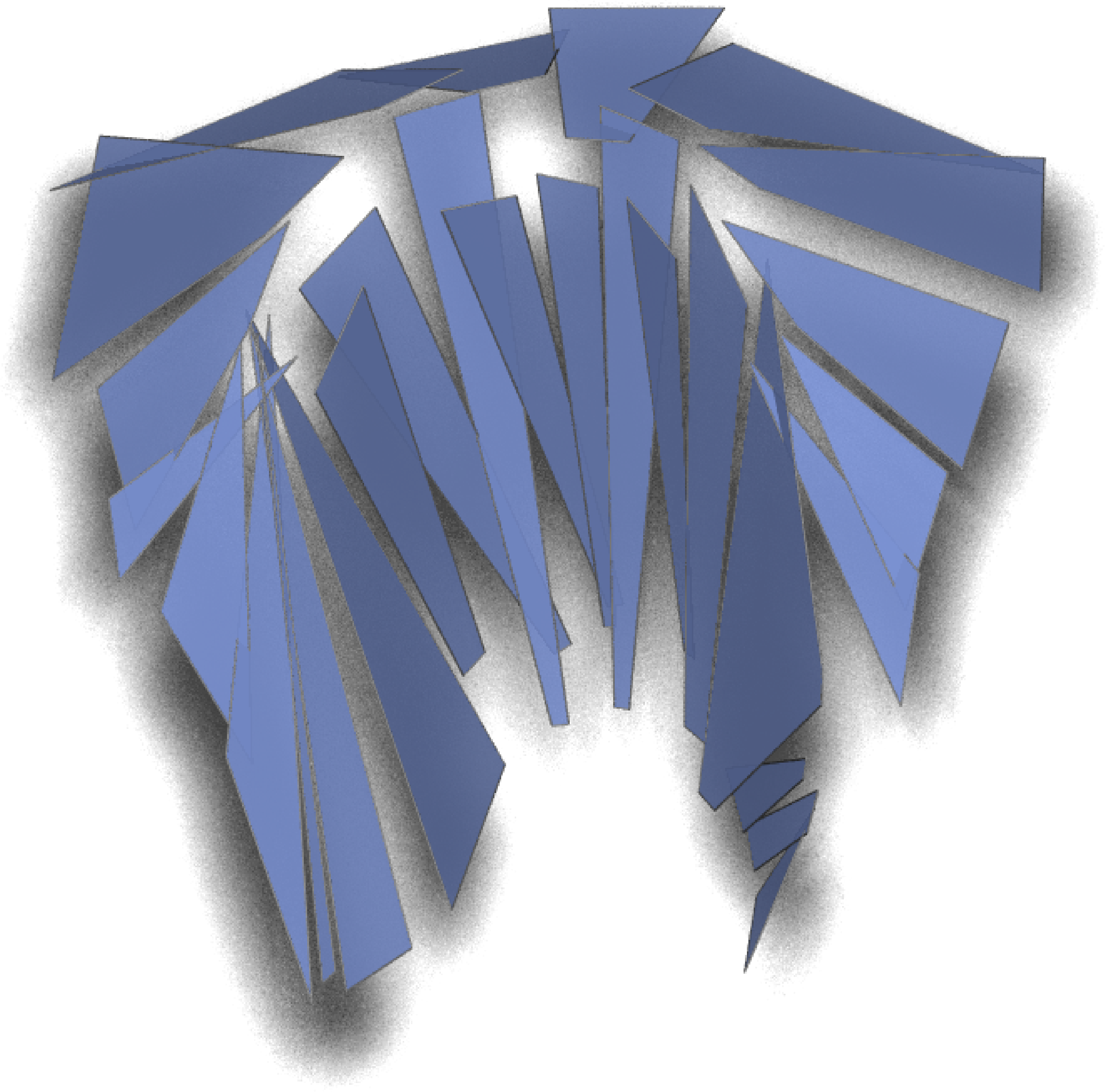
Step 2: Unfold to a Plane



Step 3: Quad Mesh Initialization

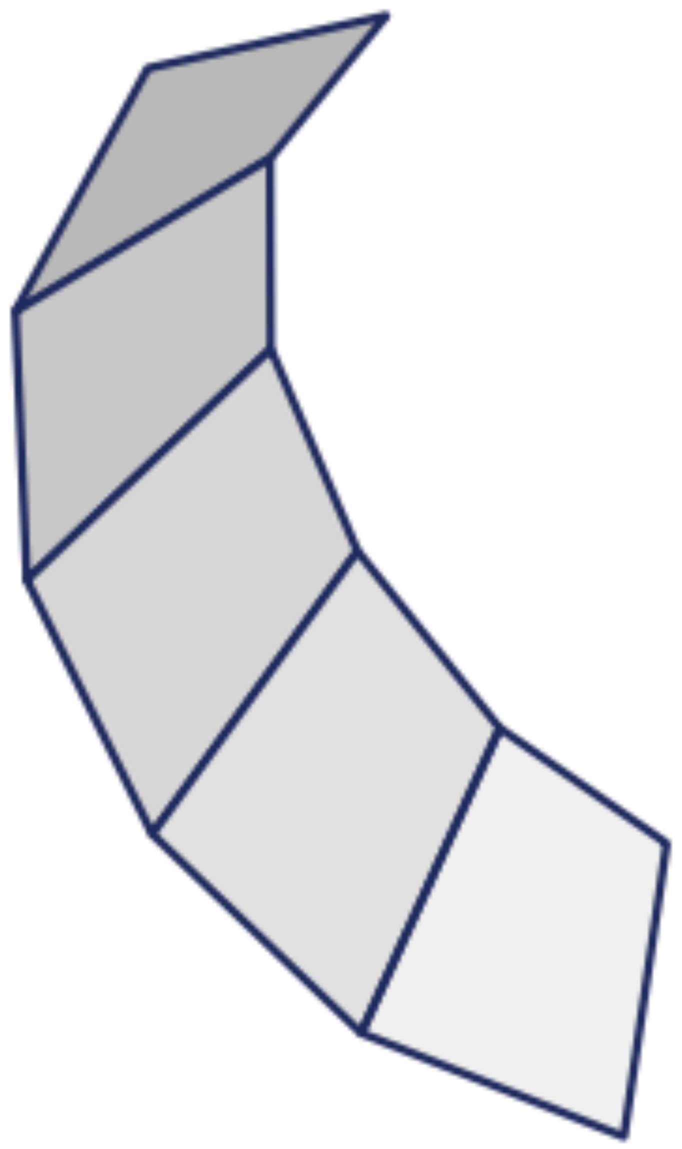


Step 4: 2D-3D Optimization



Representation

Representation



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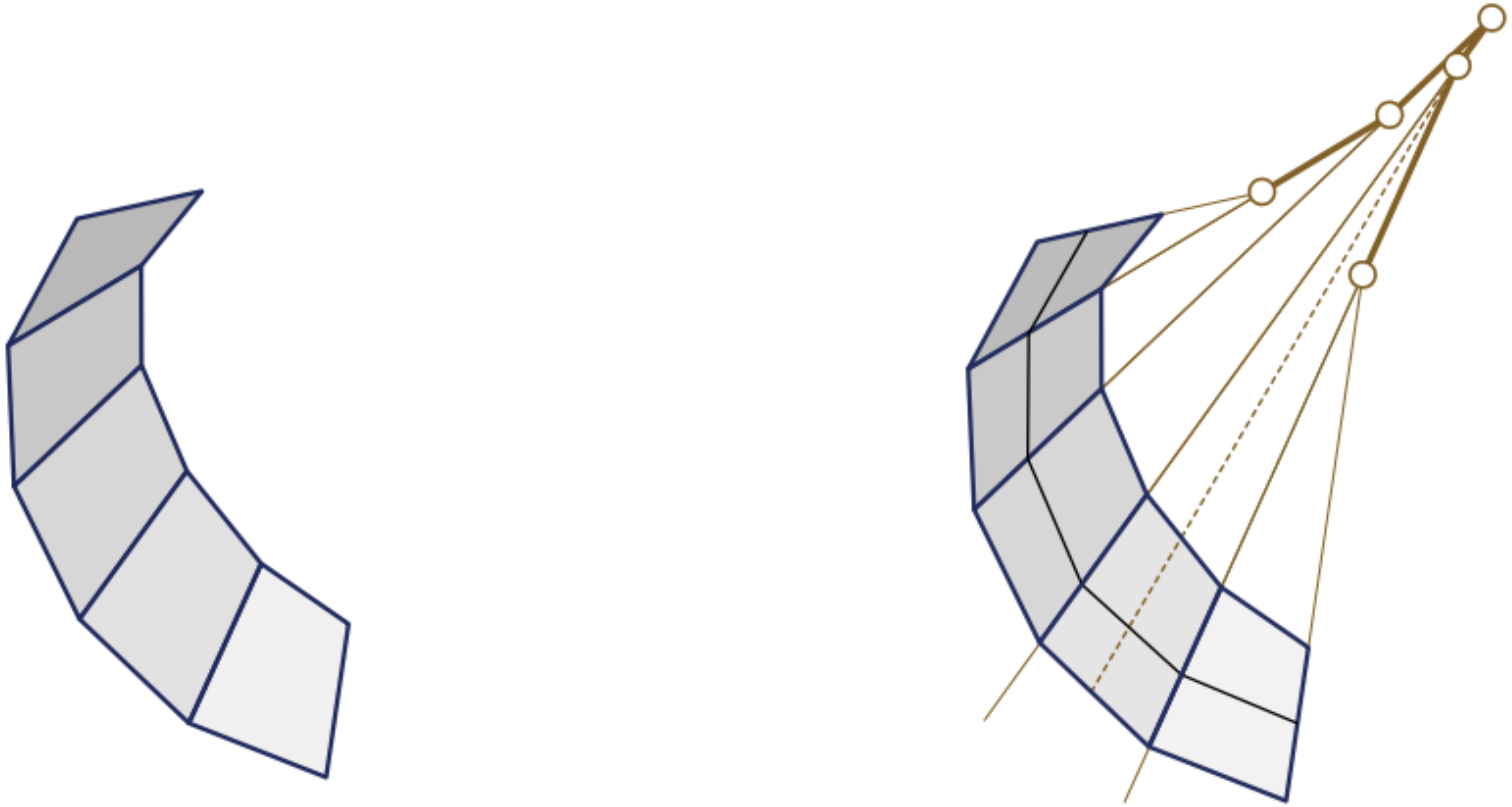
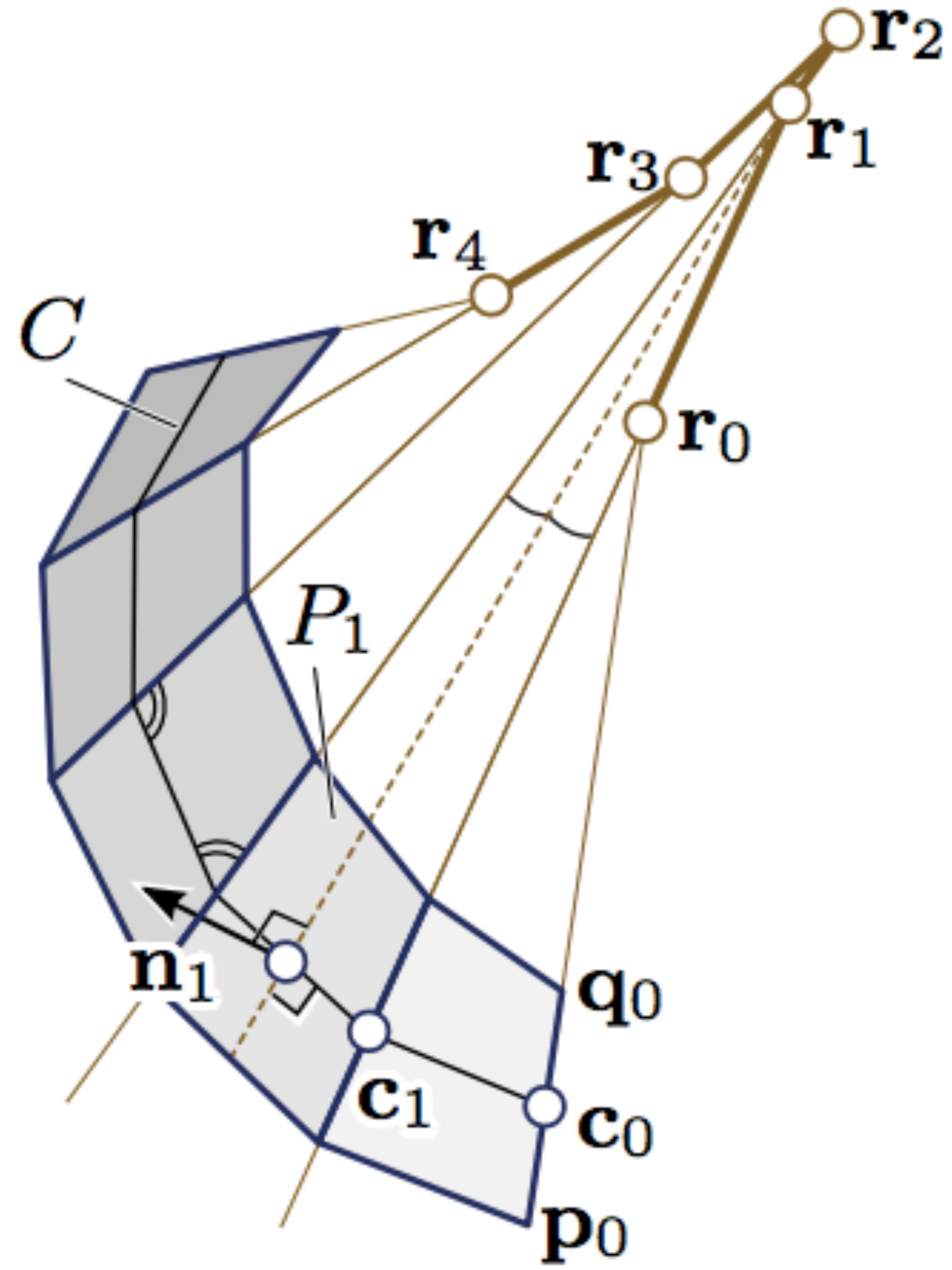
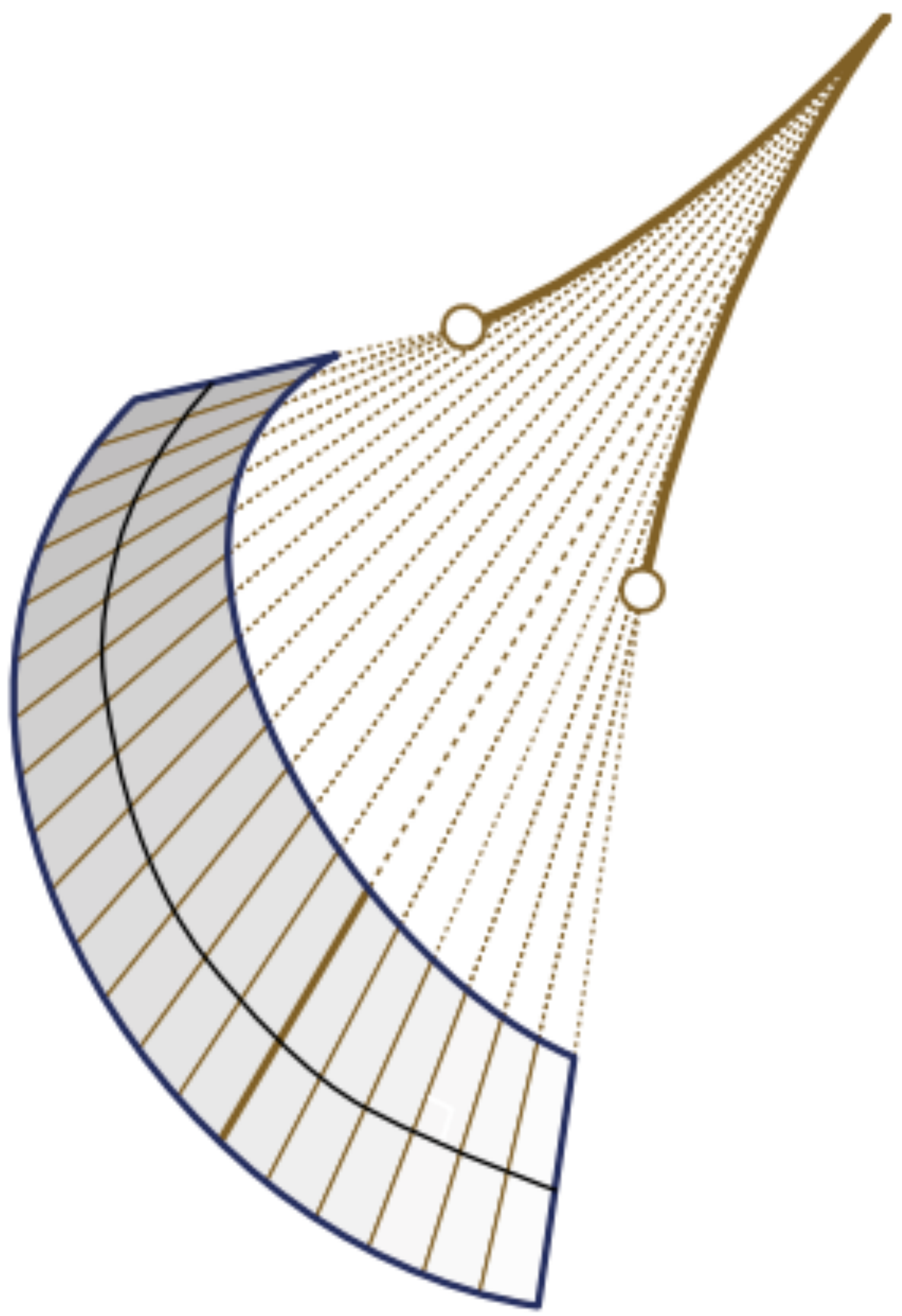
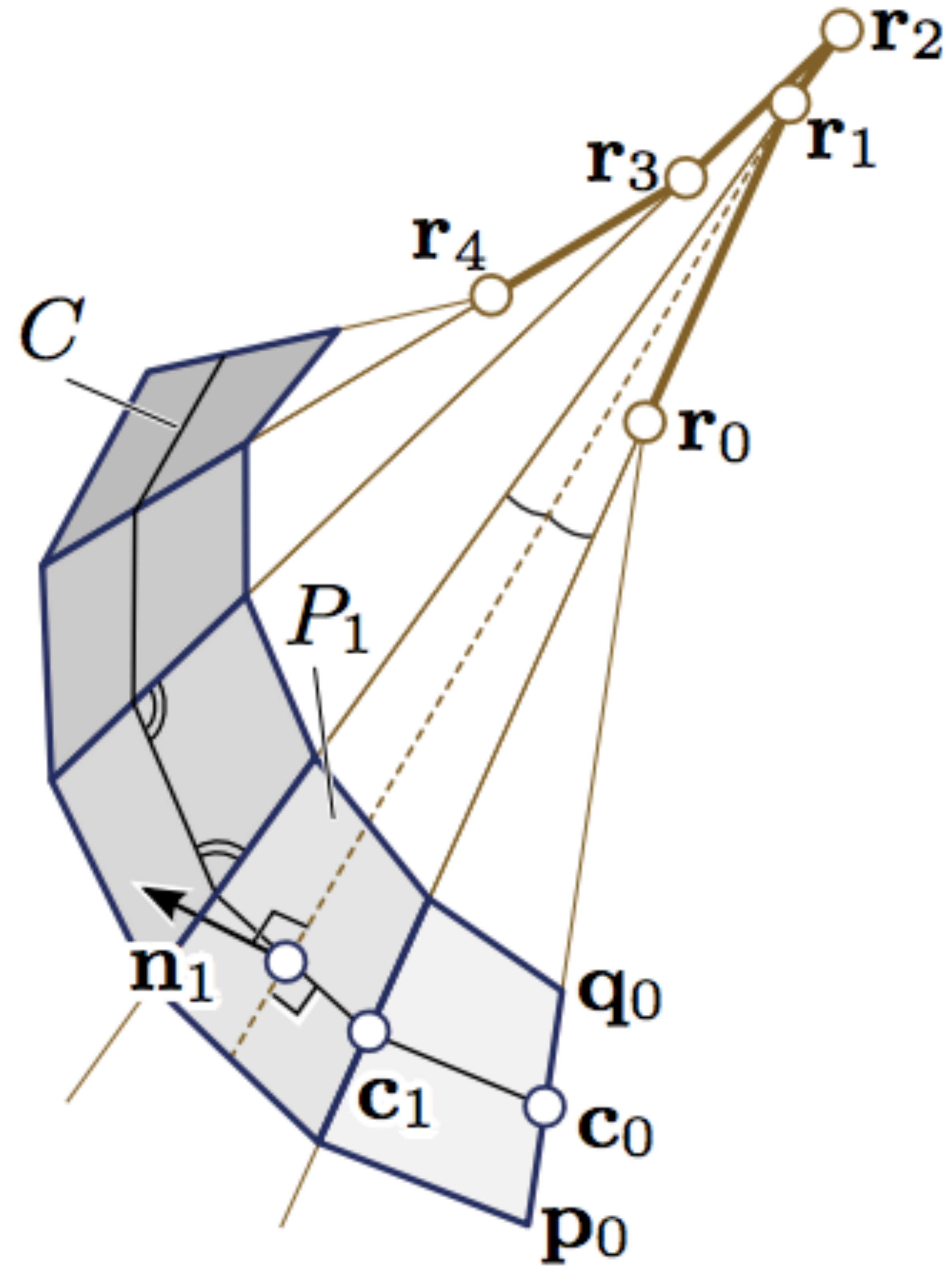


Discretizing Curvature



Discretizing Curvature





Discretizing ‘Folds’

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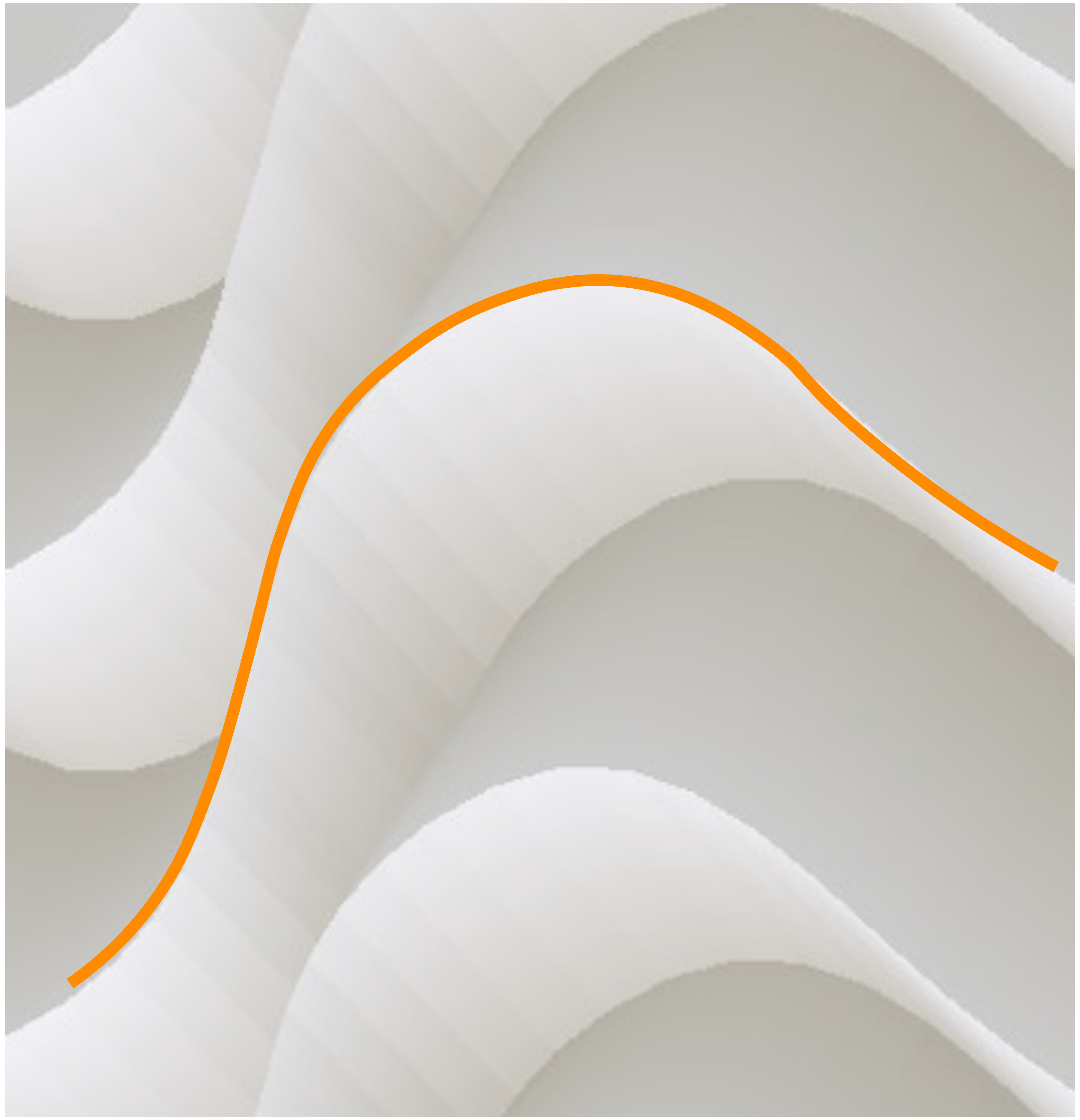
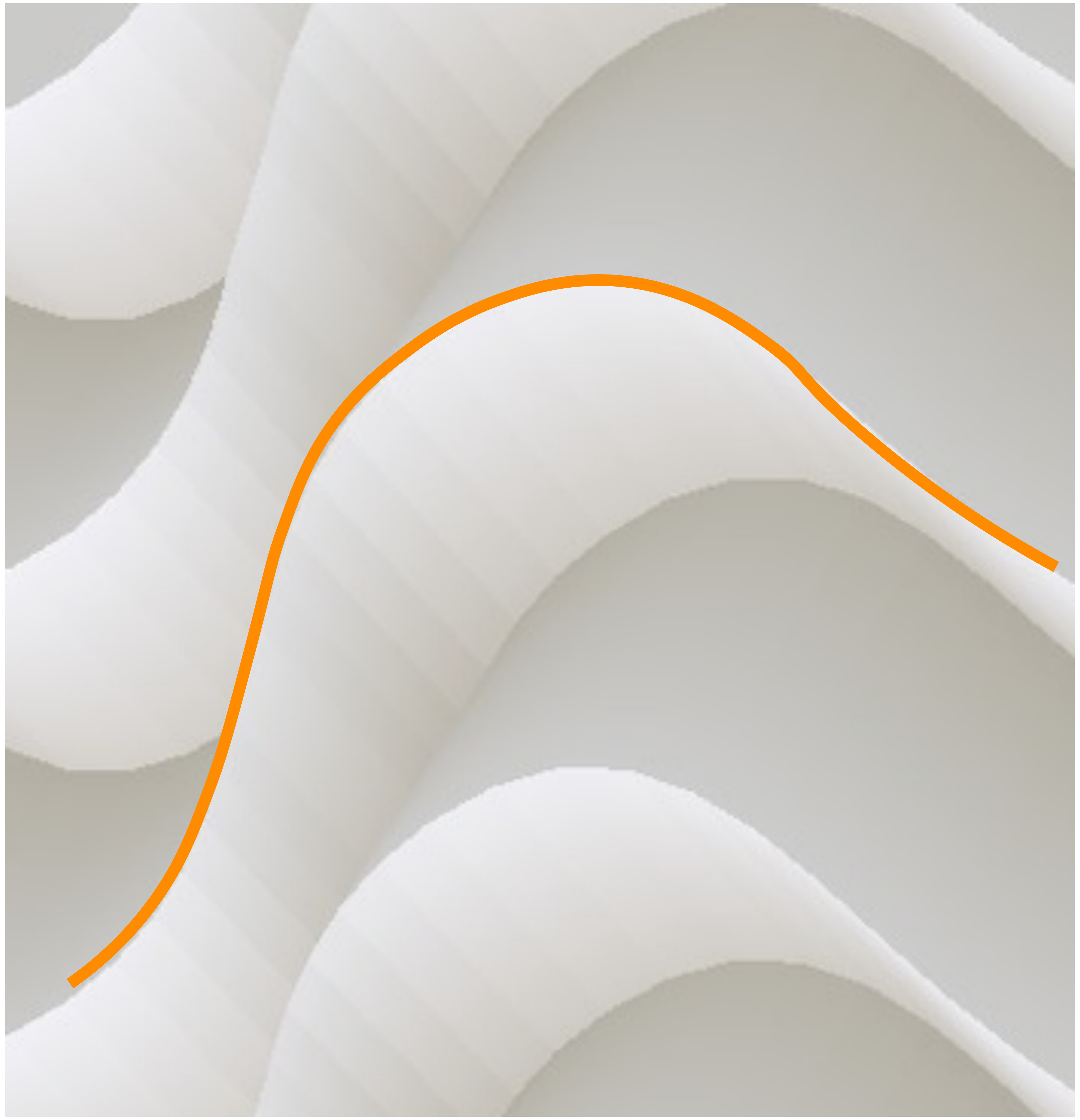
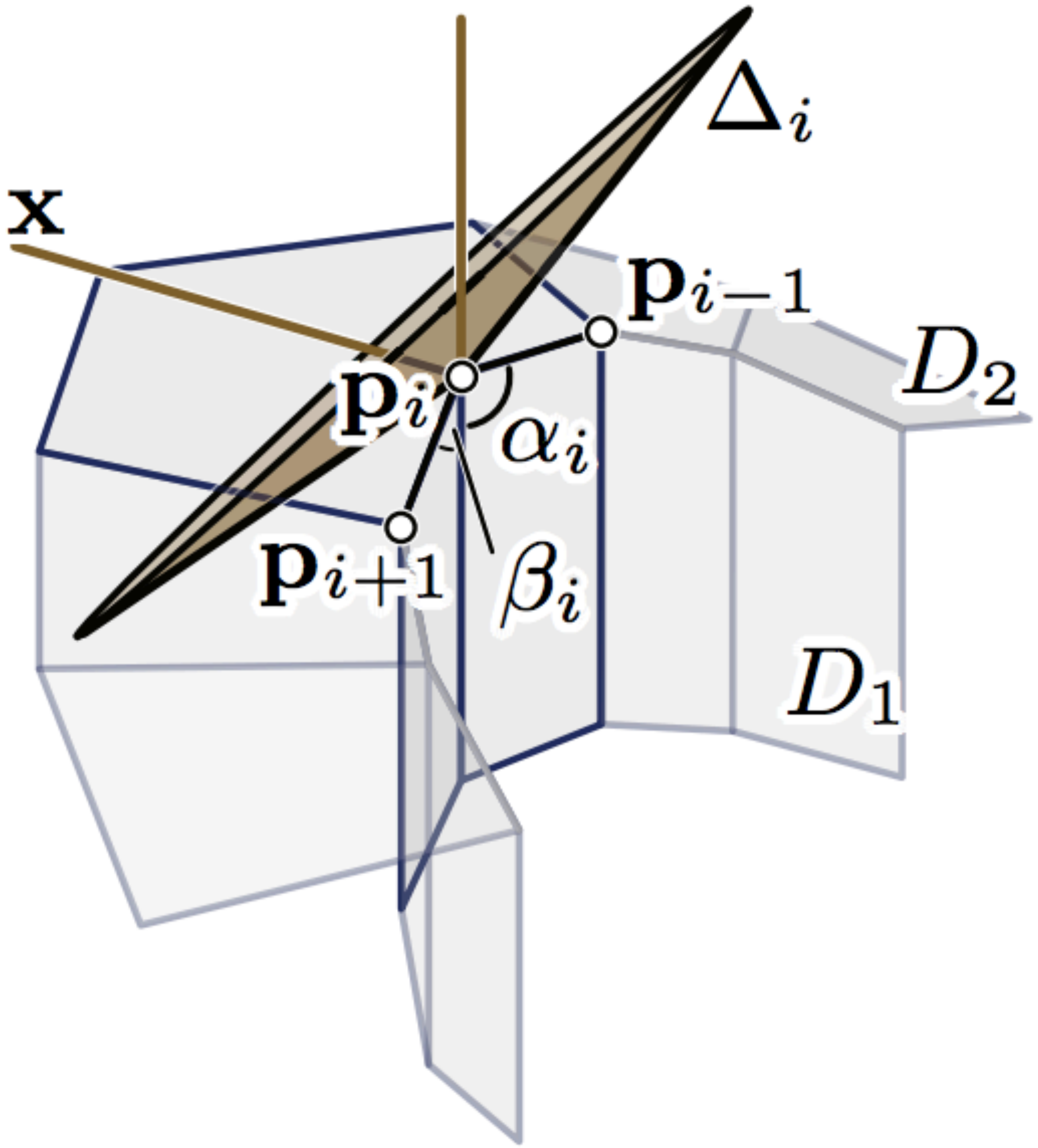
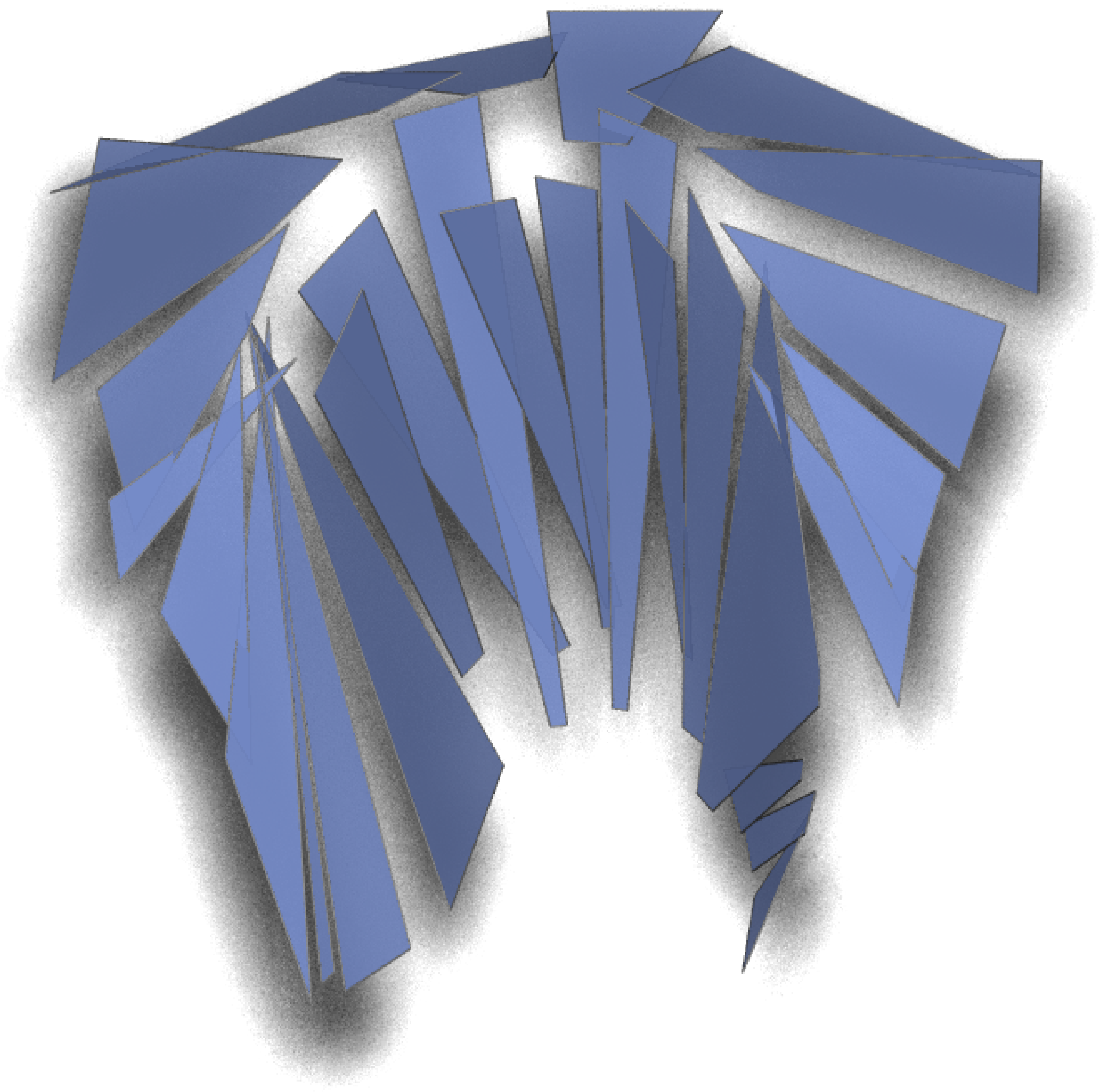


Discretizing 'Folds'





Step 4: 2D-3D Optimization



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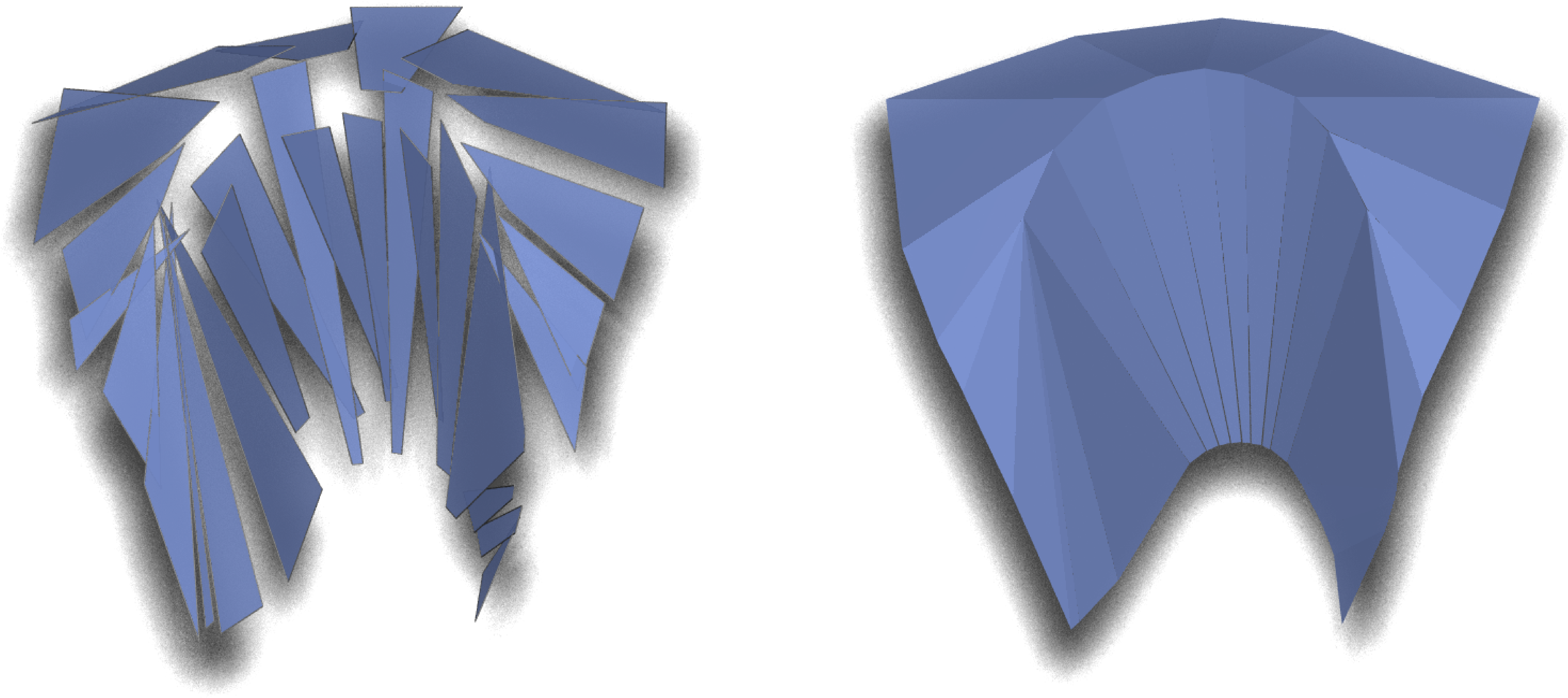

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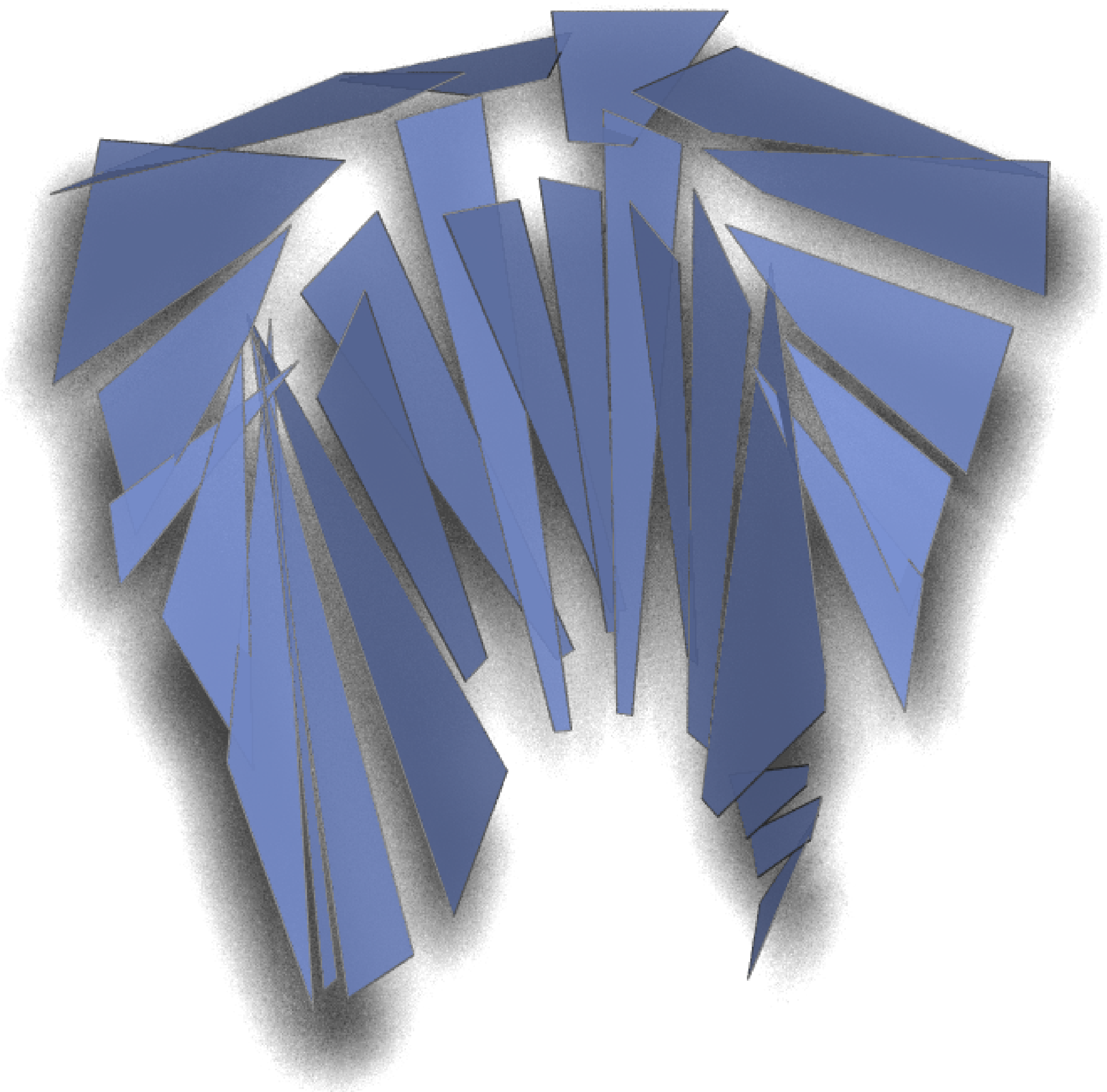

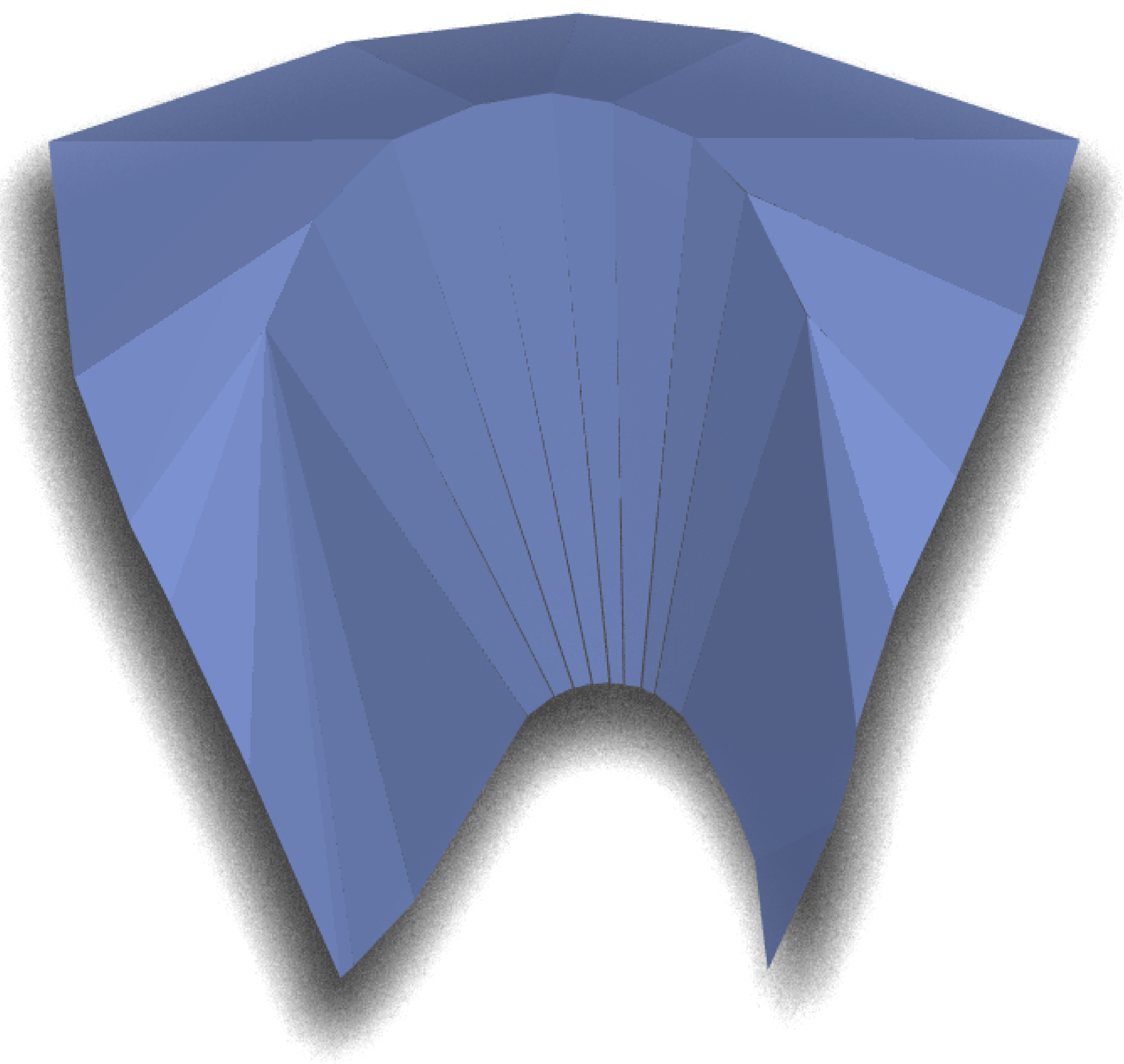

$$F_{vert} := \sum_{\mathbf{p} \in P} (\mathbf{m}_p^i - \mathbf{m}_p^j)^2$$

$$F_{fit} := \sum_{\mathbf{m} \in M} ((\mathbf{m} - \mathbf{m}_c) \cdot \mathbf{n}_c)^2$$

$$F_{fair} := \sum_{\mathbf{e}_{ij} \in E} w_{ij} (\mathbf{n}^i - \mathbf{n}^j)^2$$

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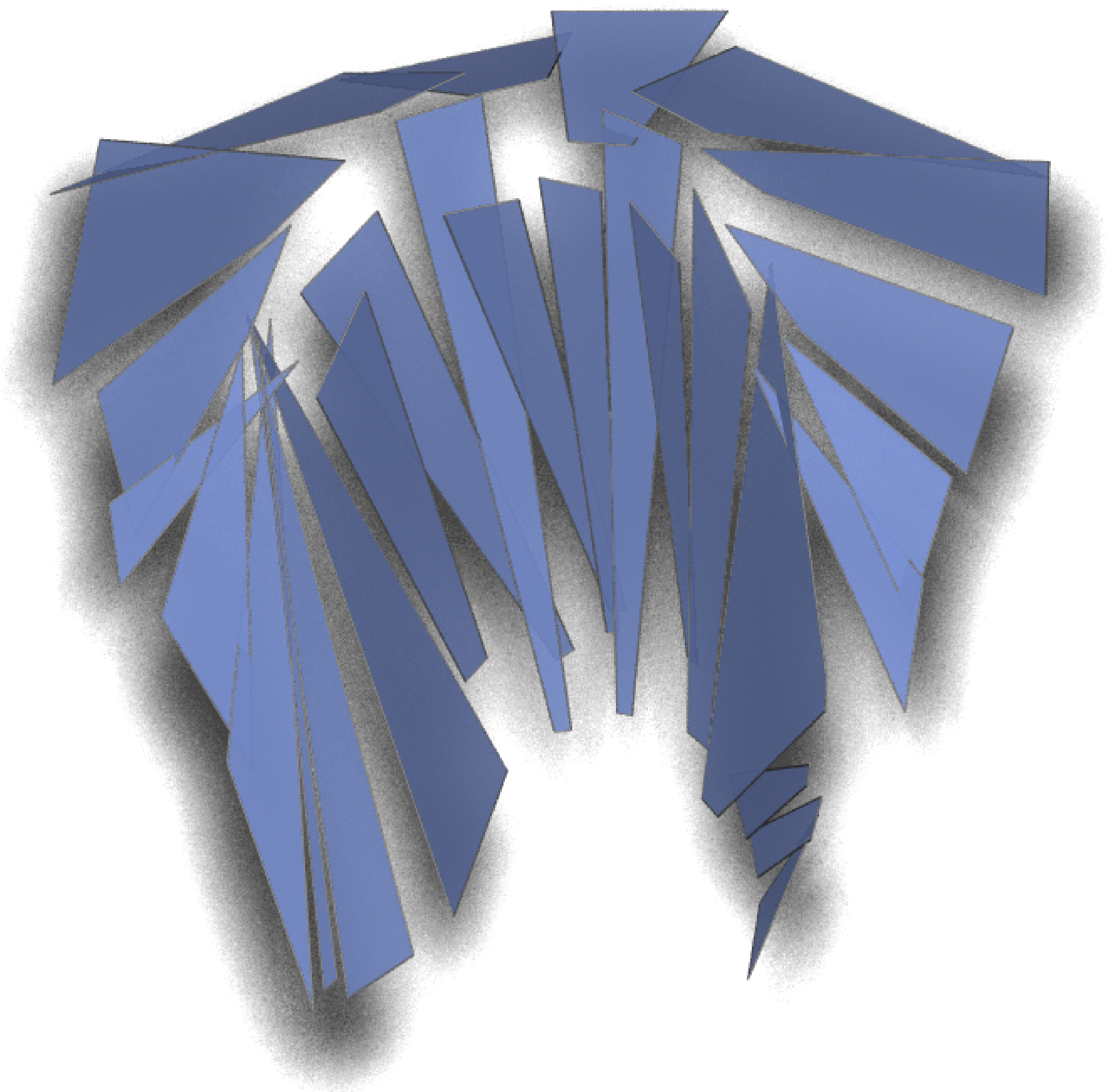

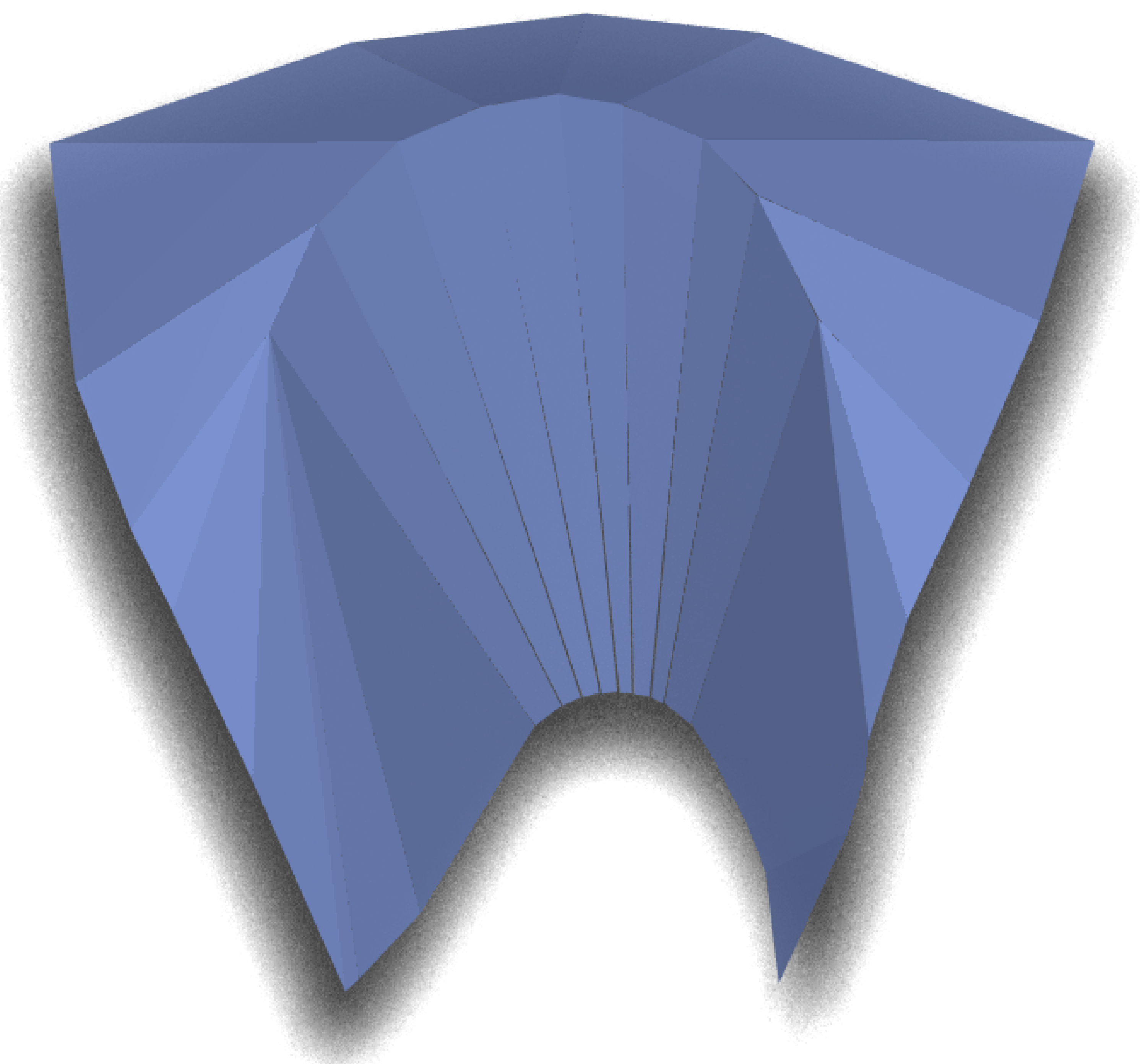

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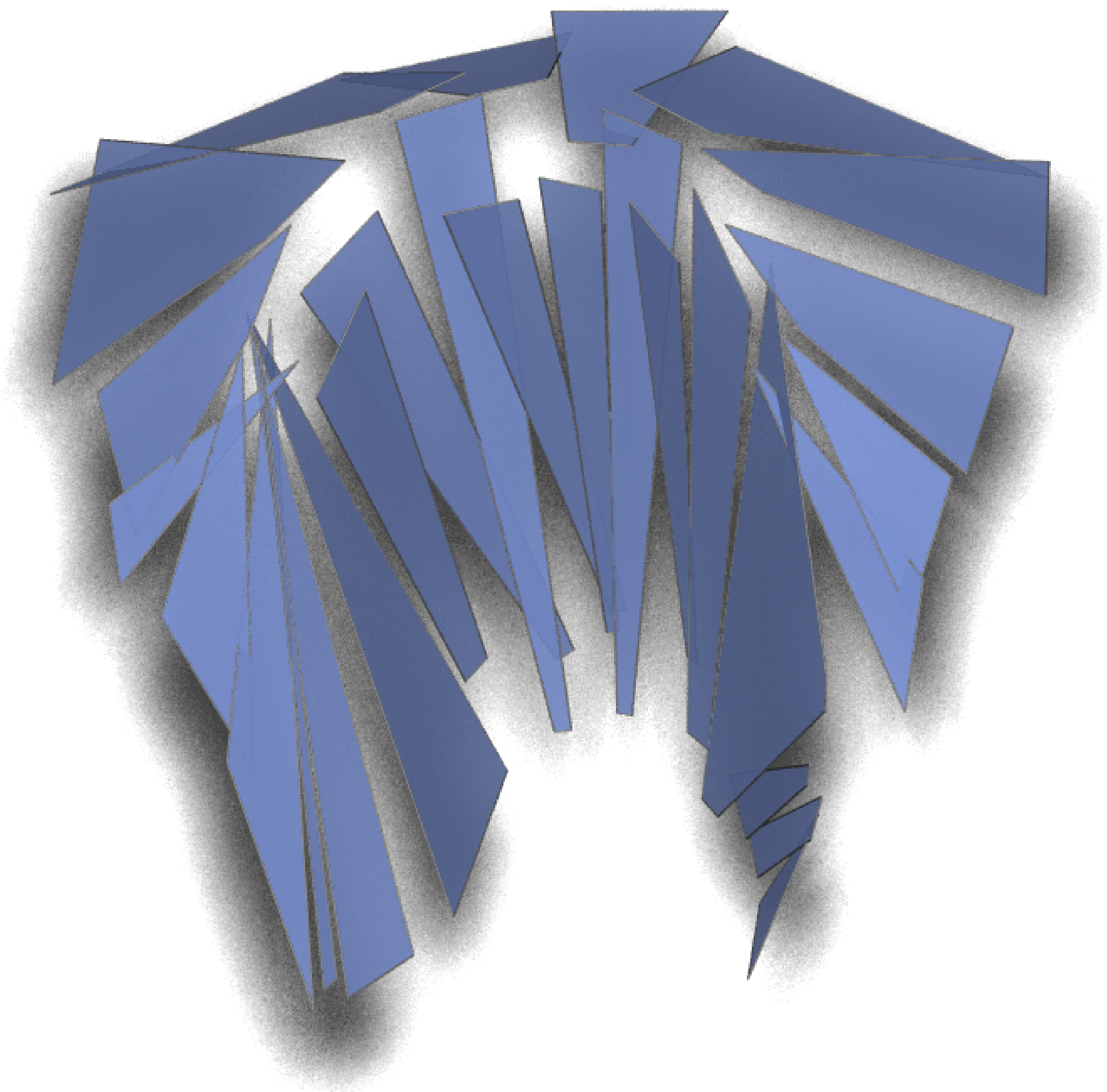
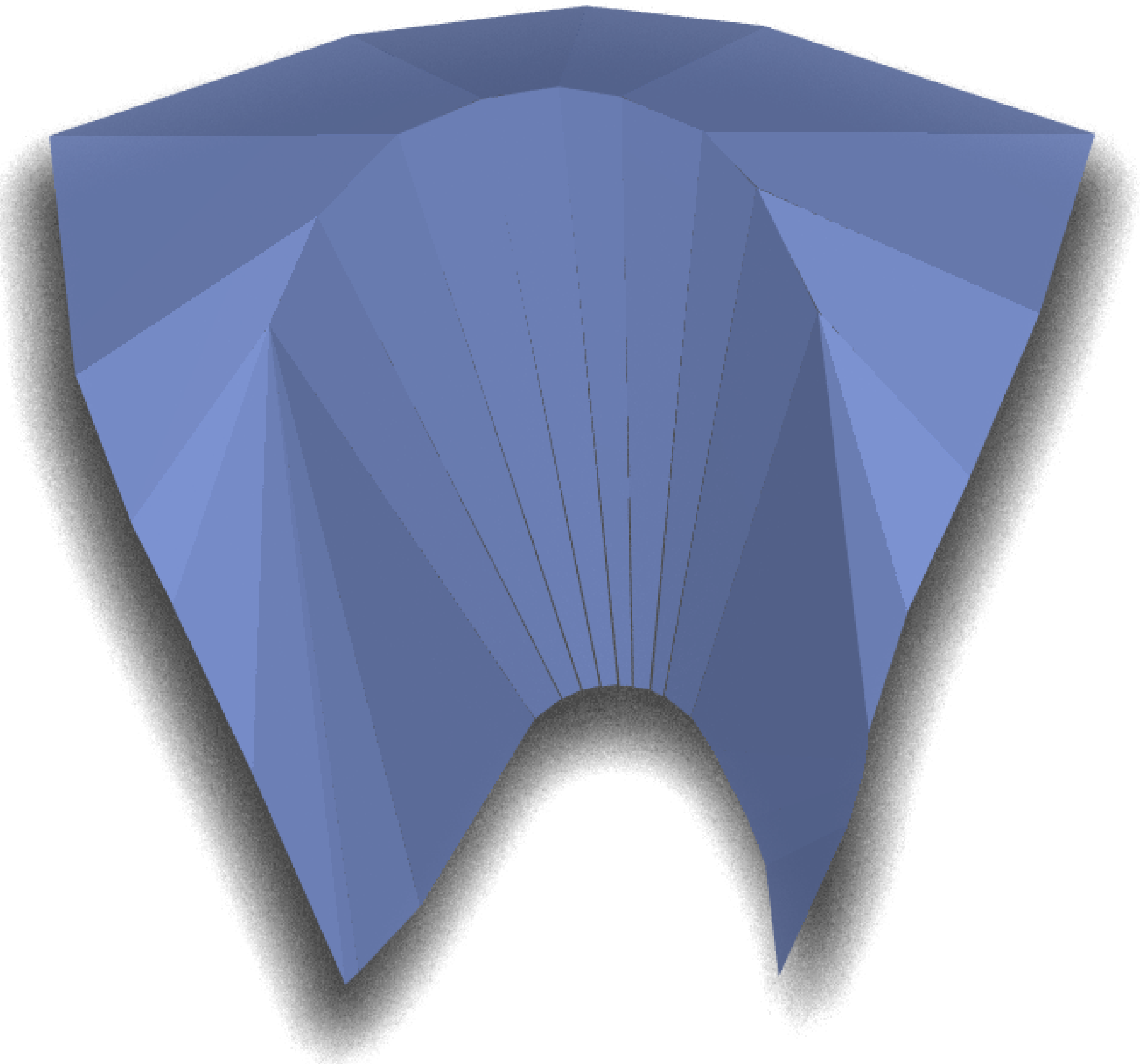
$$E = \int_S \kappa_1^2 + \kappa_2^2 dA$$

Step 4: 2D-3D Optimization





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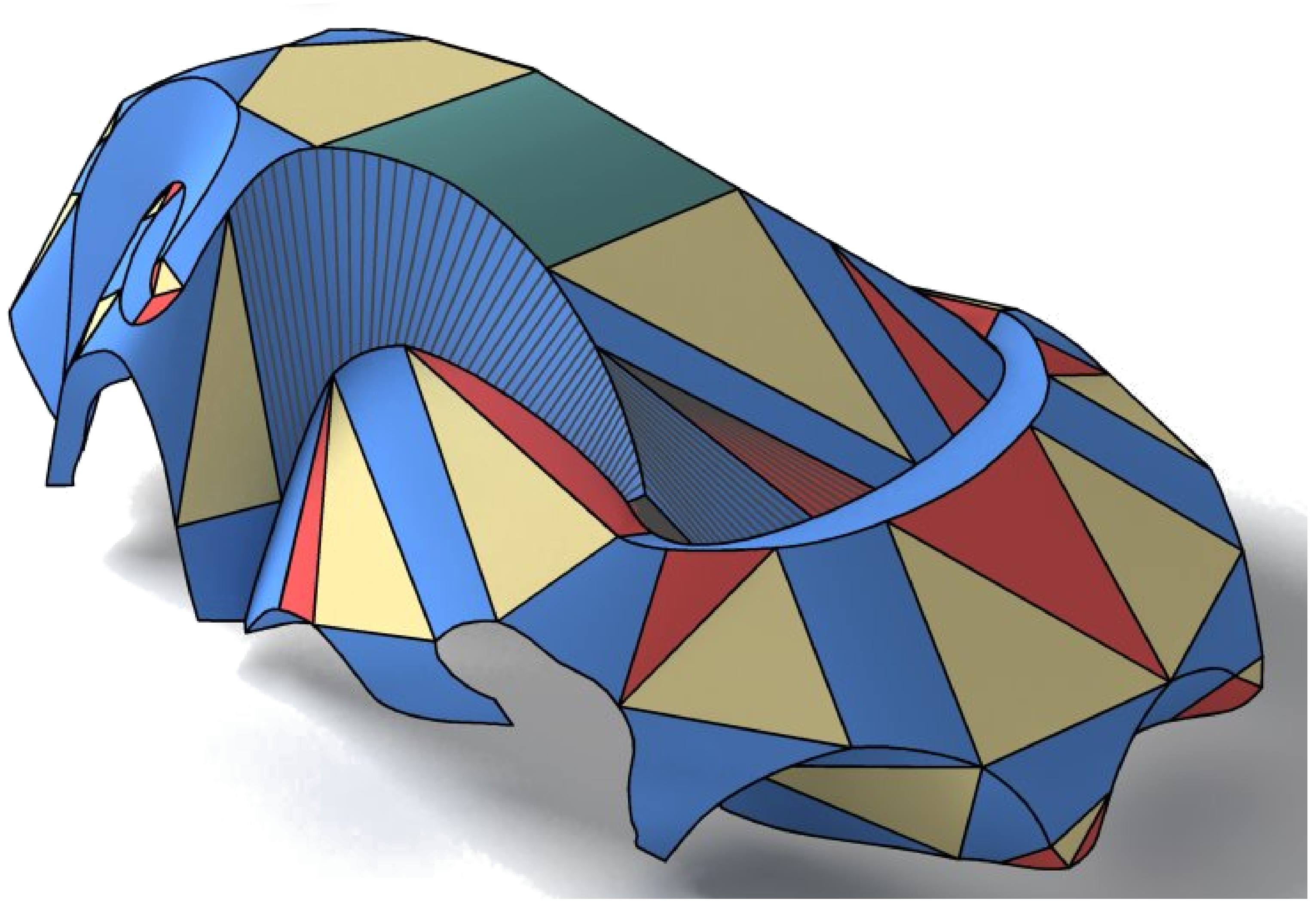
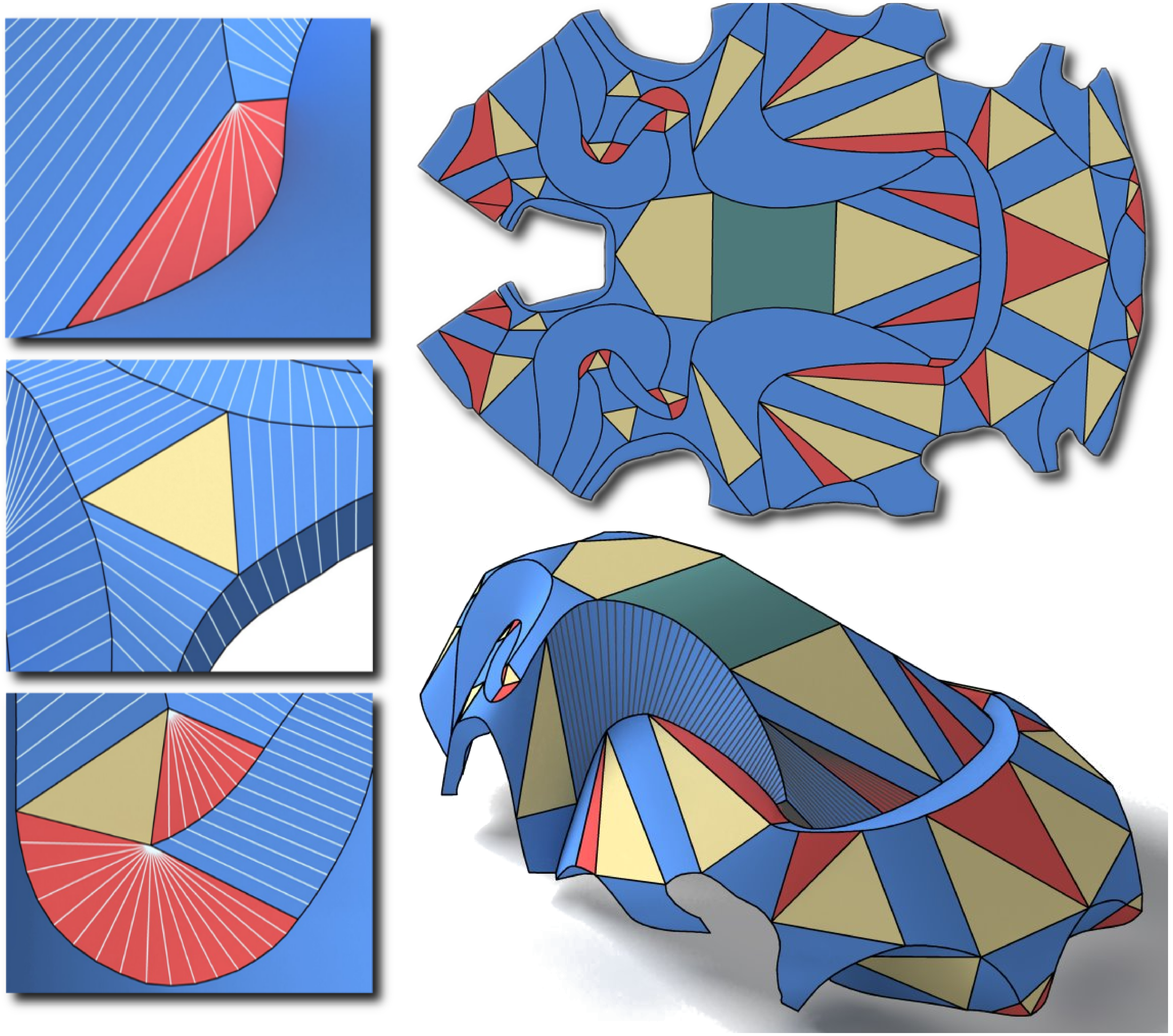
$$E = \int_S \kappa_1^2 \, dA$$

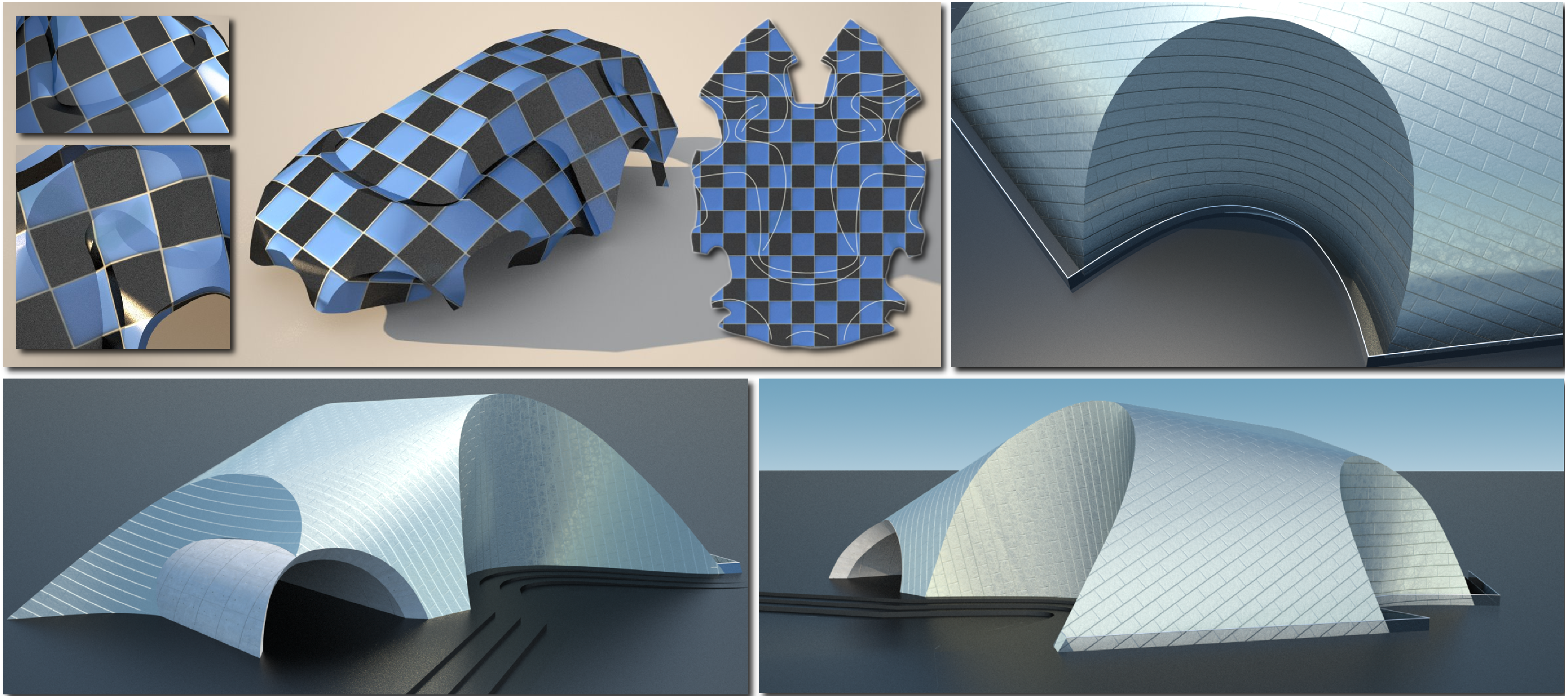
Final Output



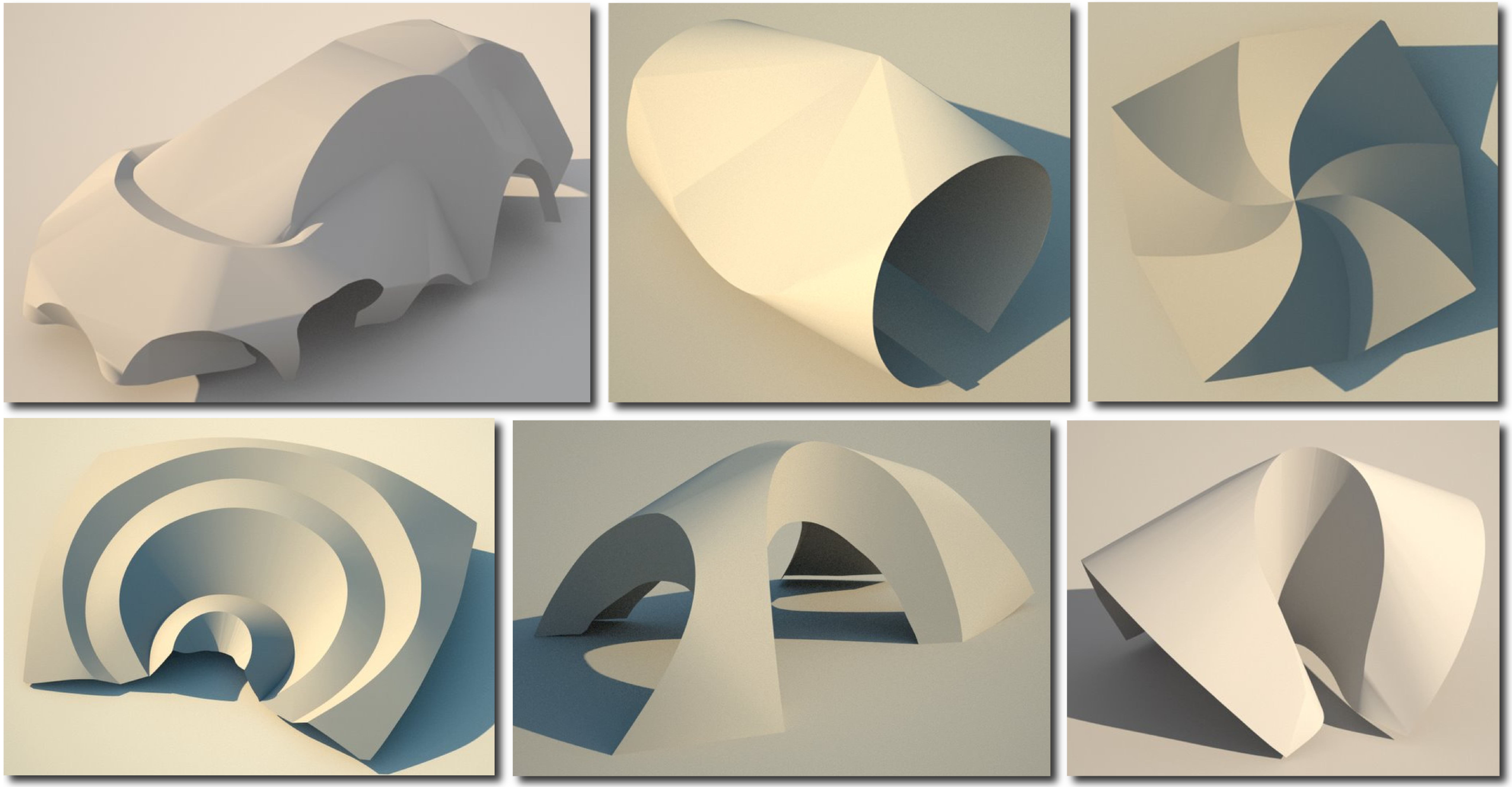
Final Output



Results



Results: Paper Models



(Constrained) Shape Space

- Mesh maps to a **point** in $\mathbb{R}^d$
- Deformation maps to a **path** in that space

[w/ Yang, Yang, Pottmann]

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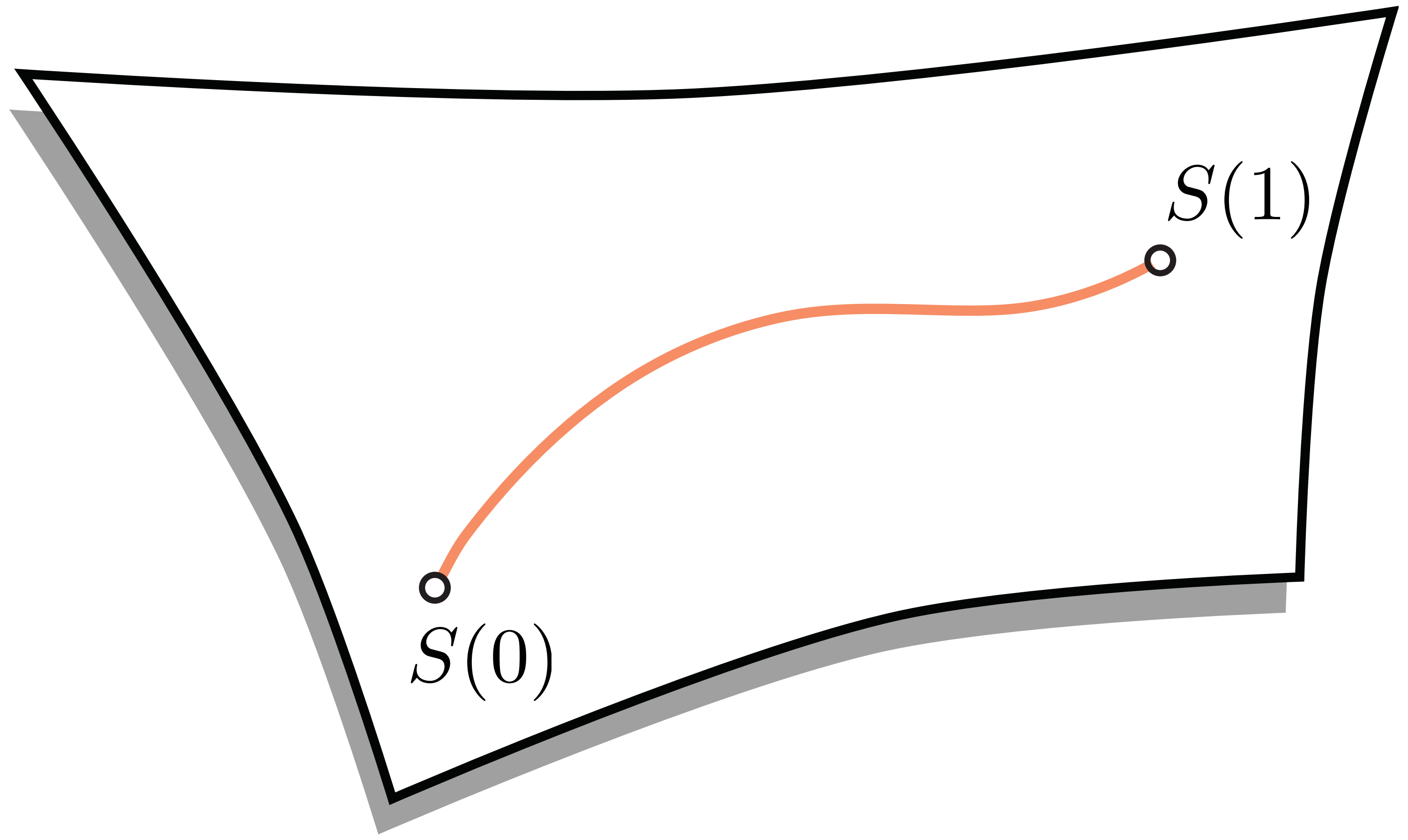

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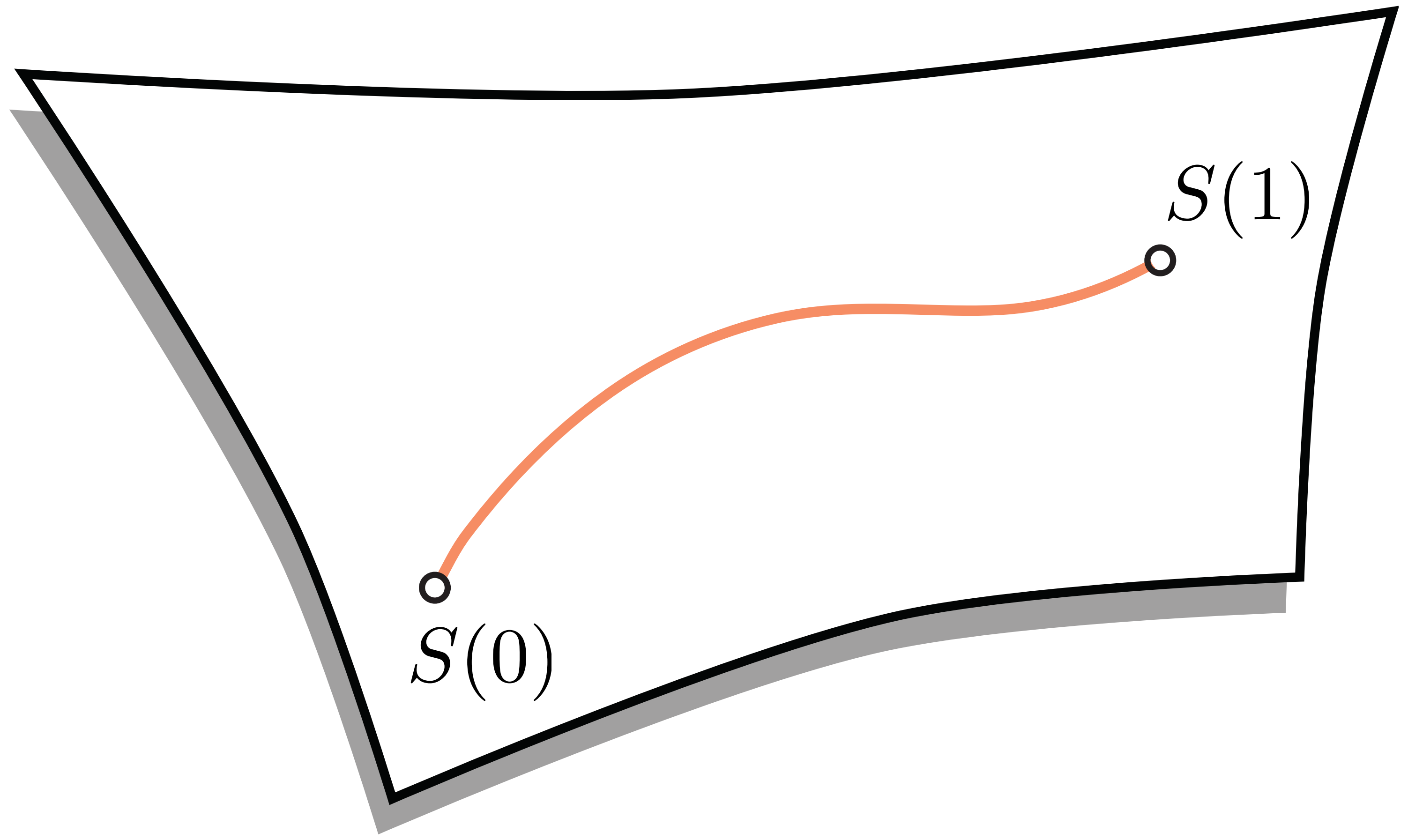

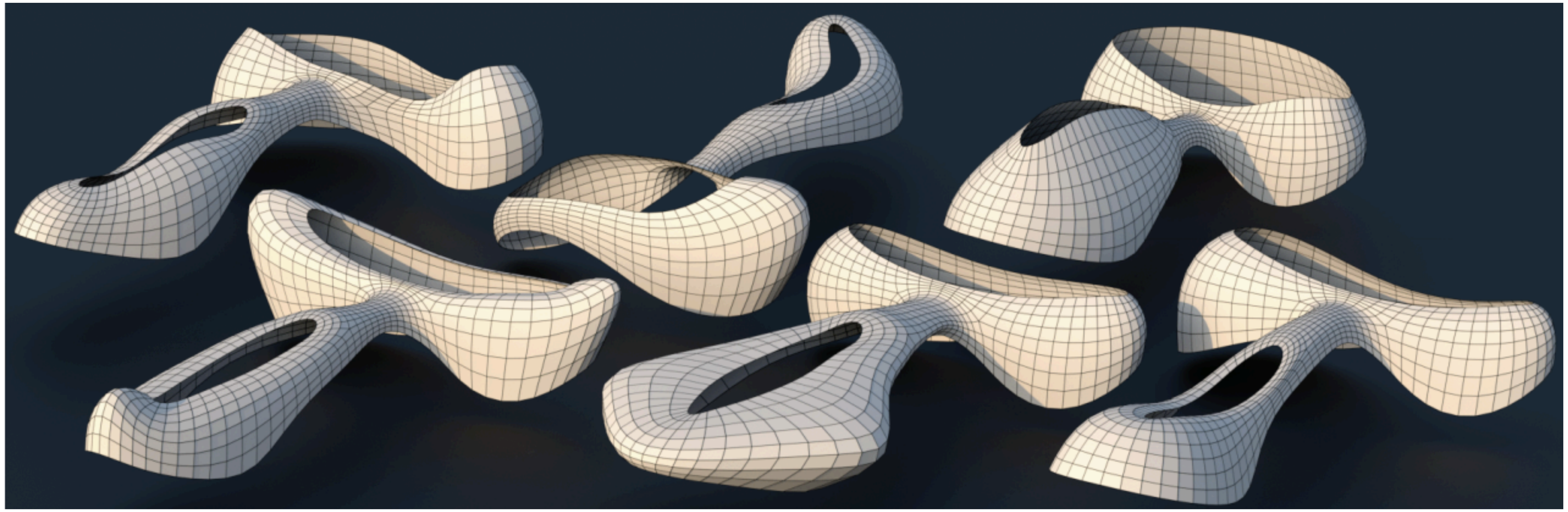

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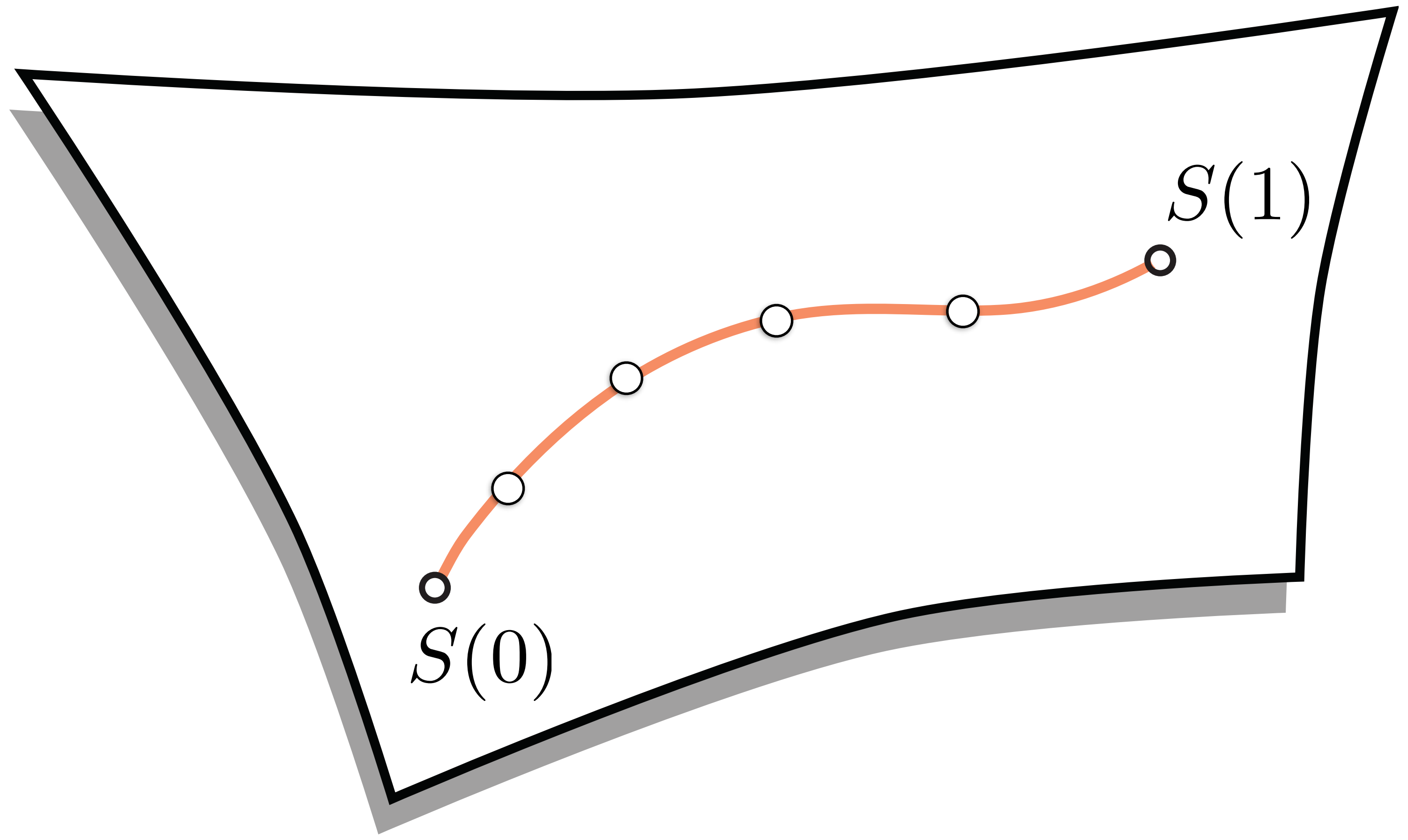

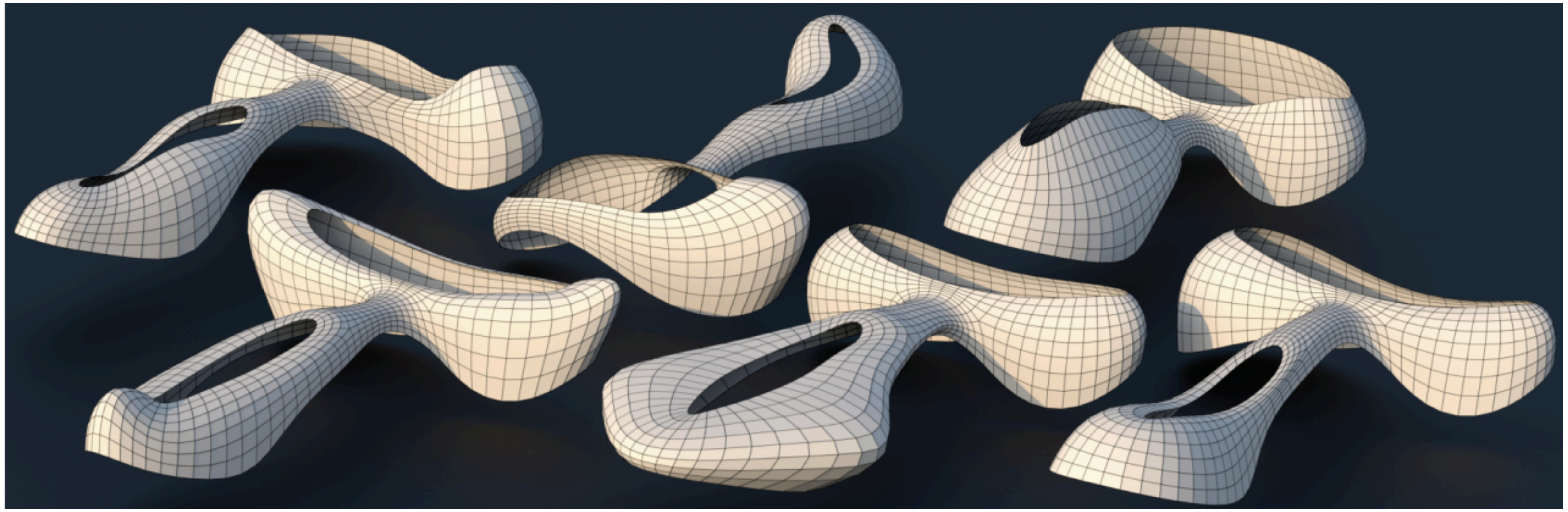

[w/ Yang, Yang, Pottmann]

Constrained Meshes

Constrained Meshes

- Given:
  single input mesh with a set of ***non-linear constraints*** in terms of mesh vertices

Constrained Meshes

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  single input mesh with a set of ***non-linear constraints*** in terms of mesh vertices

- Goal:
  - explore *neighboring* meshes respecting the prescribed constraints
  - based on different application requirements, navigate only the *desirable* meshes according to given quality measures

Map Mesh to Point

Map Mesh to Point

- The family of meshes with same combinatorics

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- Mesh to point

$$\mathbf{x} := (v_1, \ldots, v_n) \in \mathbb{R}^D$$

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$$\mathbf{d} \Rightarrow \mathbf{x}_0 + \mathbf{d}$$

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- Displacement vector to update the current mesh

$$\mathbf{d} \Rightarrow \mathbf{x}_0 + \mathbf{d}$$

- Distance measure

$$d(\mathbf{x}_1, \mathbf{x}_2) := \|\mathbf{x}_1 - \mathbf{x}_2\|$$

Constrained Mesh Manifold

- Constrained mesh manifold *M*:
  - represents all the meshes under a set of non-linear constraints
- Individual constraint
  - $E(\mathbf{x}_i) = 0$ defines a hypersurface in $\mathbb{R}^D$

Constrained Mesh Manifold

- Involving *m* constraints in $\mathbb{R}^D$

$$\Gamma_i = \{\mathbf{x} \in \mathbb{R}^D | E_i(\mathbf{x}) = 0\}, i = 1, \ldots, m$$

Constrained Mesh Manifold

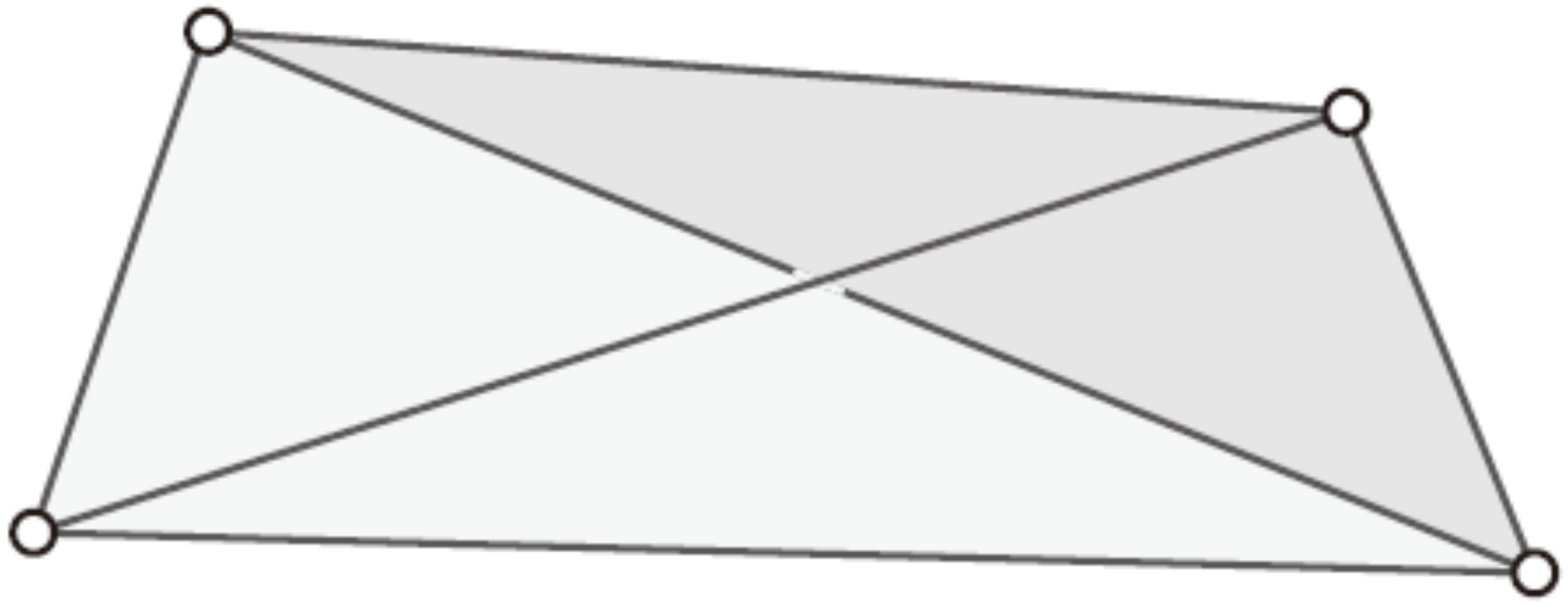
- Involving *m* constraints in $\mathbb{R}^D$

$$\Gamma_i = \{\mathbf{x} \in \mathbb{R}^D | E_i(\mathbf{x}) = 0\}, i = 1, \ldots, m$$

- *M* is the intersection of m hypersurfaces
  - dimension ***D-m*** (tangent space)
  - codimension ***m*** (normal space)

Example: PQ Mesh Manifold

- PQ mesh manifold *M*: $f_i \rightarrow E_i$
- Constraints (planarity per face)
  - each face $|E_i| \leq \epsilon$ (signed diagonal distance)



  - *deviation* from planarity
  - 10mm allowance for 2m x 2m panels

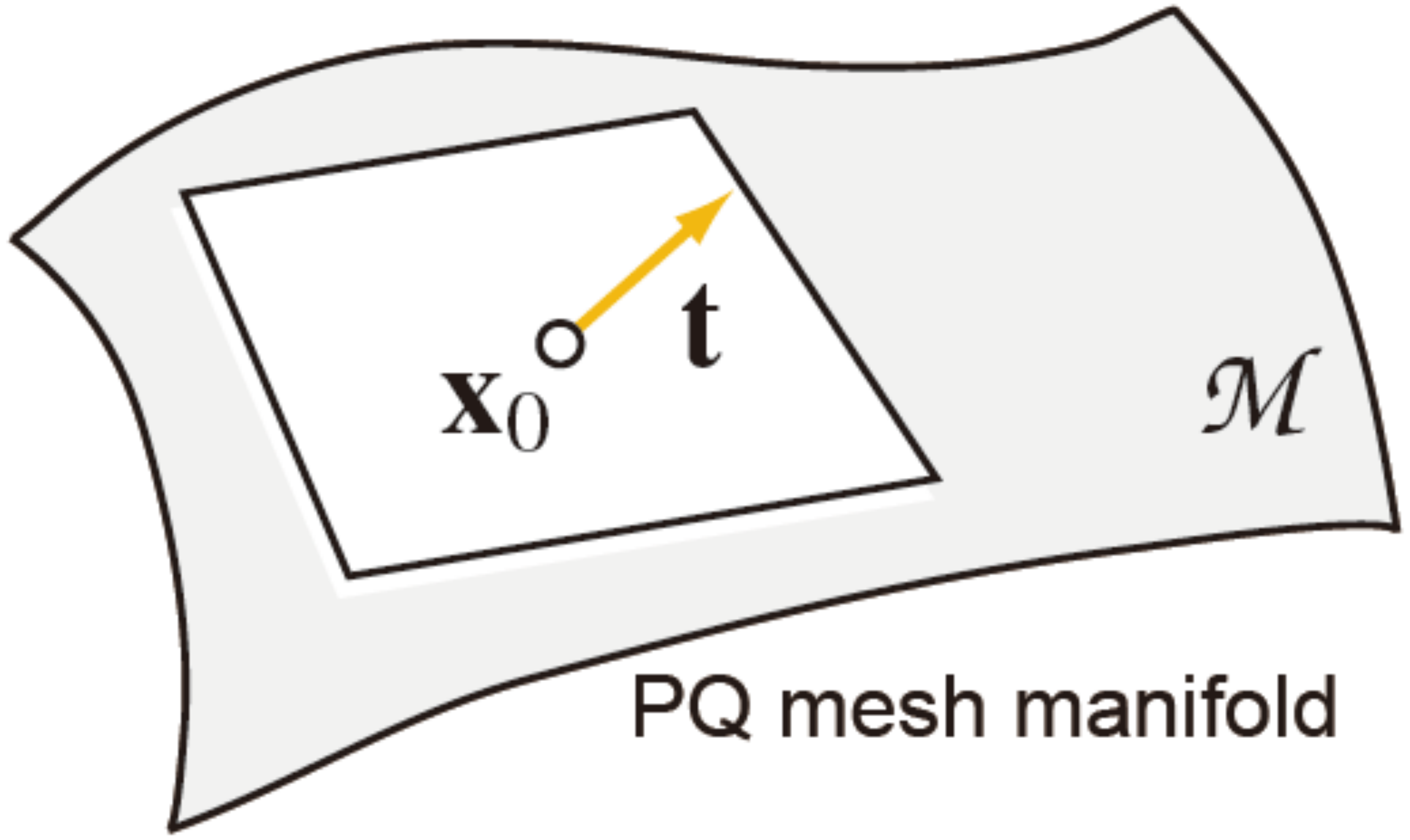
Tangent Space

- Starting mesh $\mathbf{x}_0$ is PQ

$$\mathrm{T}_{\mathcal{M}}(\mathbf{x}_0) := \{\mathbf{x}_0 + \mathbf{t} \mid \nabla E_i^T(\mathbf{x}_0) \cdot \mathbf{t} = 0 \; \forall \; i = 1, \ldots, m\}.$$

- Geometrically, intersection of all the tangent planes of the hypersurfaces

Walking on the Tangent Space



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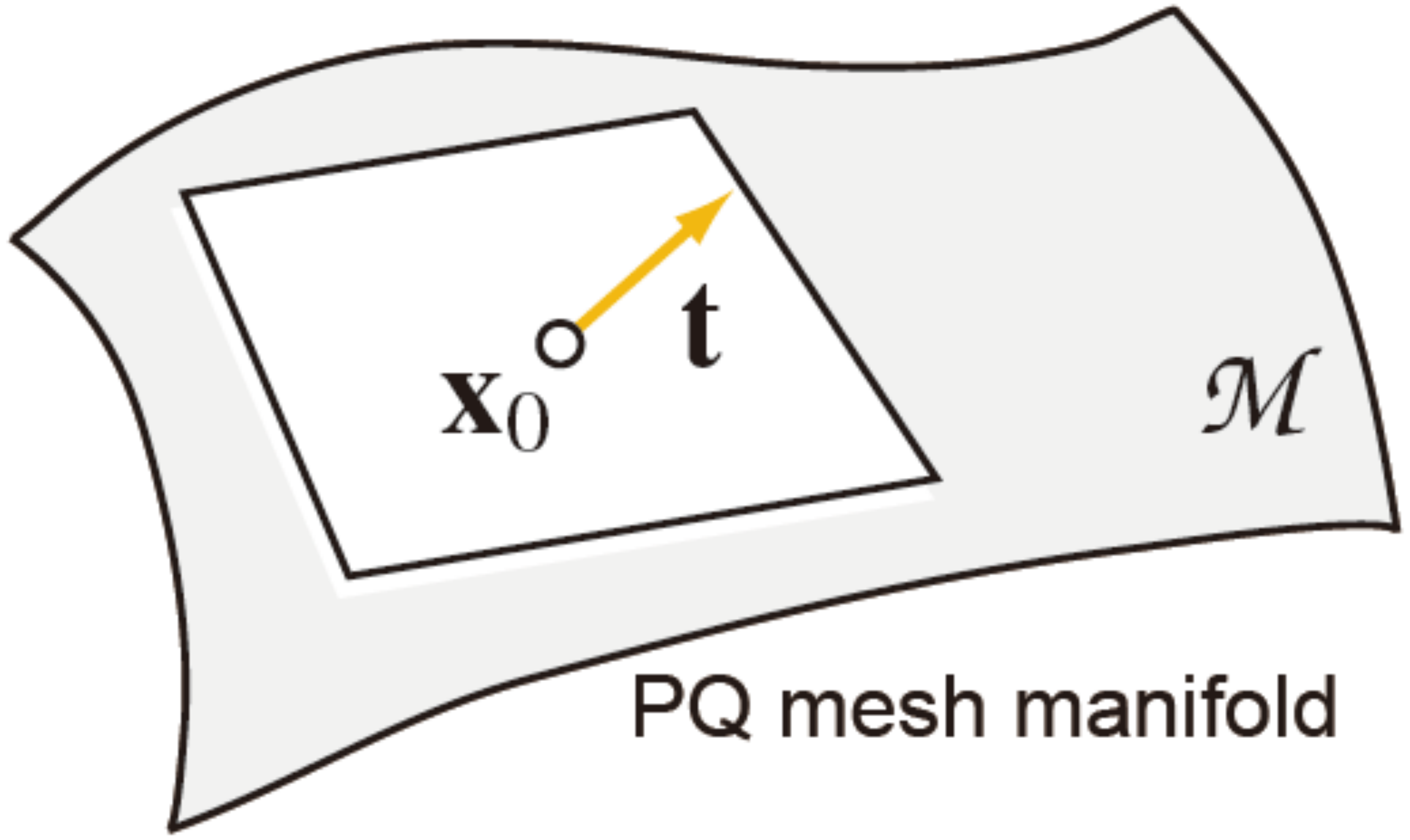

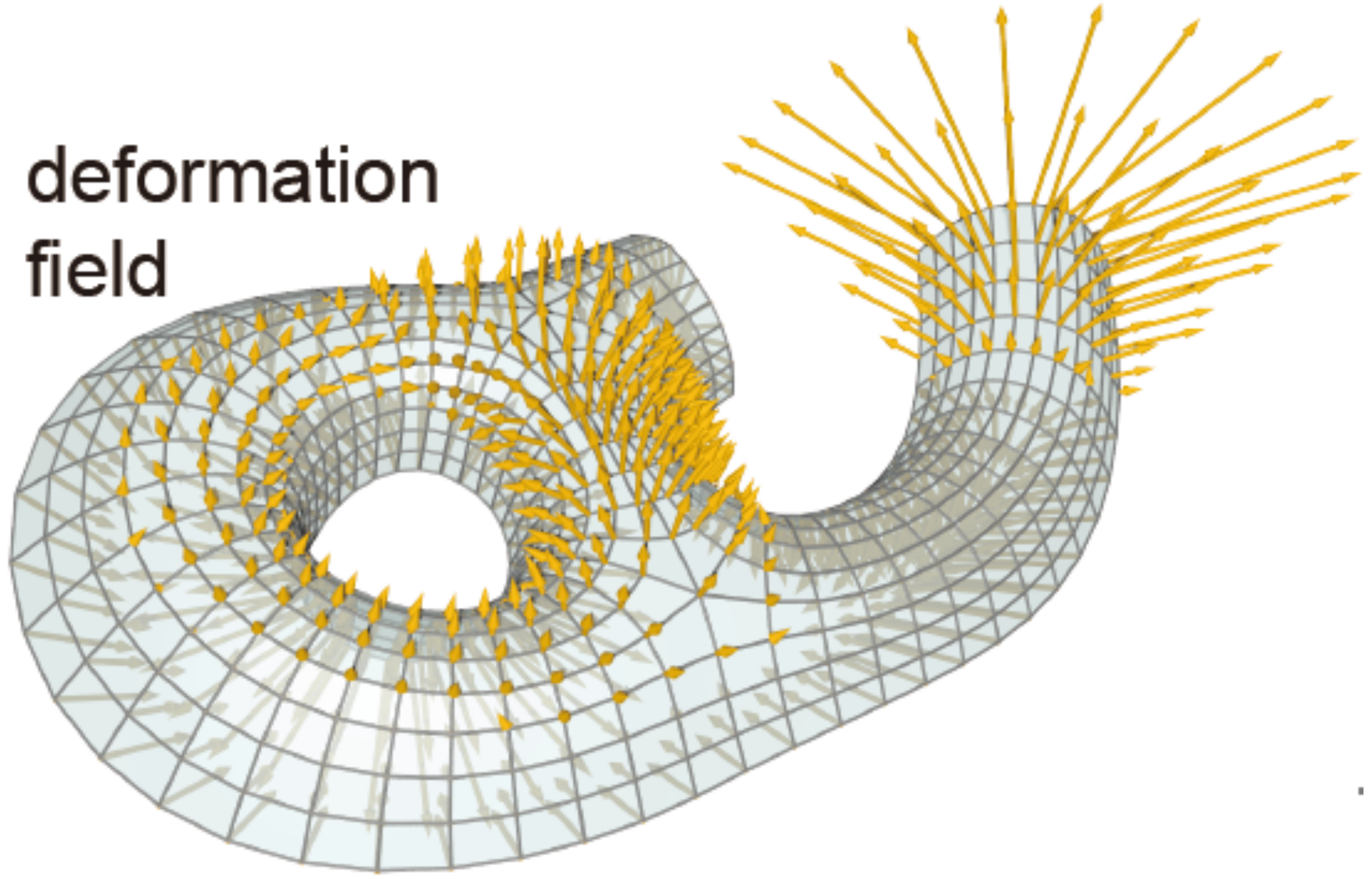

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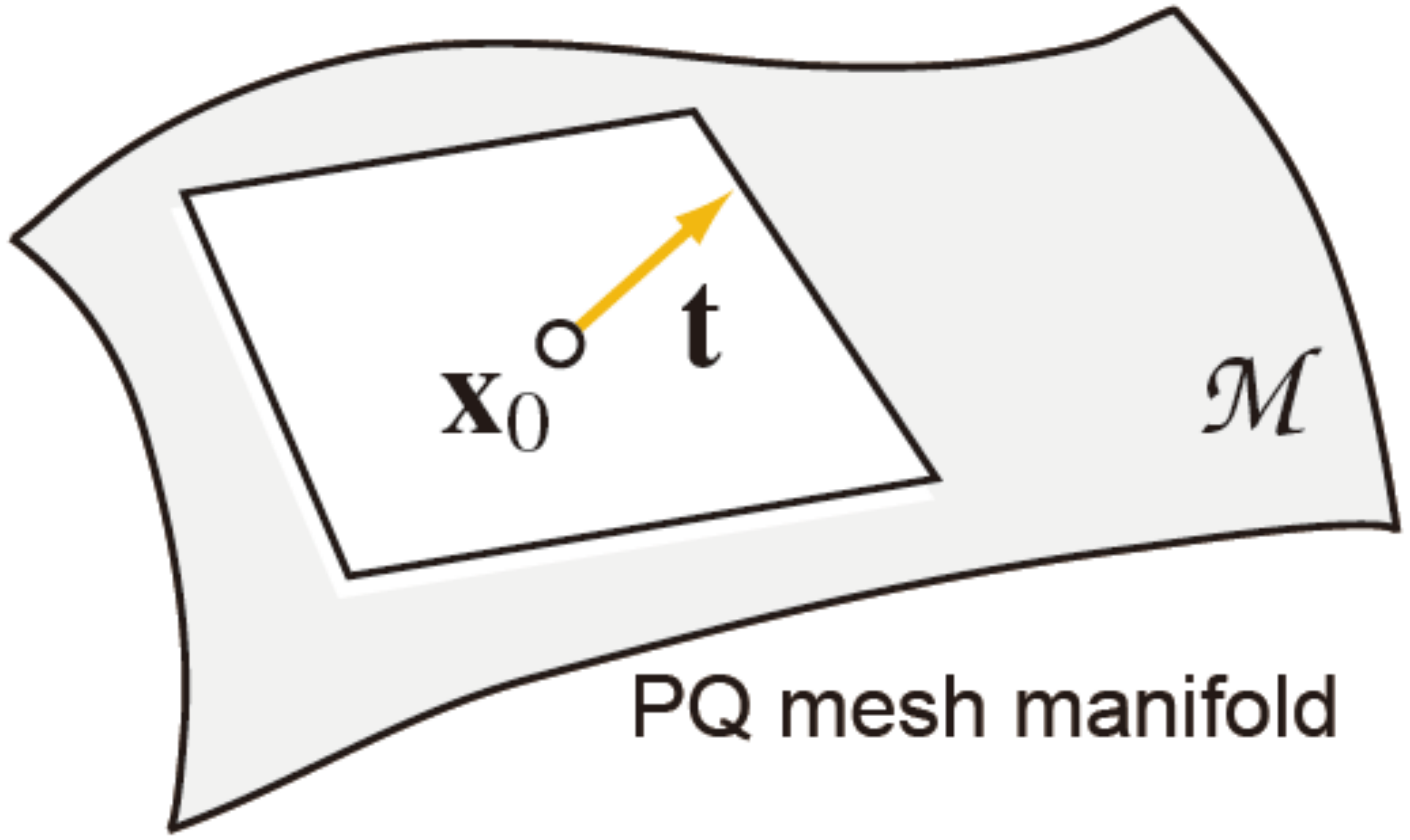

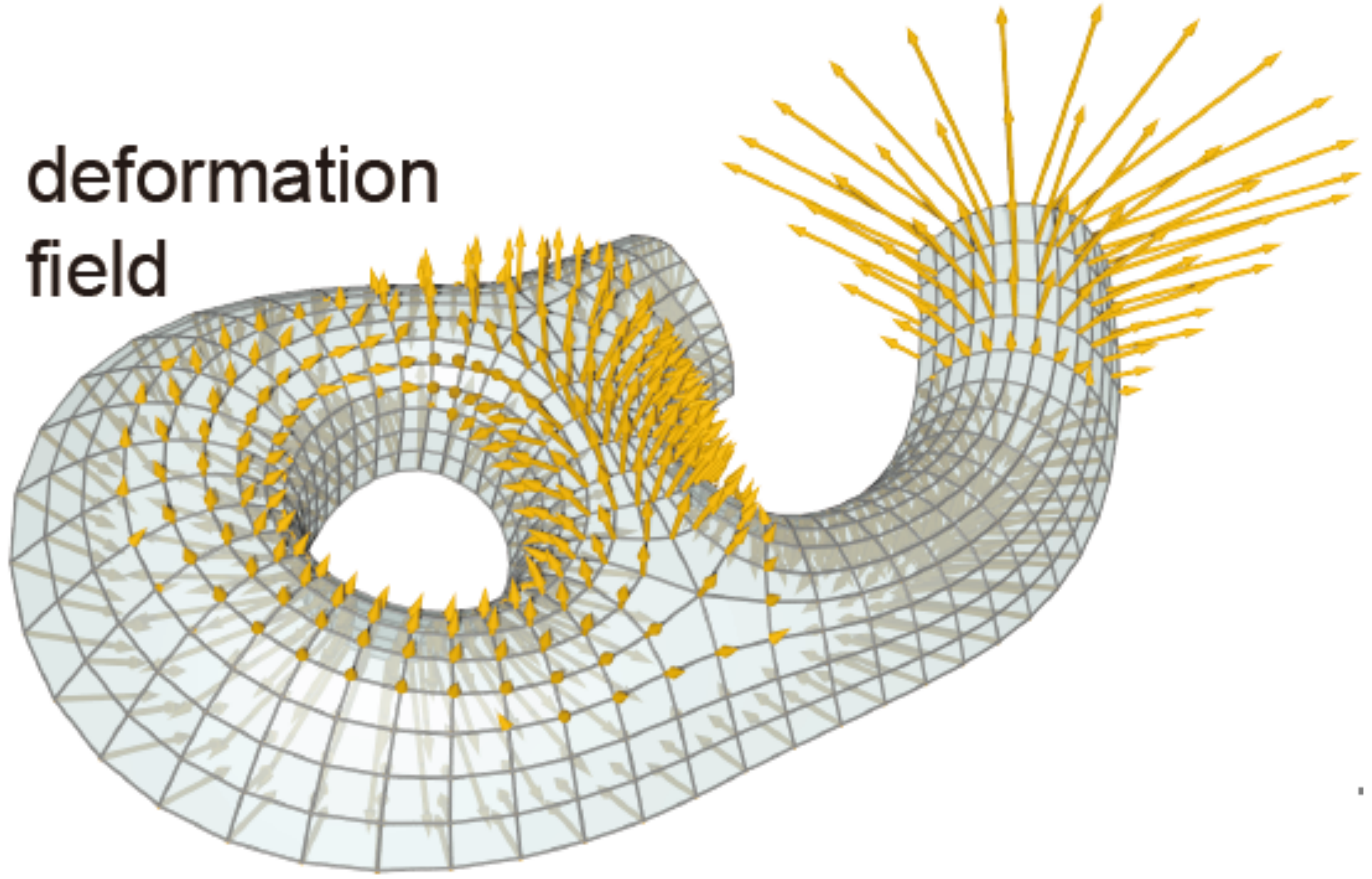

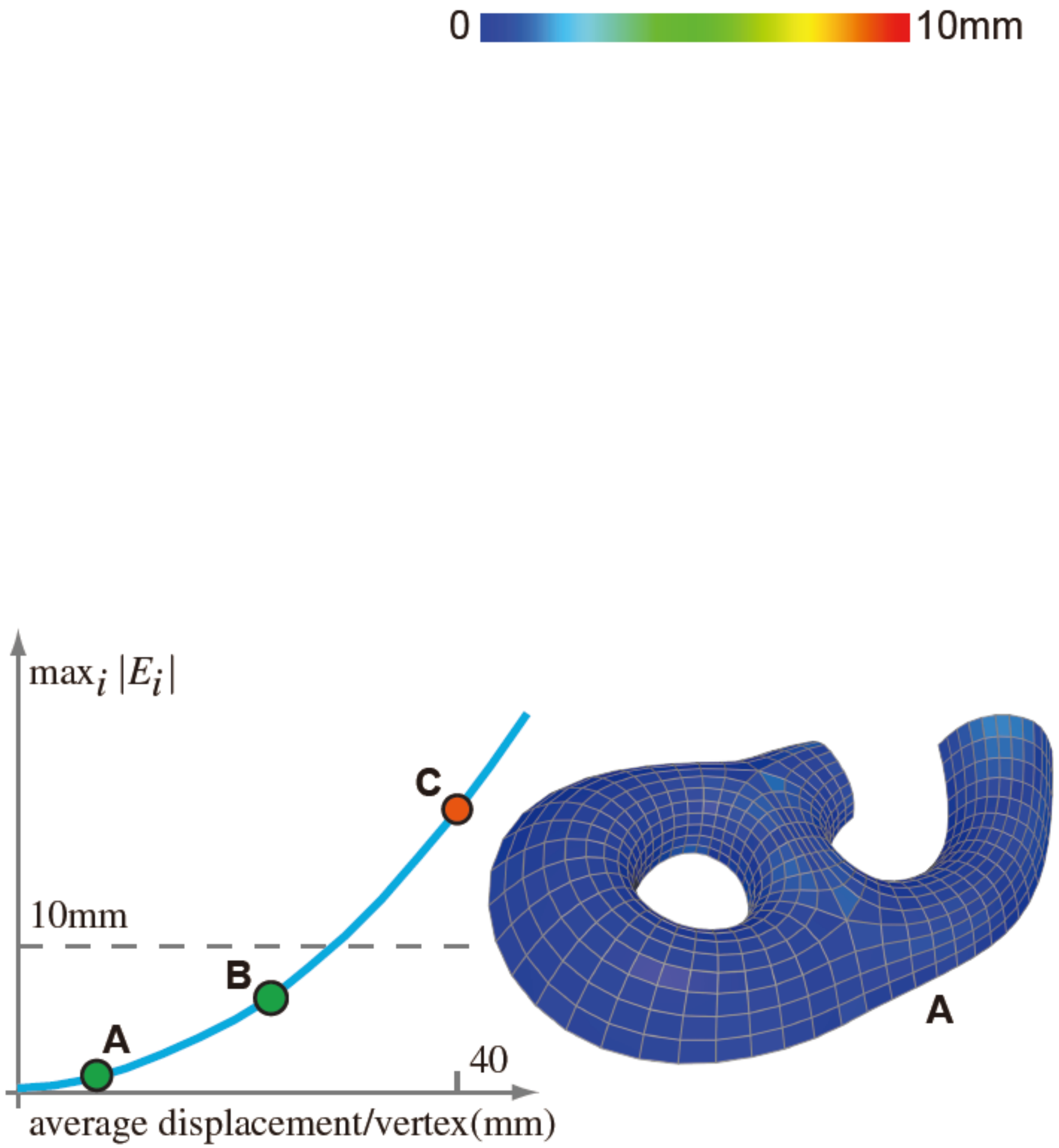

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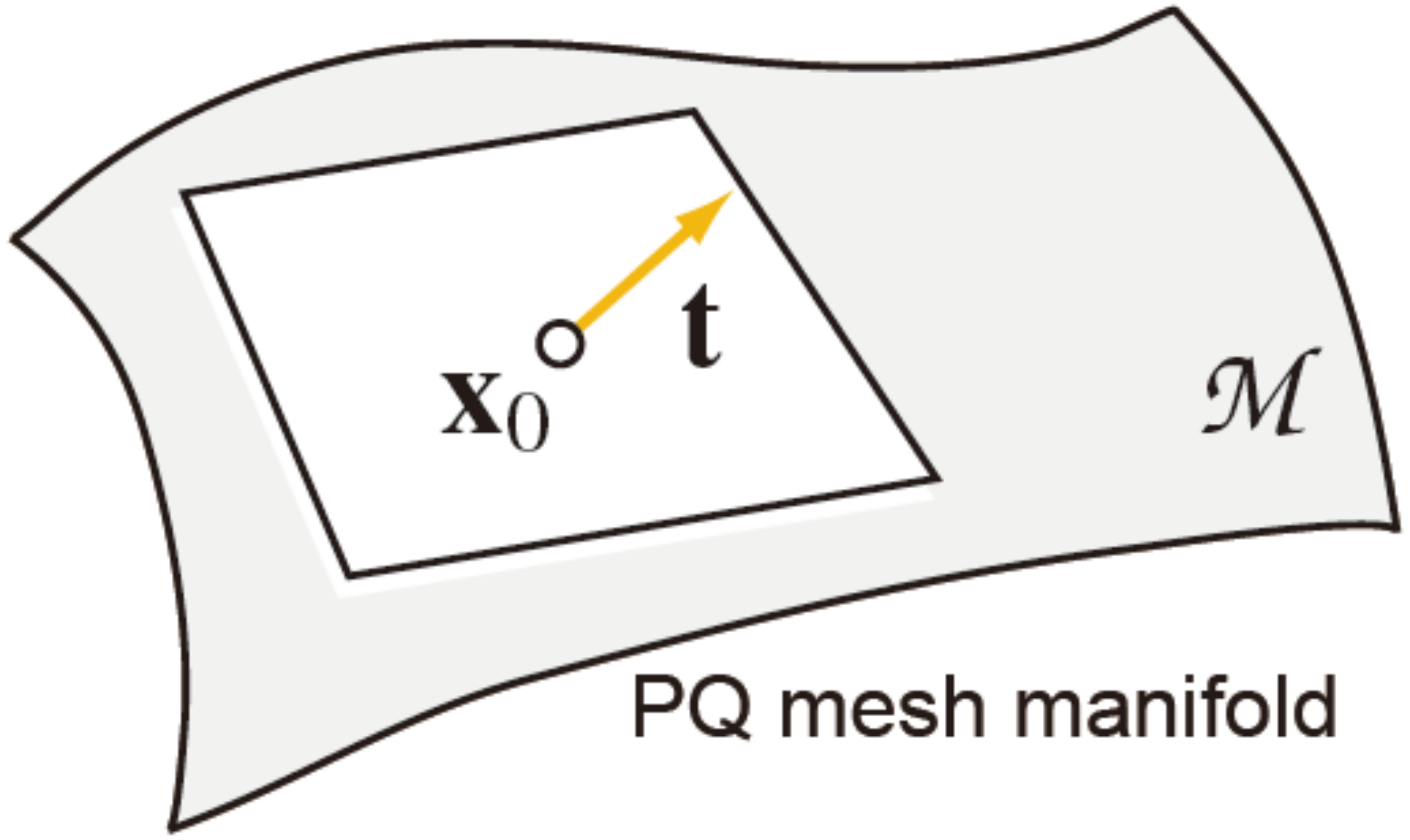

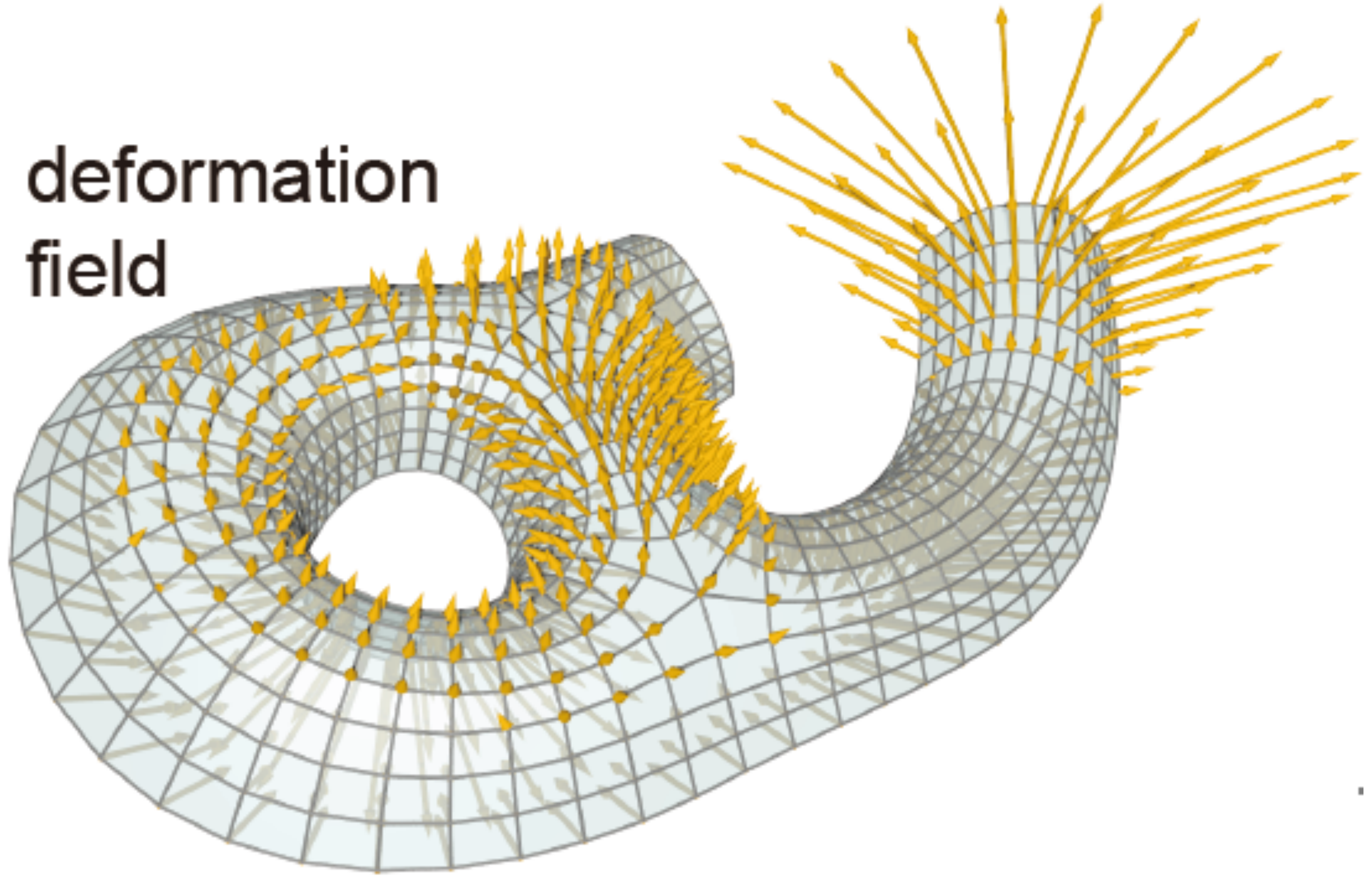

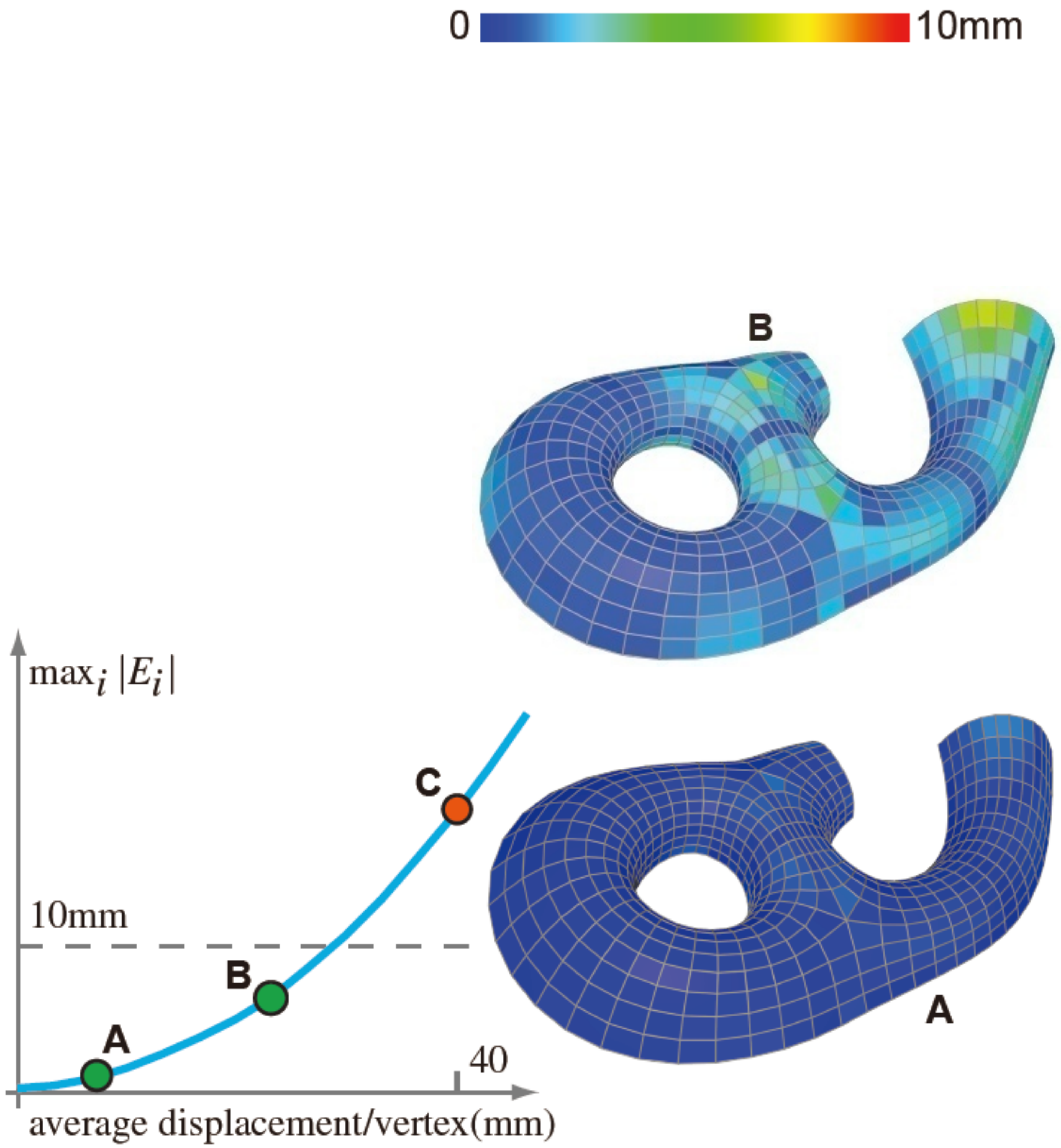

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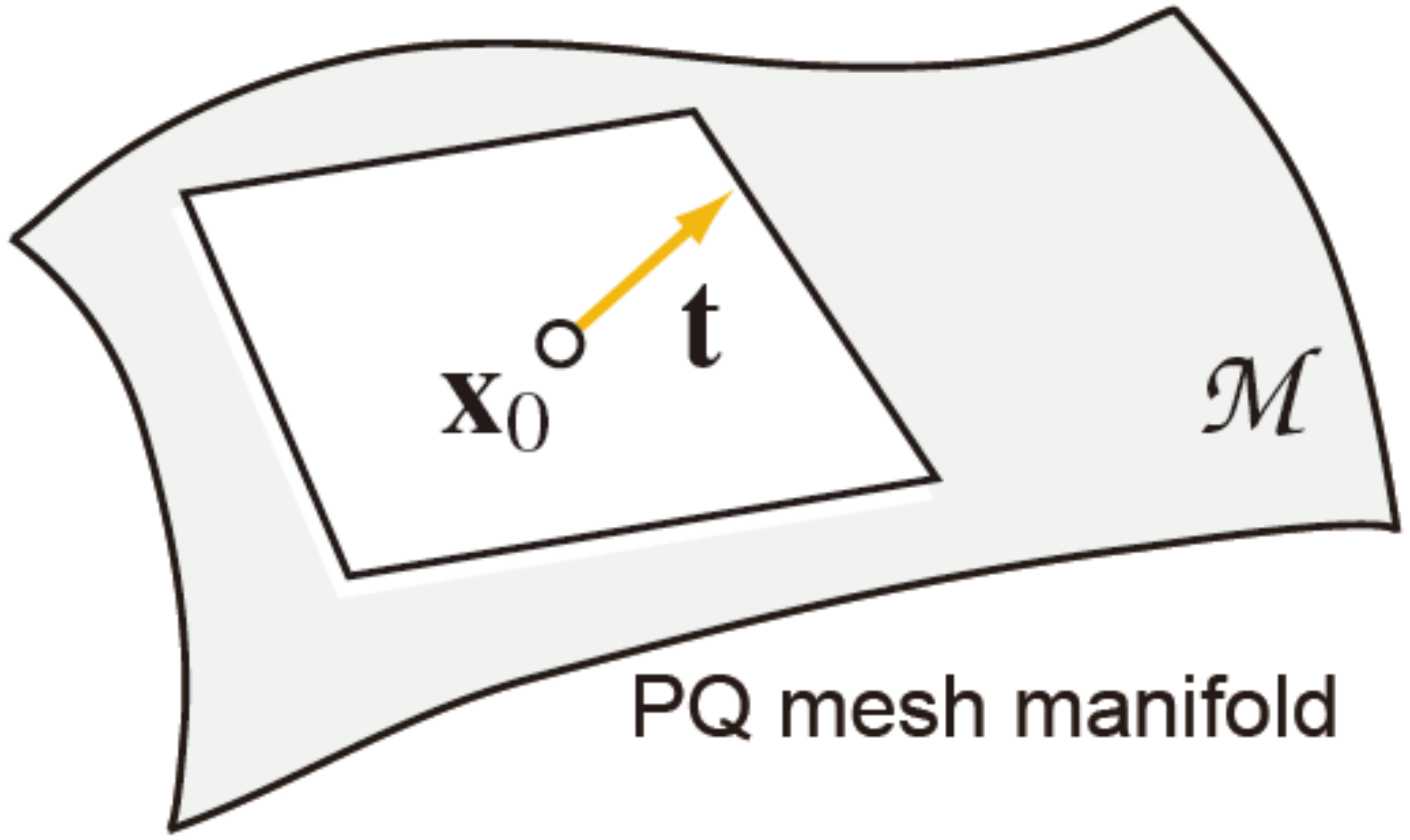
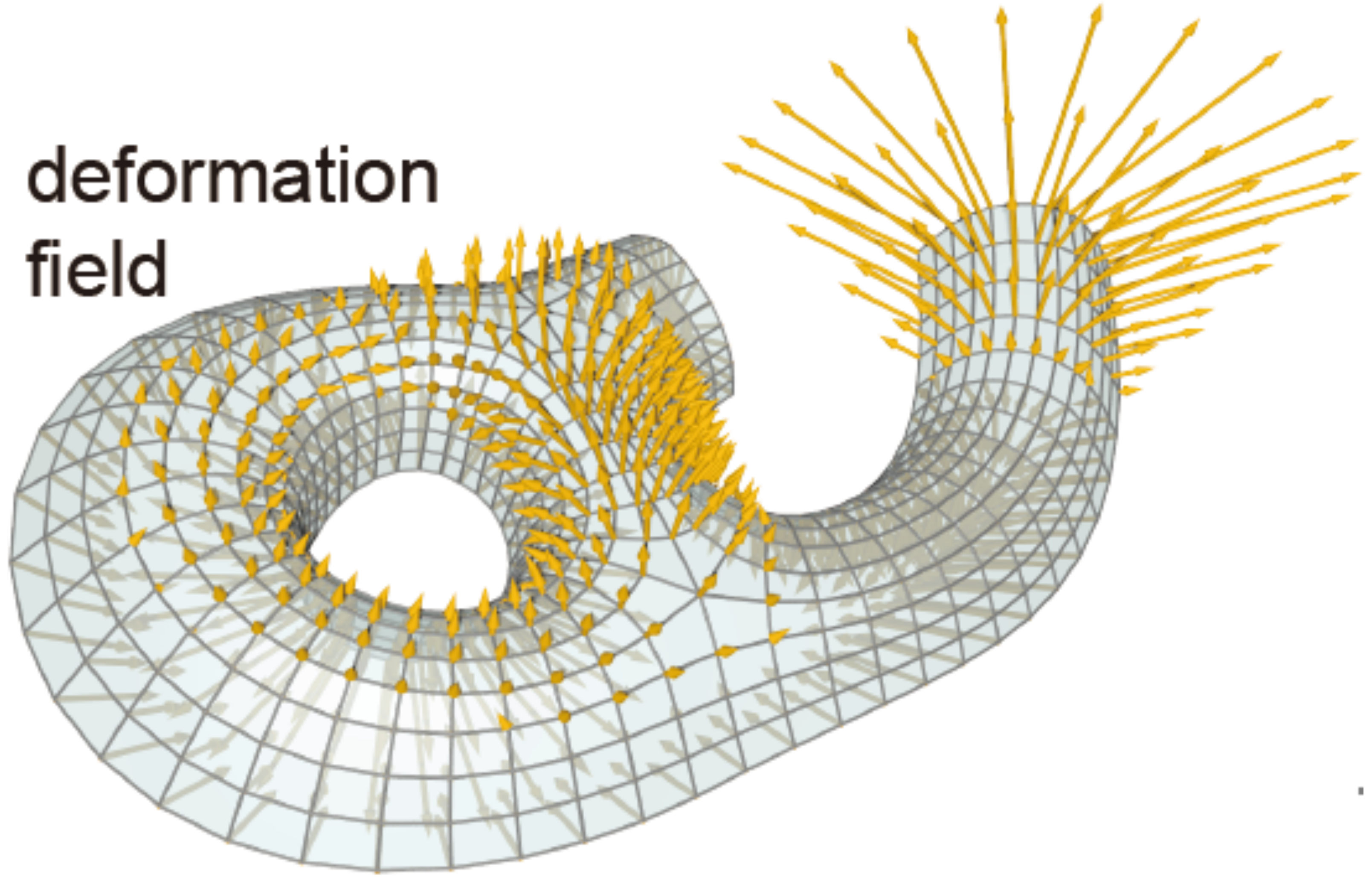
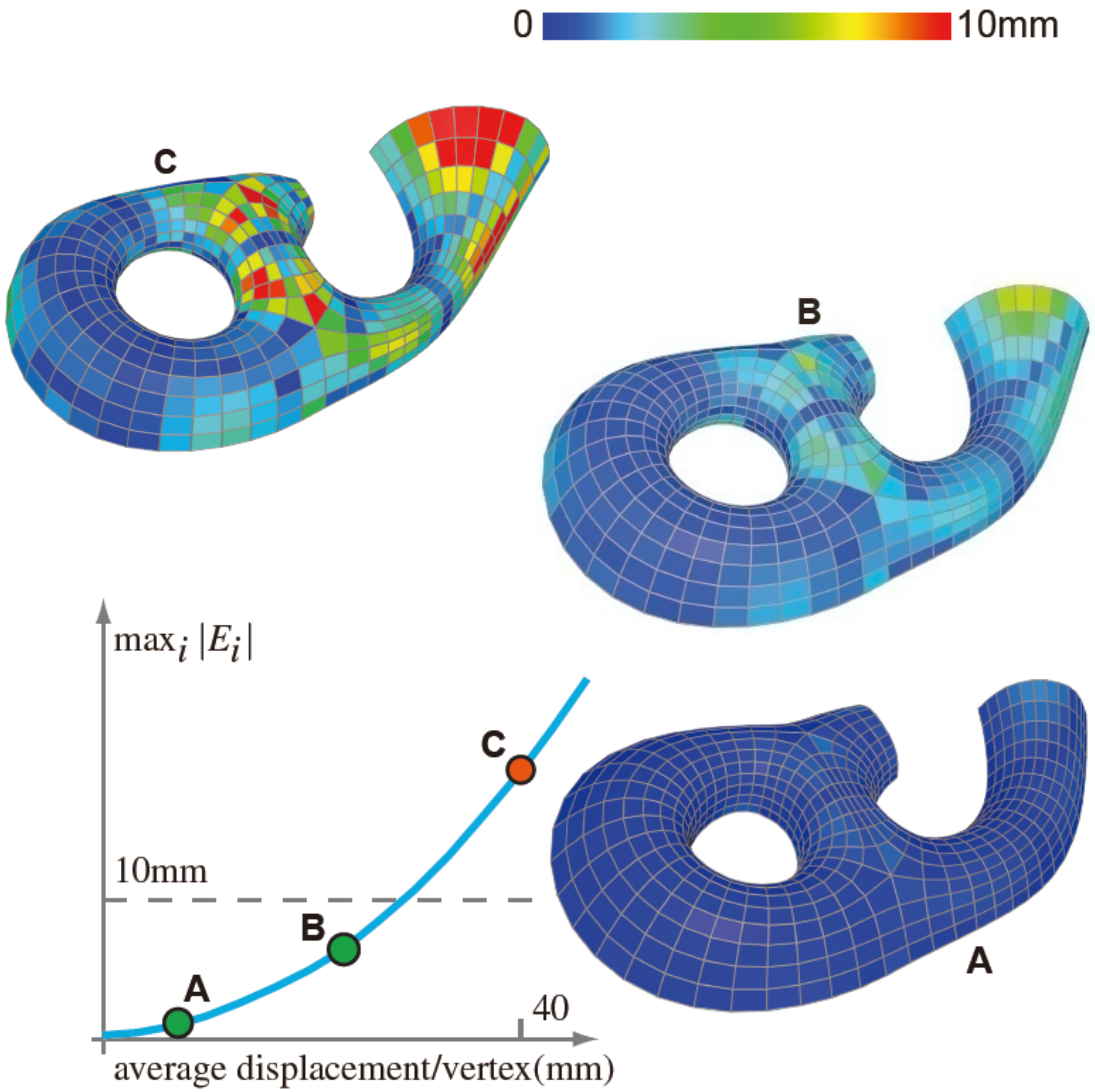
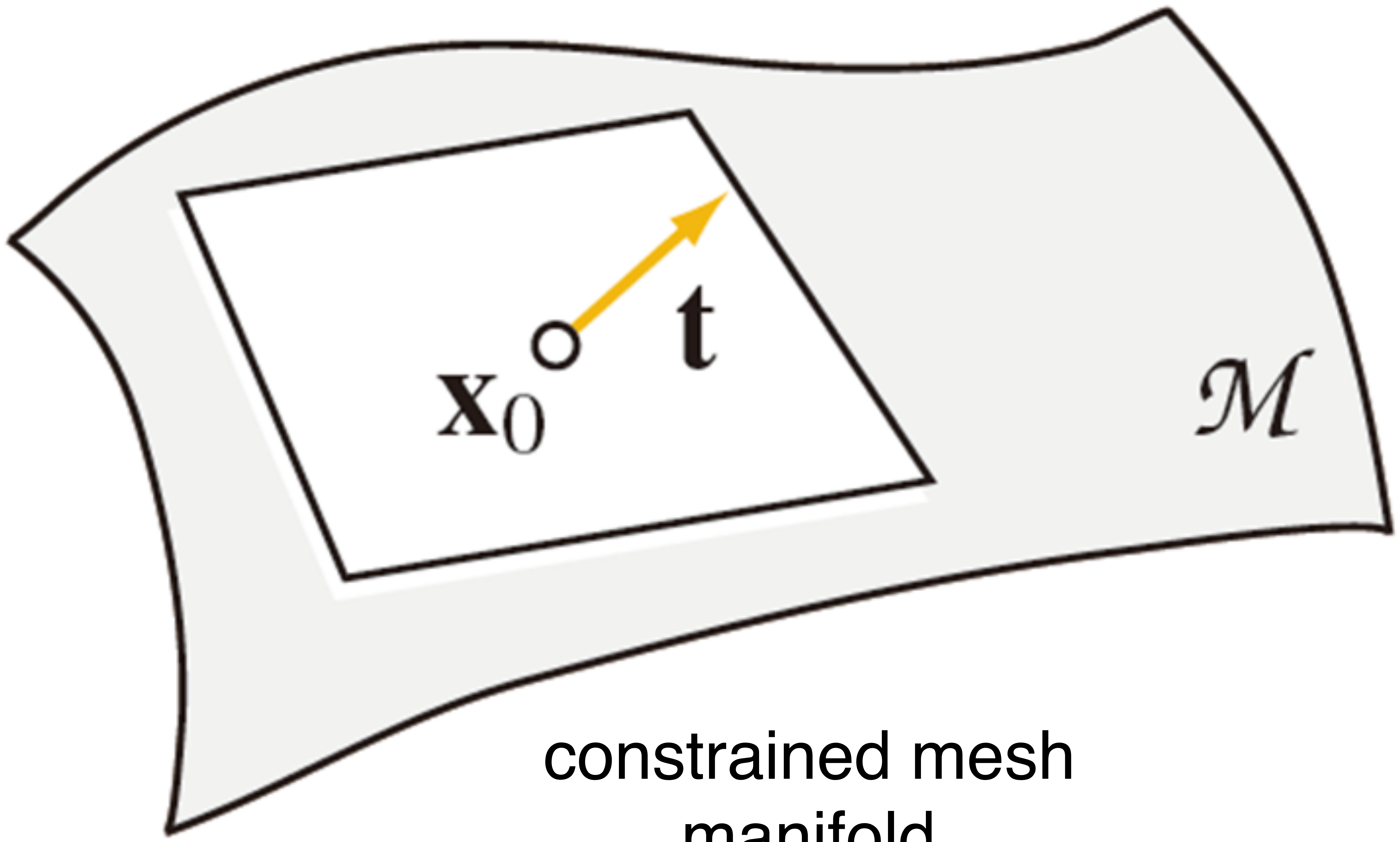






Better Approximation?

- Tangent space - 1st order approximant



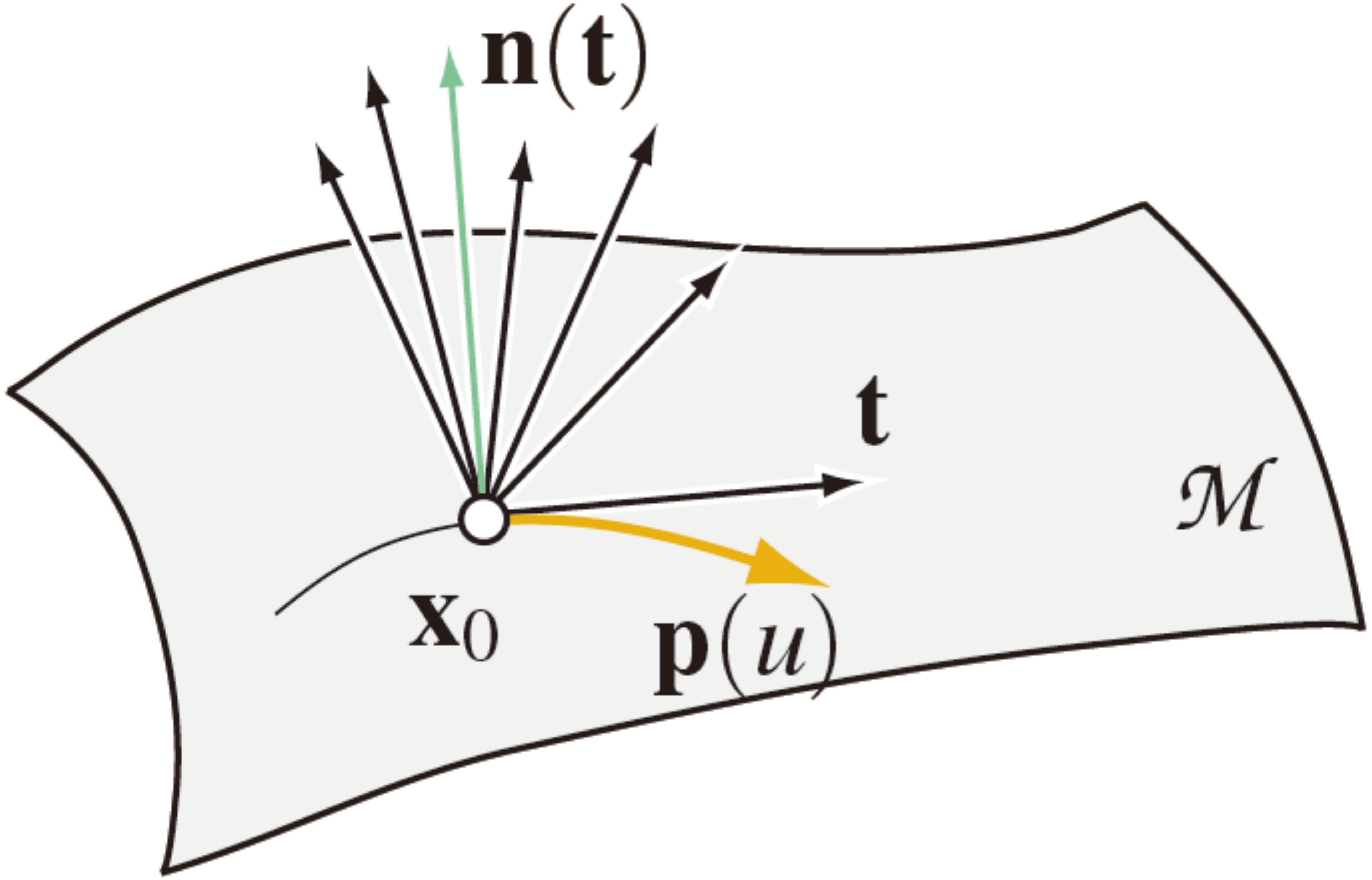
constrained mesh
manifold

straight path

ignores the curvature of the manifold

Better Approximation?

- Better approximation - 2nd order approximant



curved path

considers the curvature of the manifold

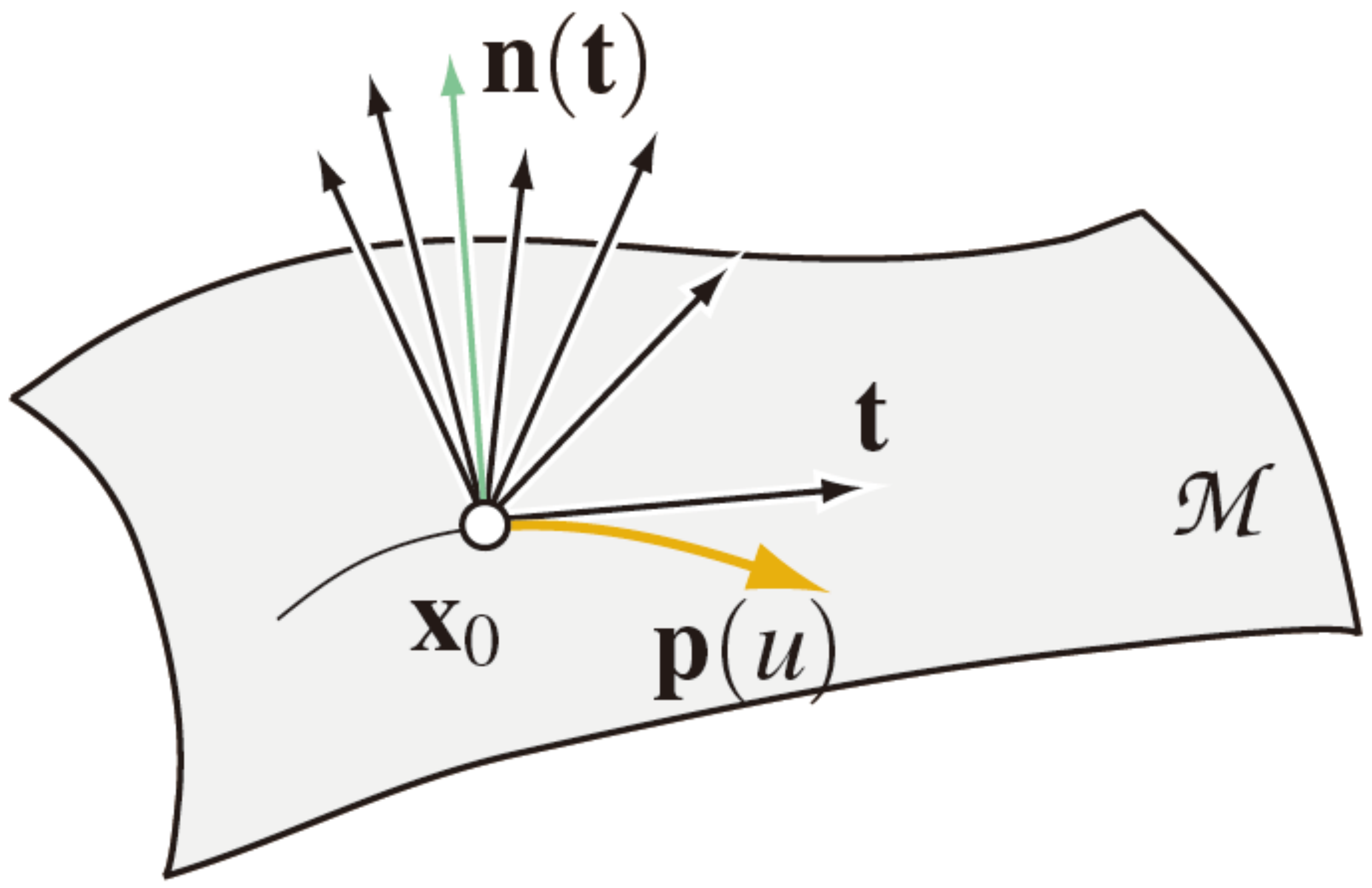
Compute Osculant

- Generalization of the osculating paraboloid in 3D
- Has the following form:

$$\mathbf{S}(\mathbf{u}) = \mathbf{x}_0 + \sum_{i=1}^{D-m} u_i \mathbf{e}_i + \frac{1}{2} \sum_{j=1}^{m} (\mathbf{u}^T \cdot \boxed{A_j} \cdot \mathbf{u}) \mathbf{n}_j$$

- Second order contact with *each* of the constraint

Walking on the Osculant



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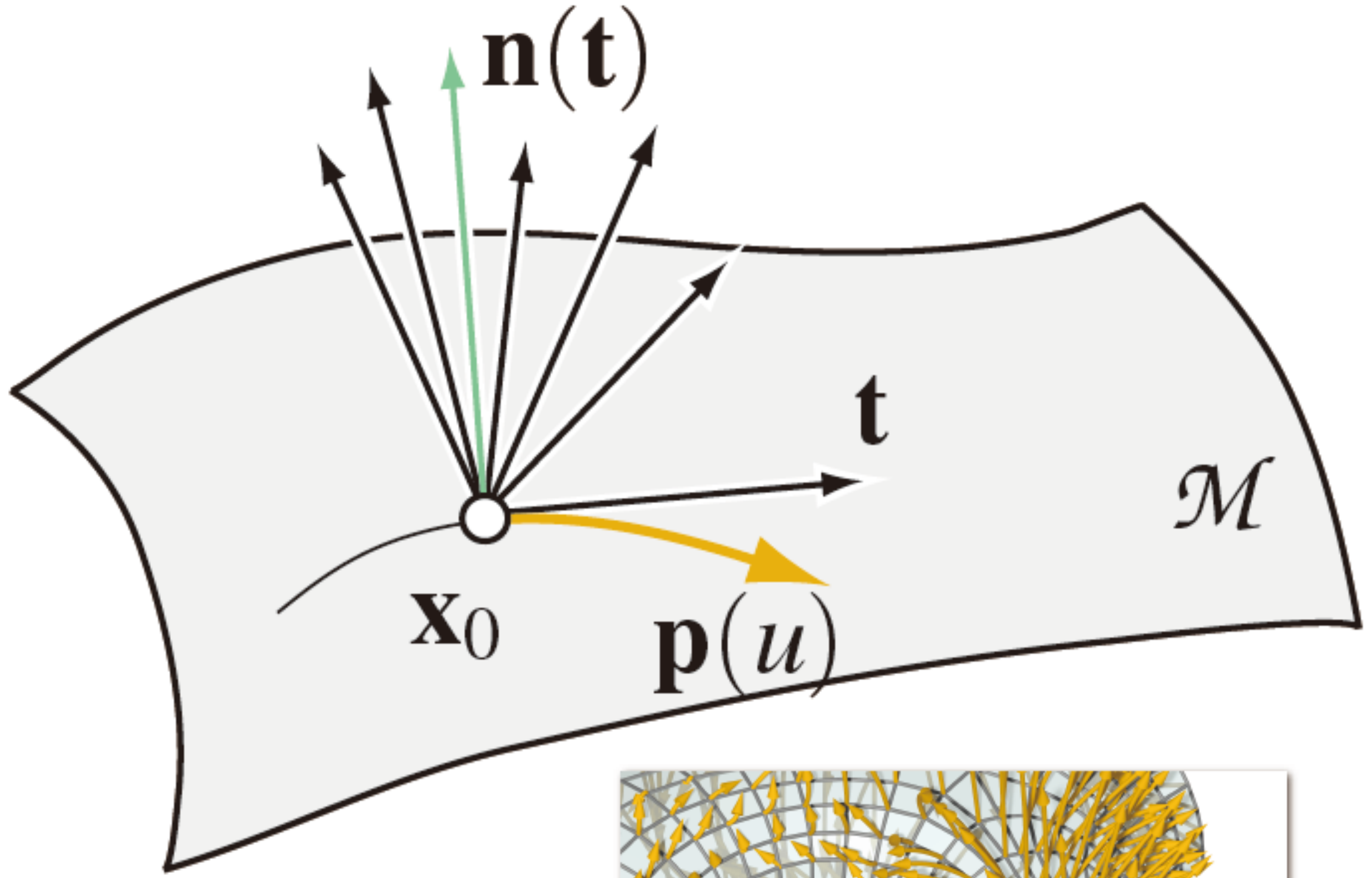
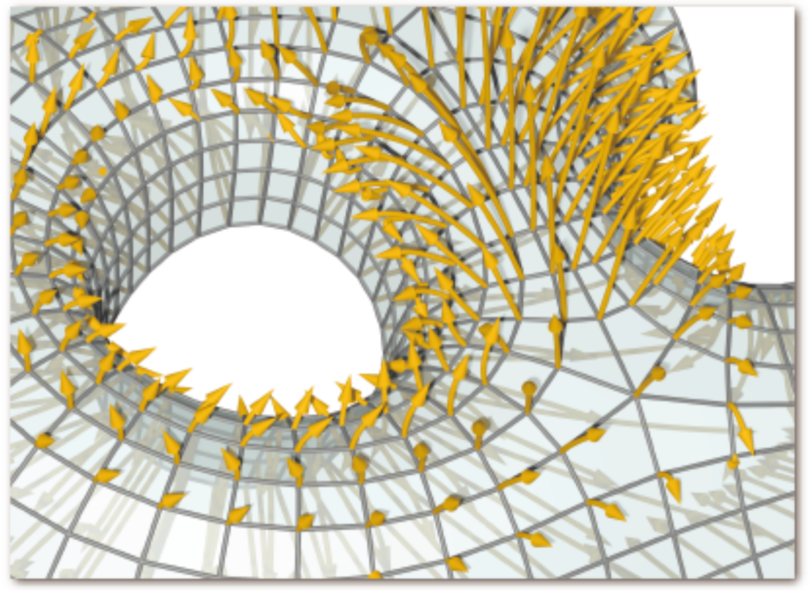
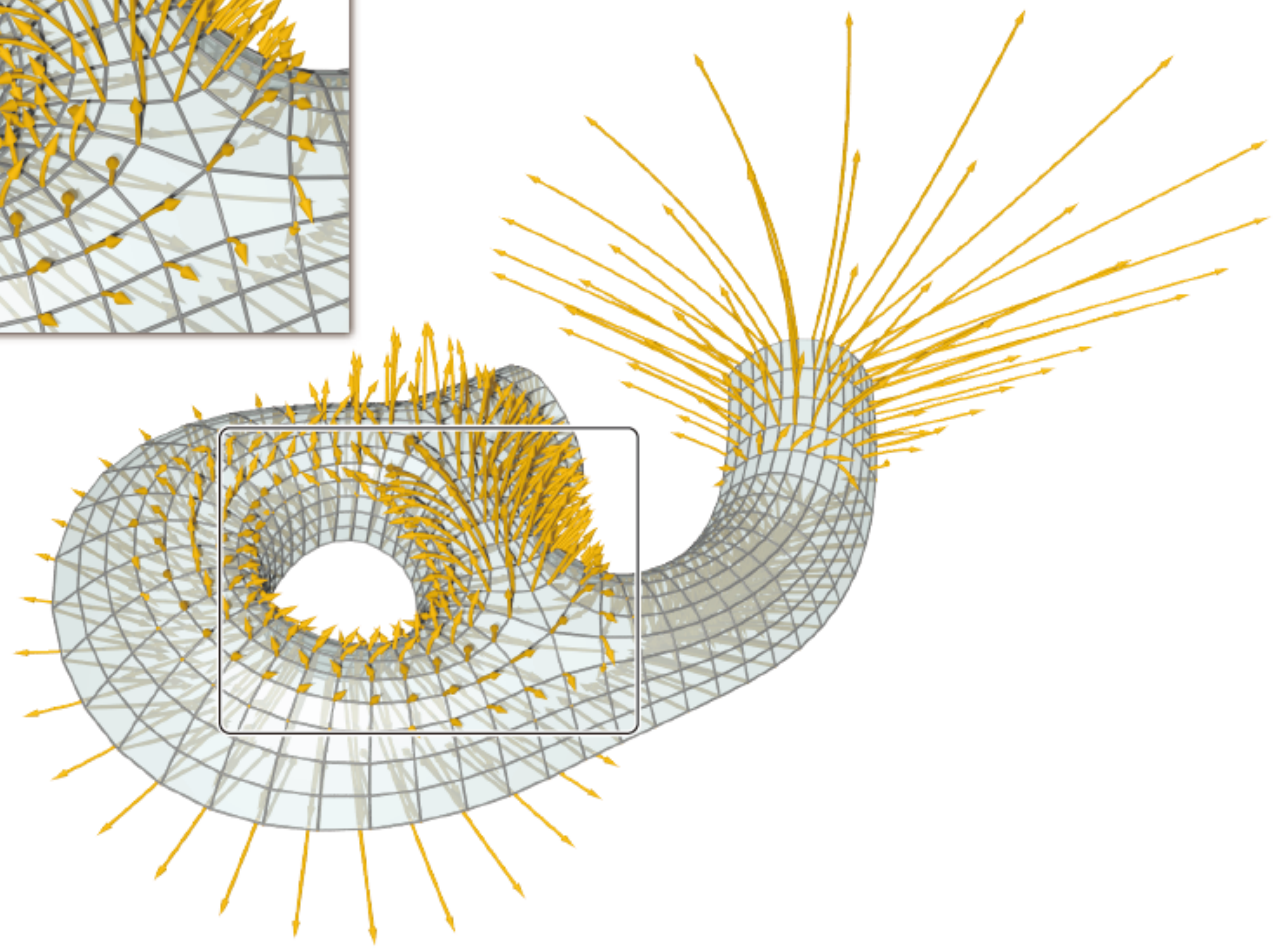




osculant
deformation
field



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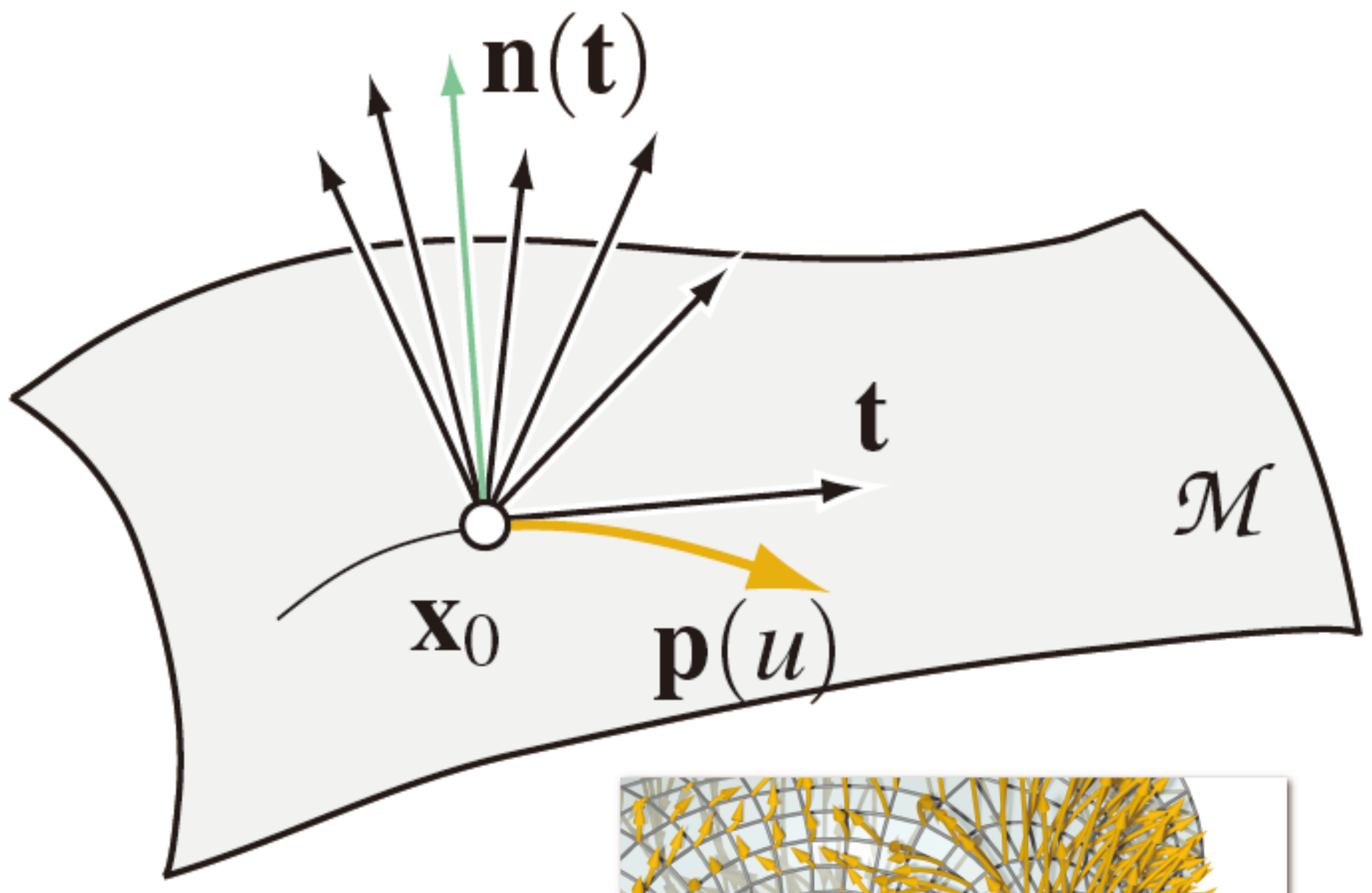

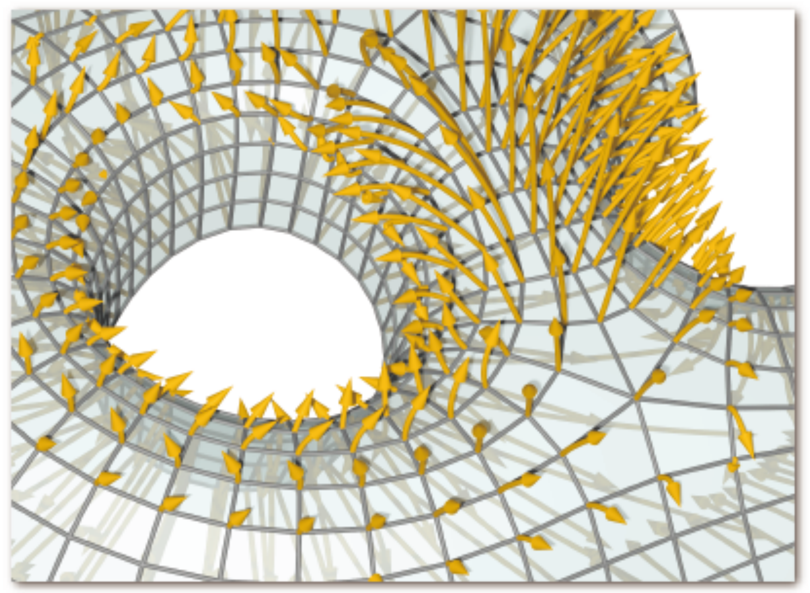

osculant
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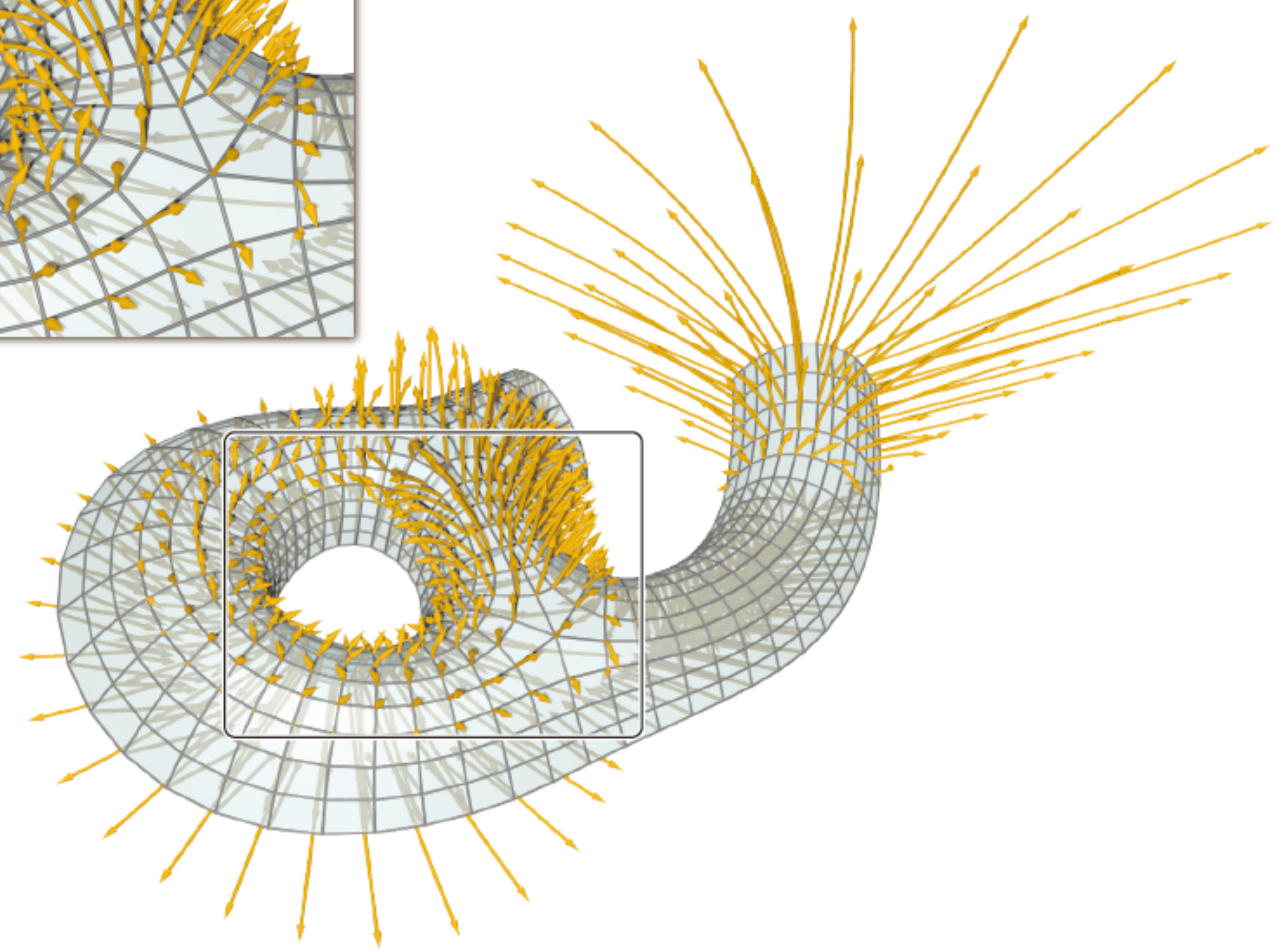

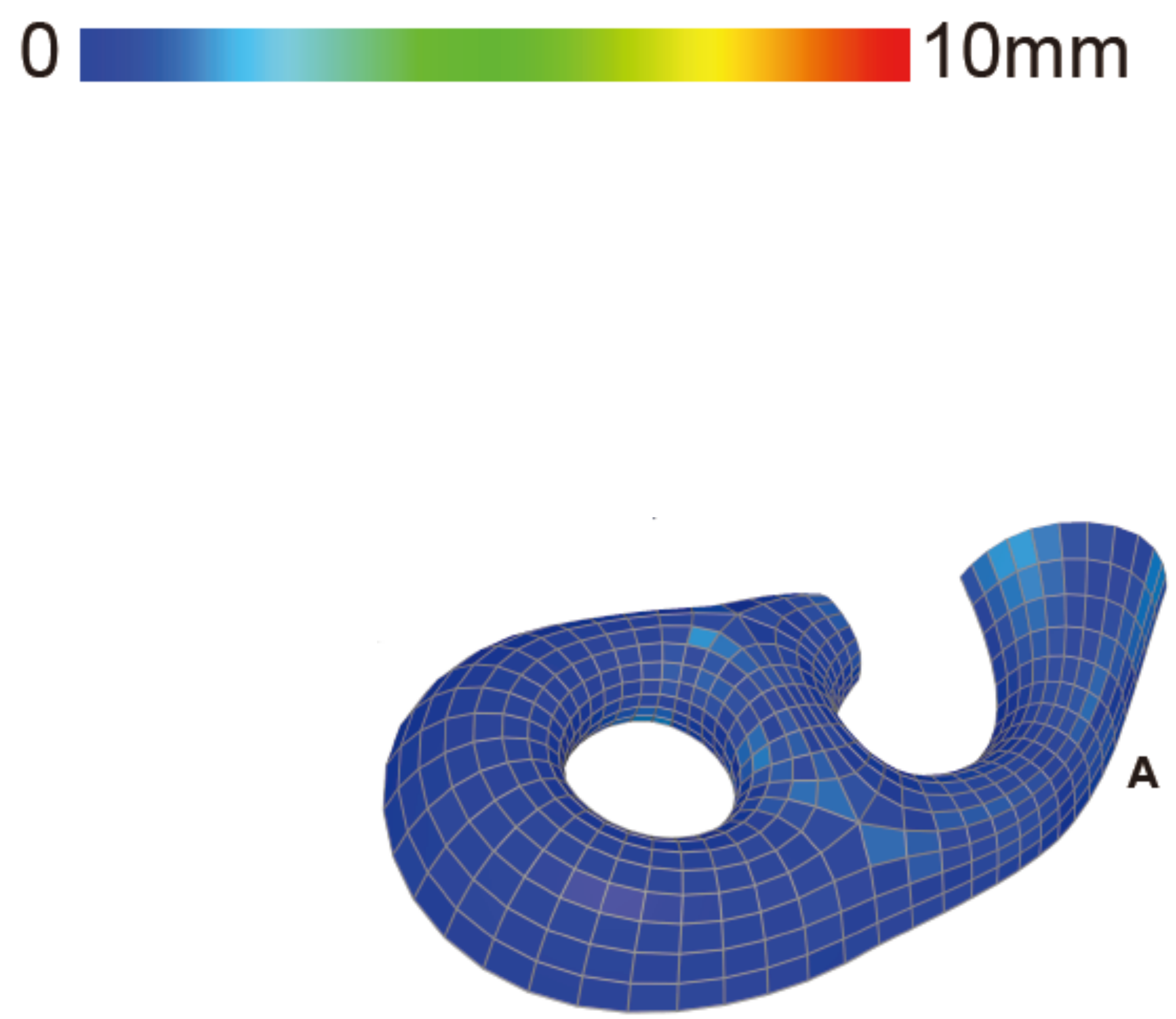

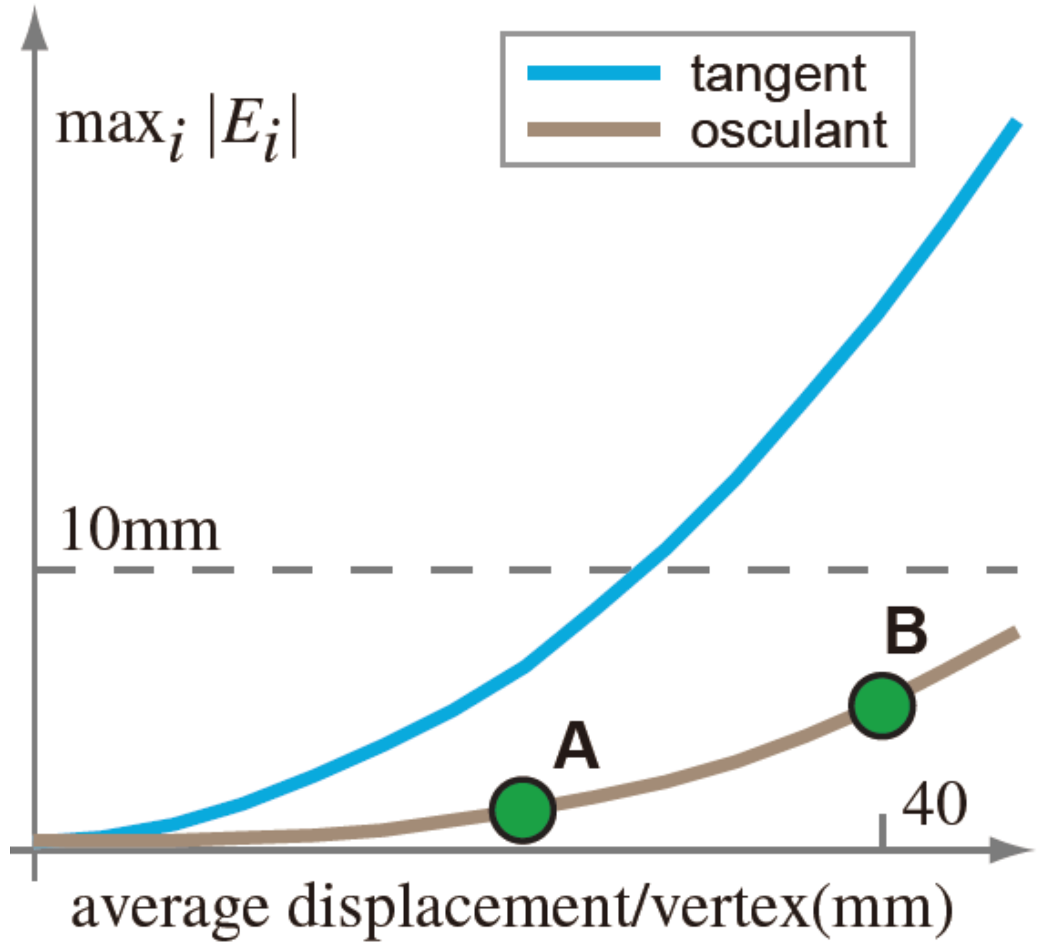

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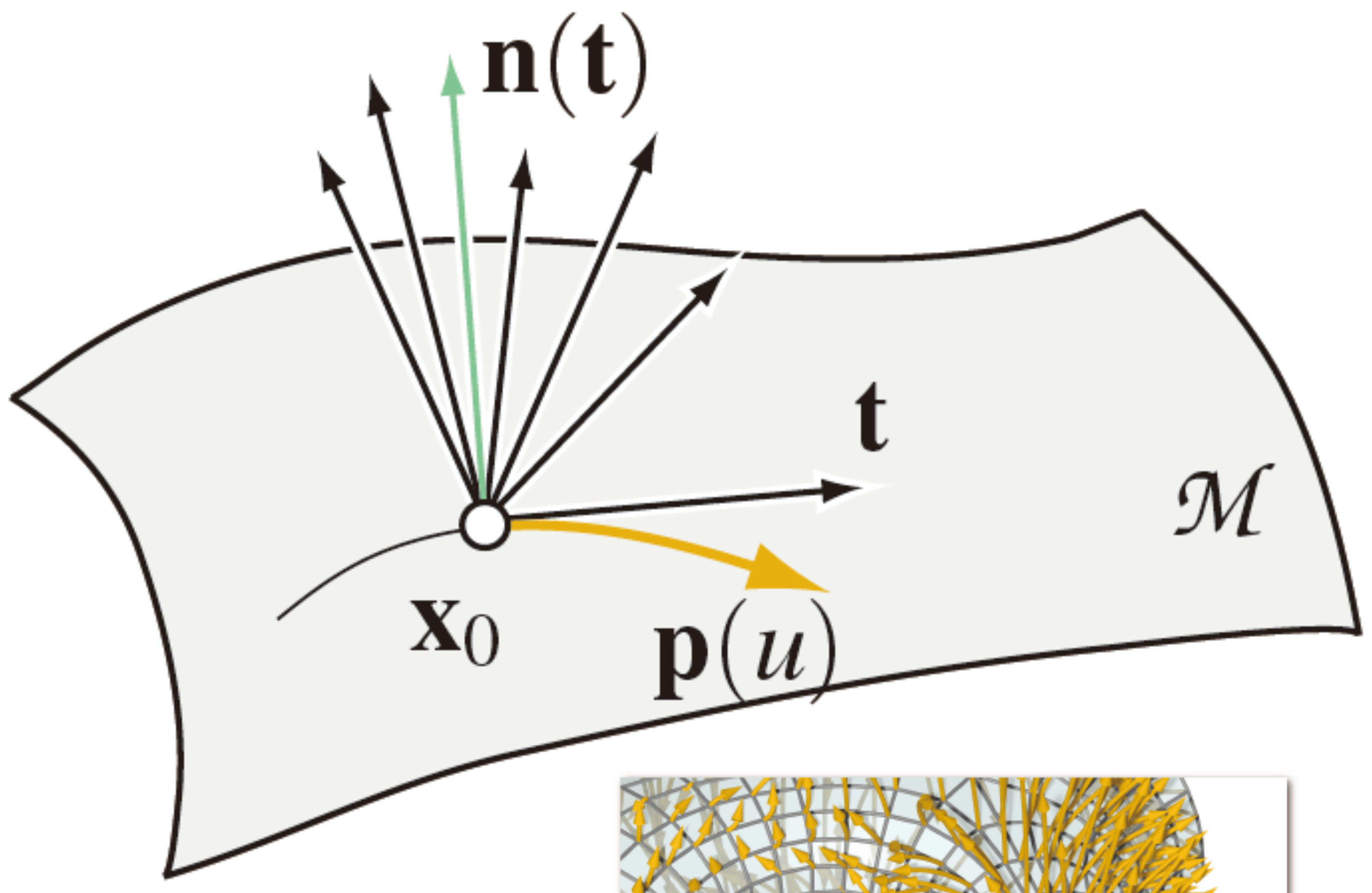
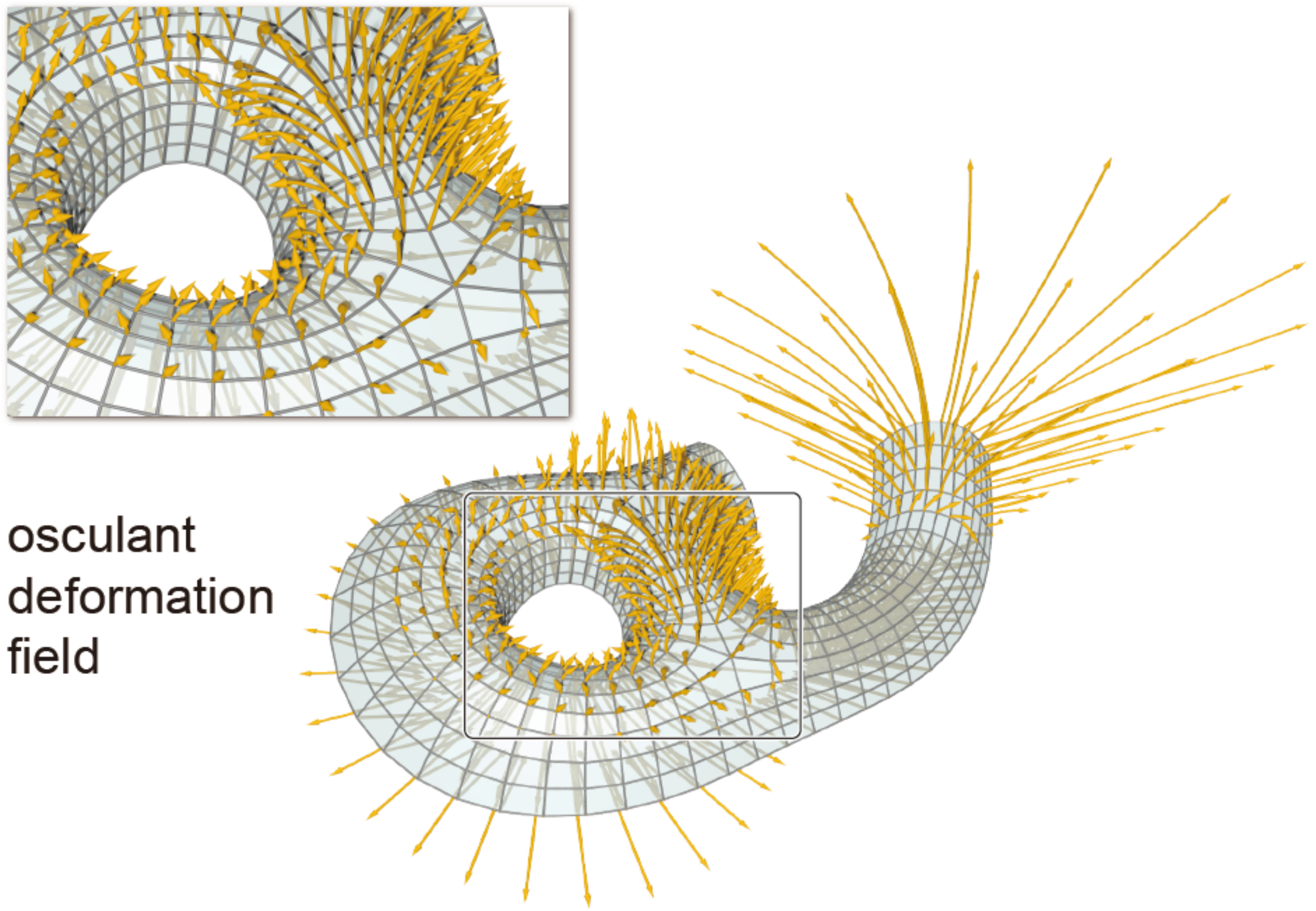
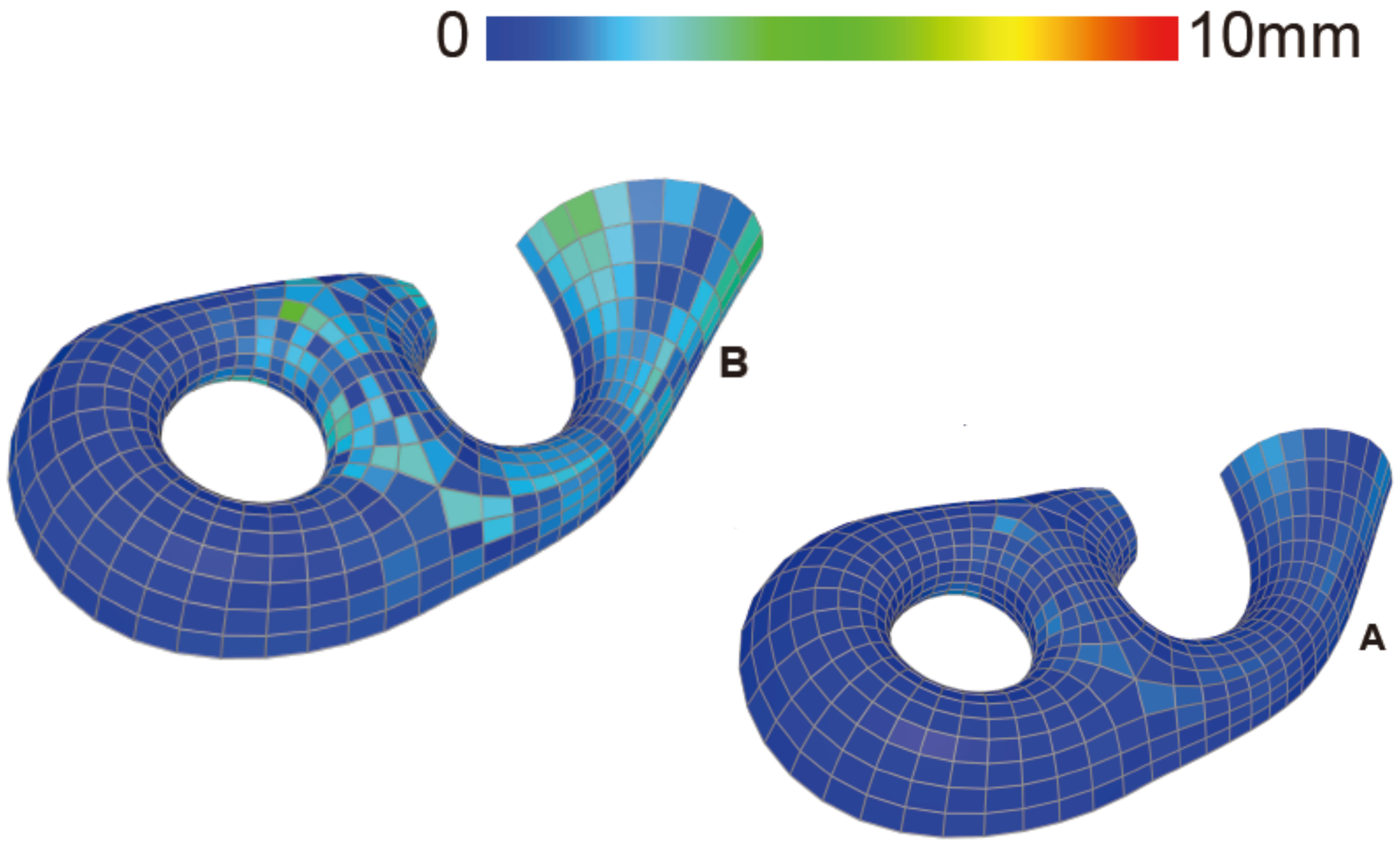
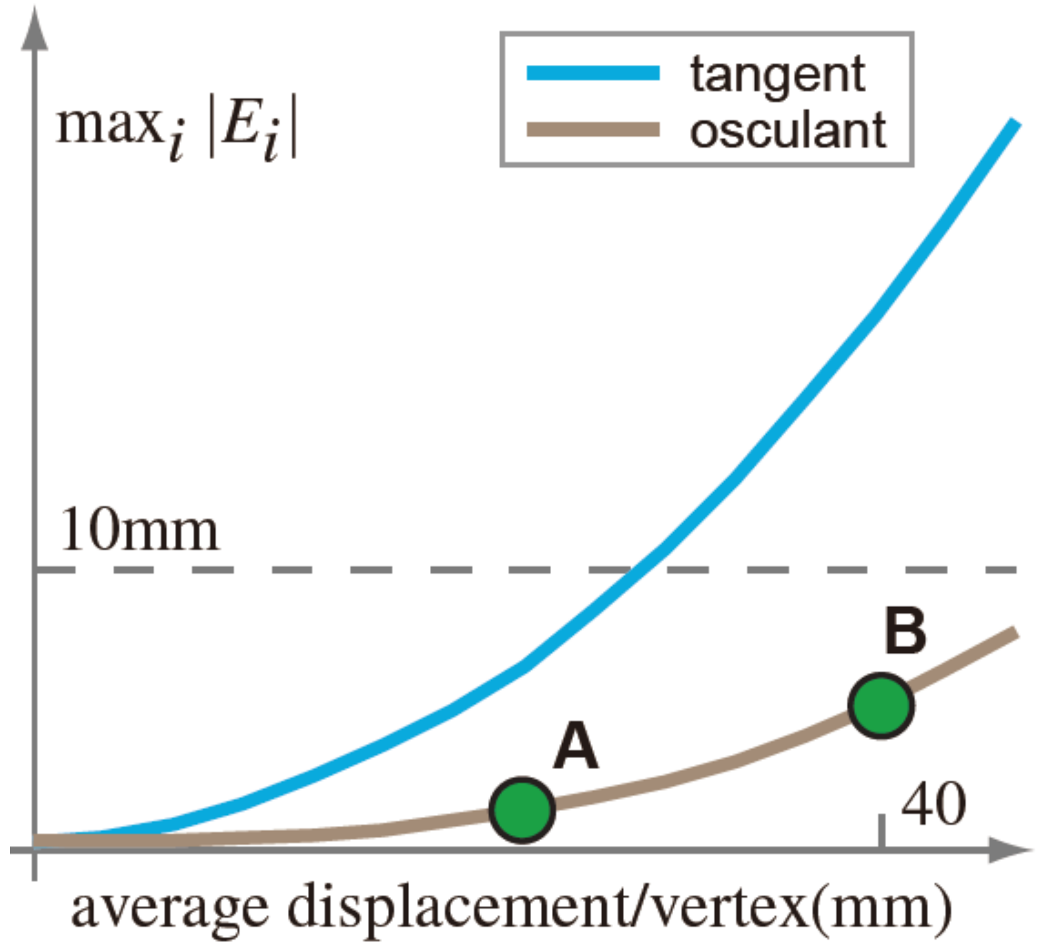




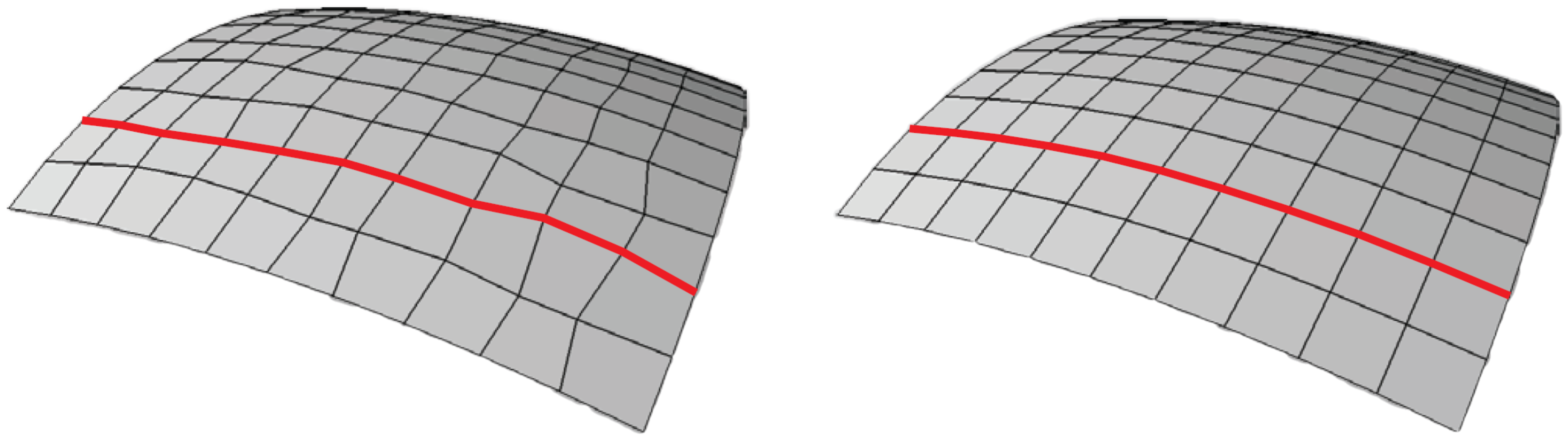
osculant deformation field





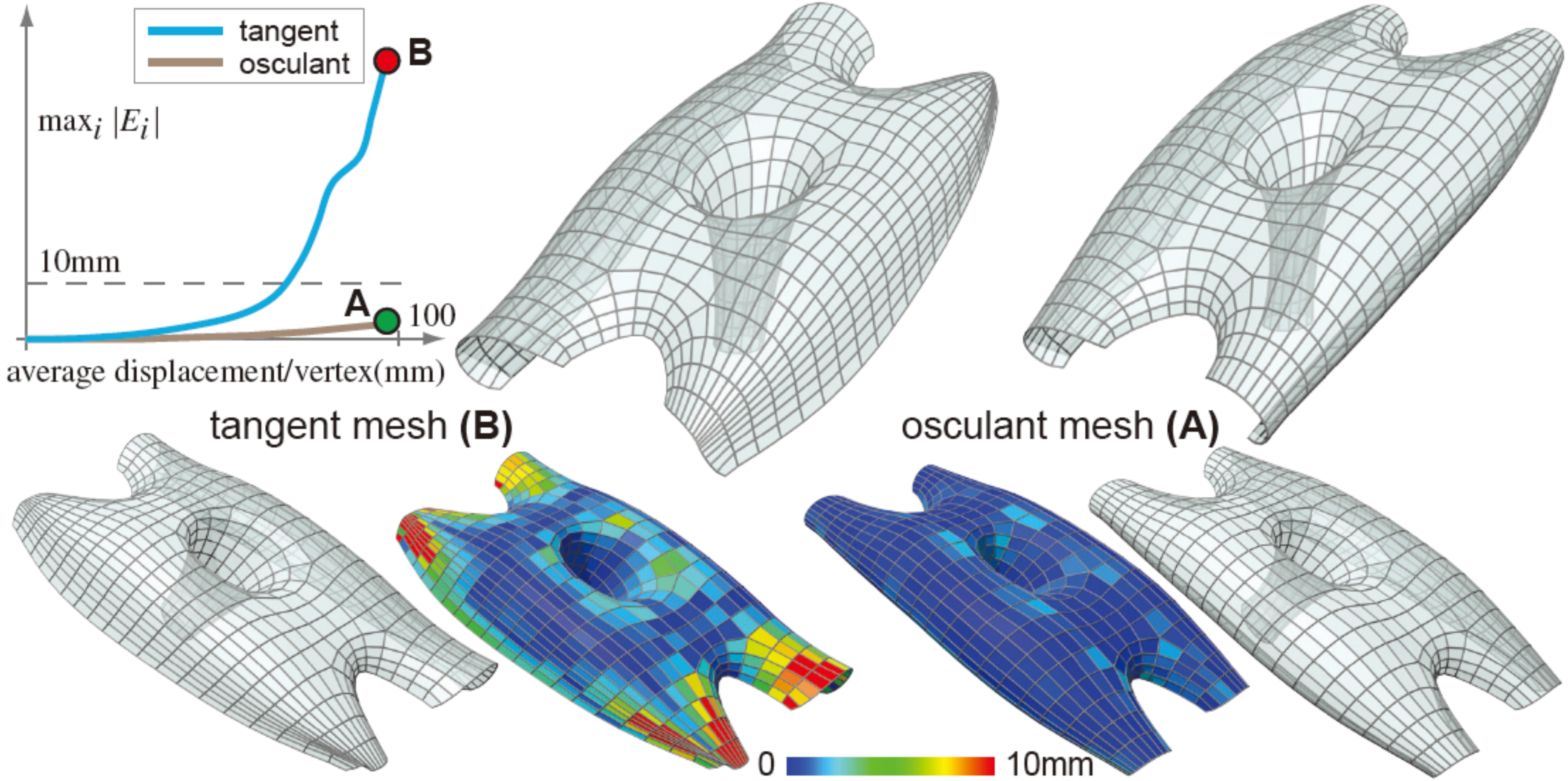
Mesh Quality?

- Osculant concerns only constraints
- Quality measures based on application
  - fairness – meshes with beautiful structure



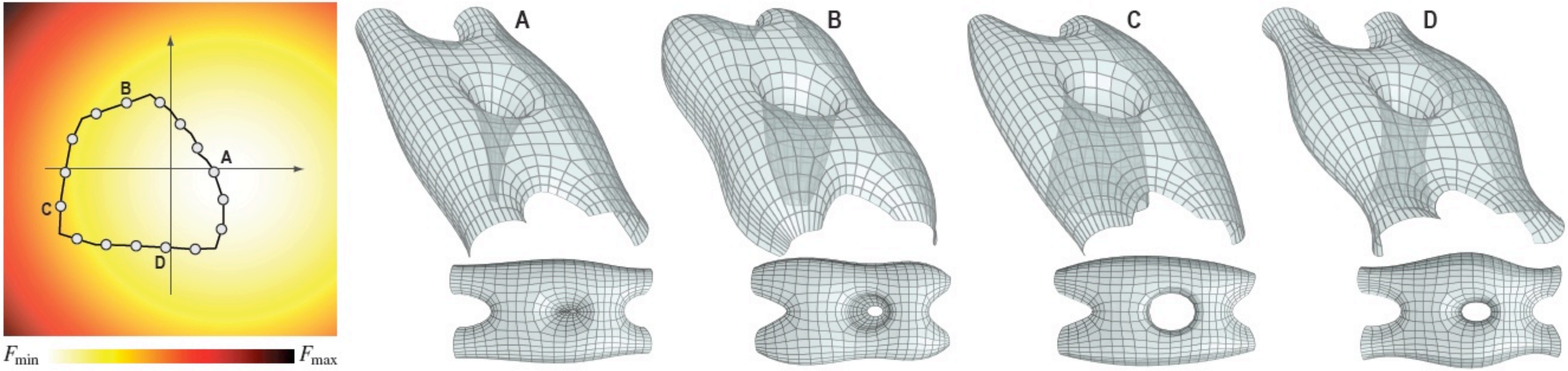
- Extract the *meaningful* part of the manifold

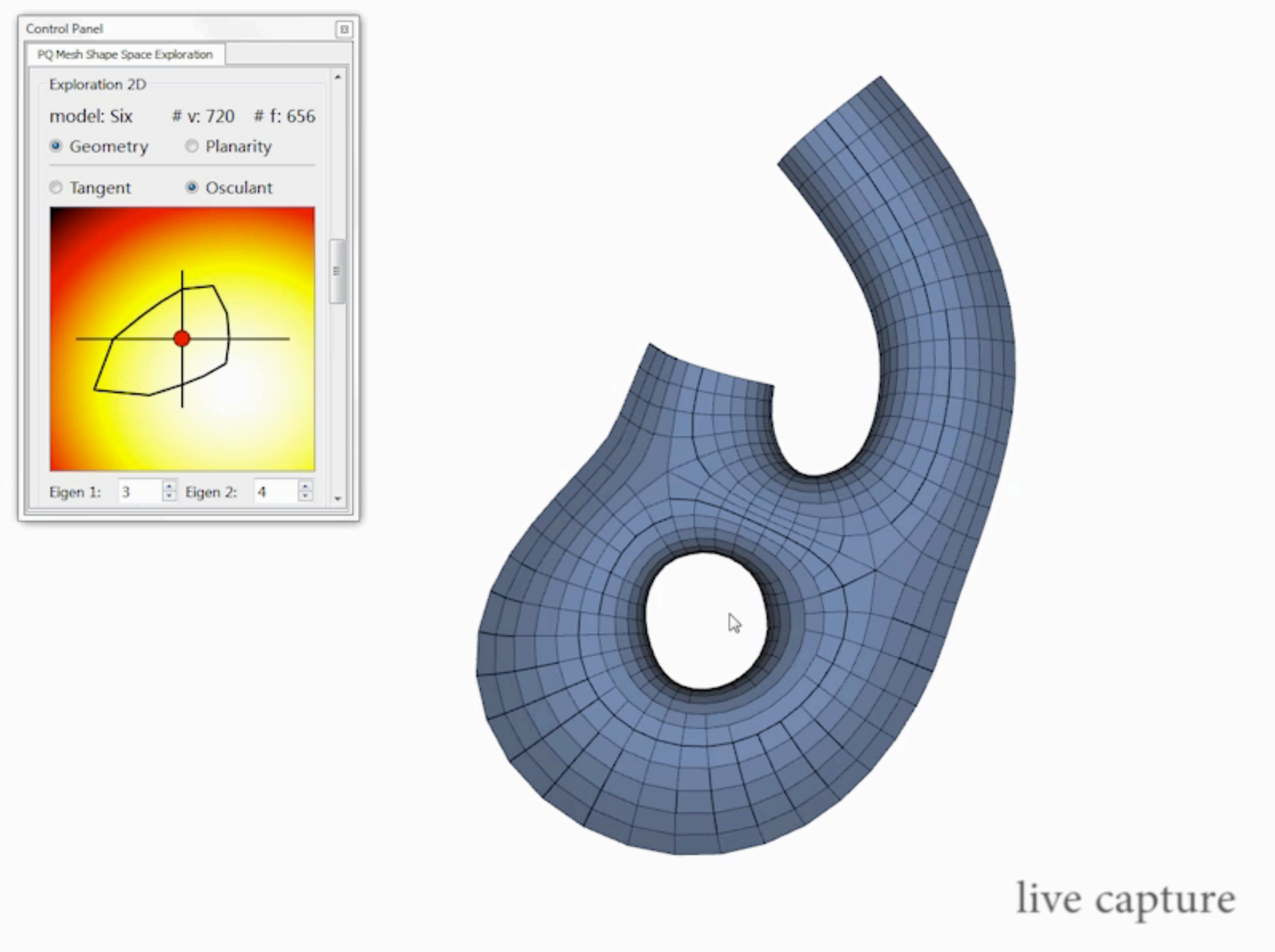
What Do We Gain?



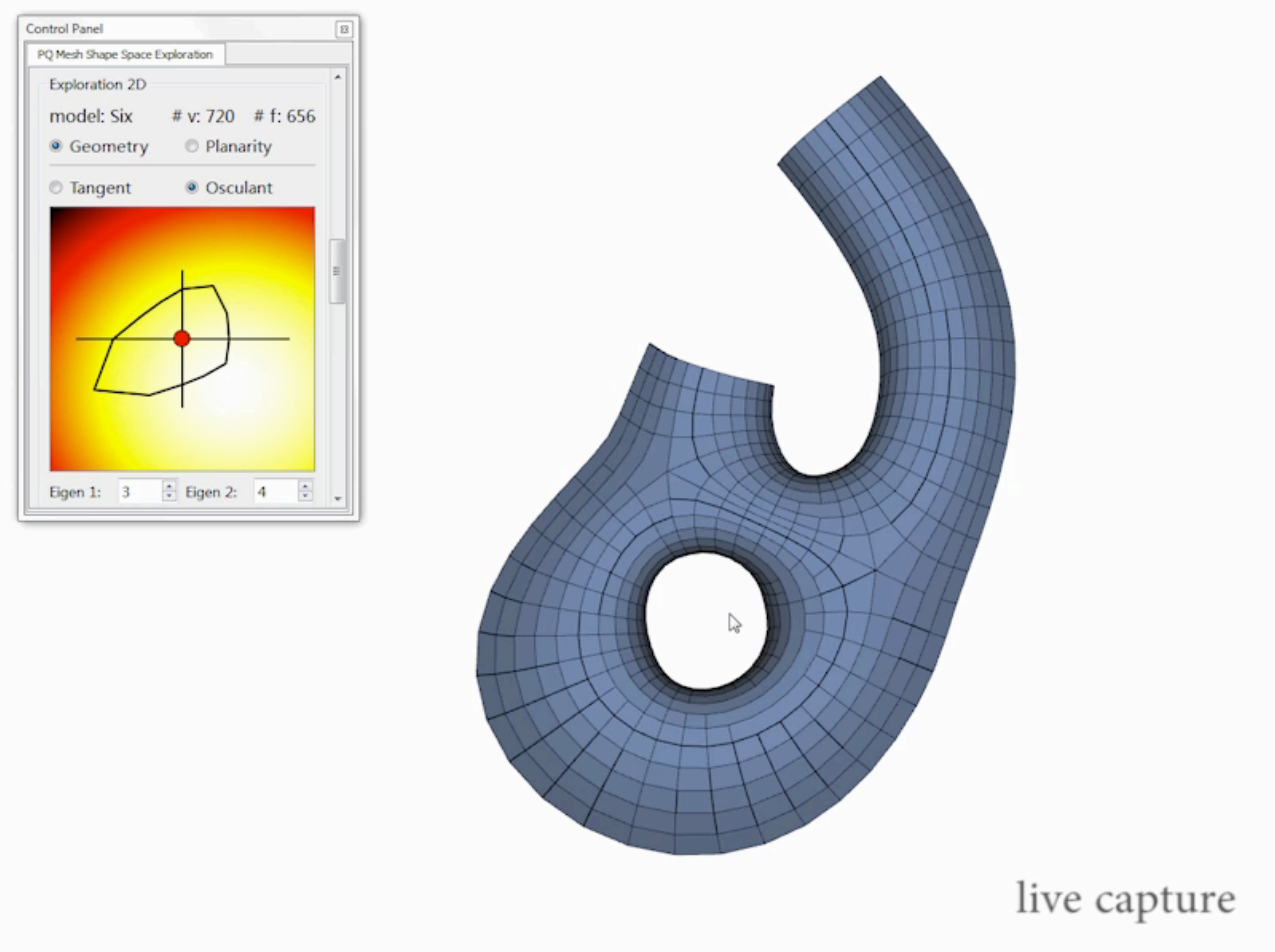
Spectral Analysis

- Good (desirable) subspaces to explore
- 2d-slice of design space





Control Panel
PQ Mesh Shape Space Exploration
Exploration 2D
model: Six
v: 720
f: 656
Geometry
Planarity
Tangent
Osculant
Eigen 1: 3
Eigen 2: 4
live capture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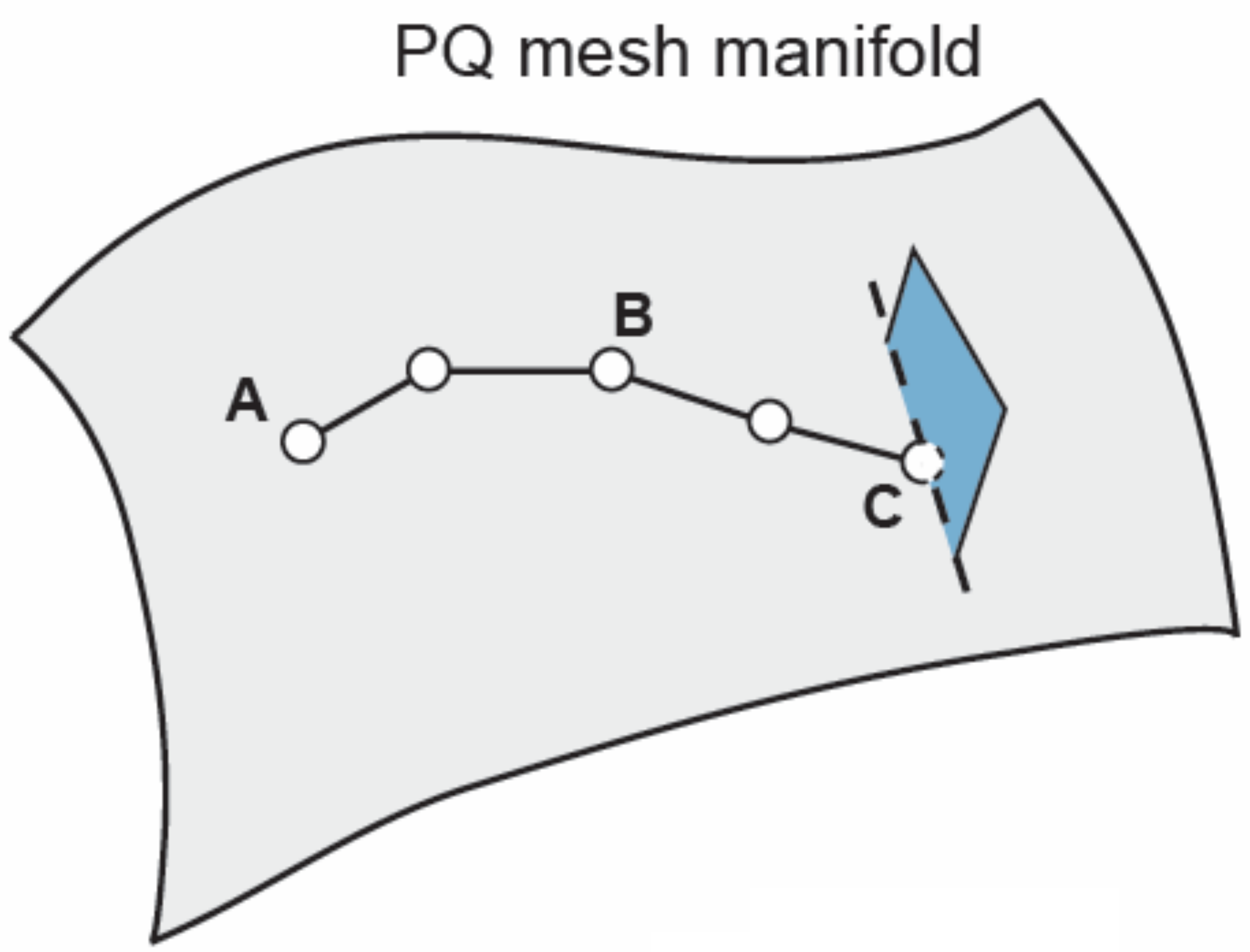
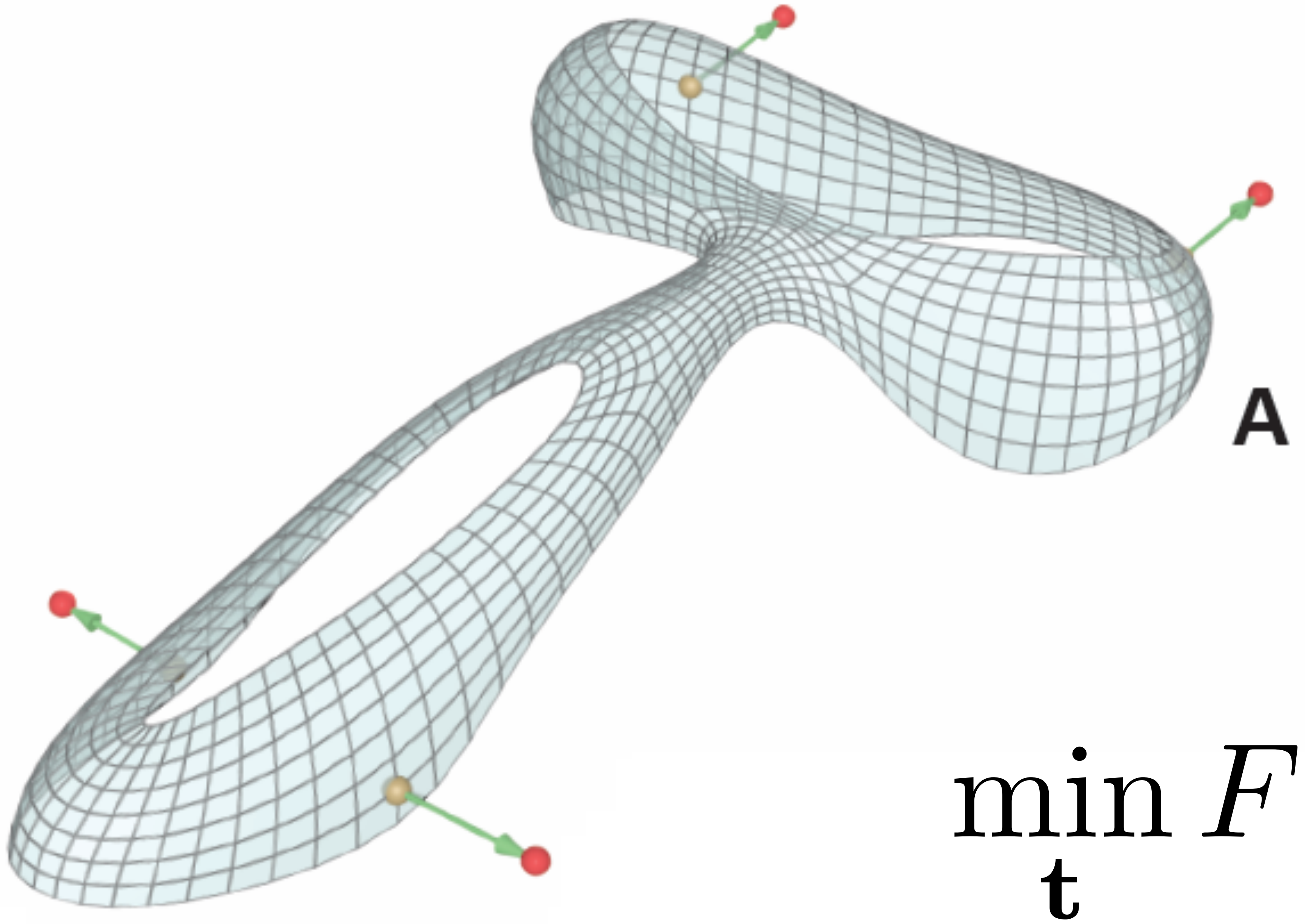
Handle Driven Exploration



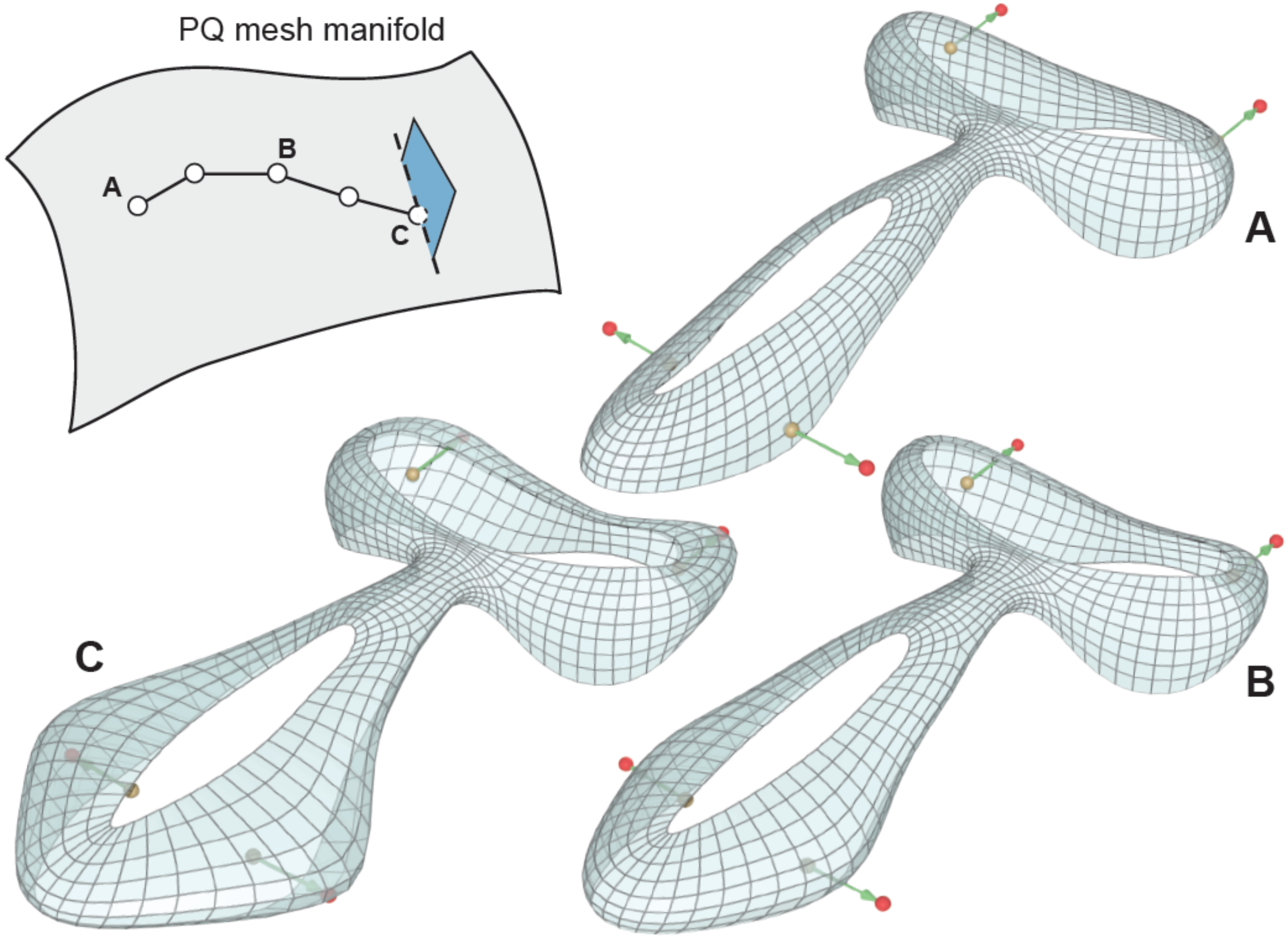


$$\min_{\mathbf{t}} F(\mathbf{x}_0 + \mathbf{t}) \text{ such that,}$$

$$\nabla E_i^T \cdot \mathbf{t} = 0,\ \forall i = 1, \ldots, m;$$

$$t_j = v'_j - v_j$$

Handle Driven Exploration

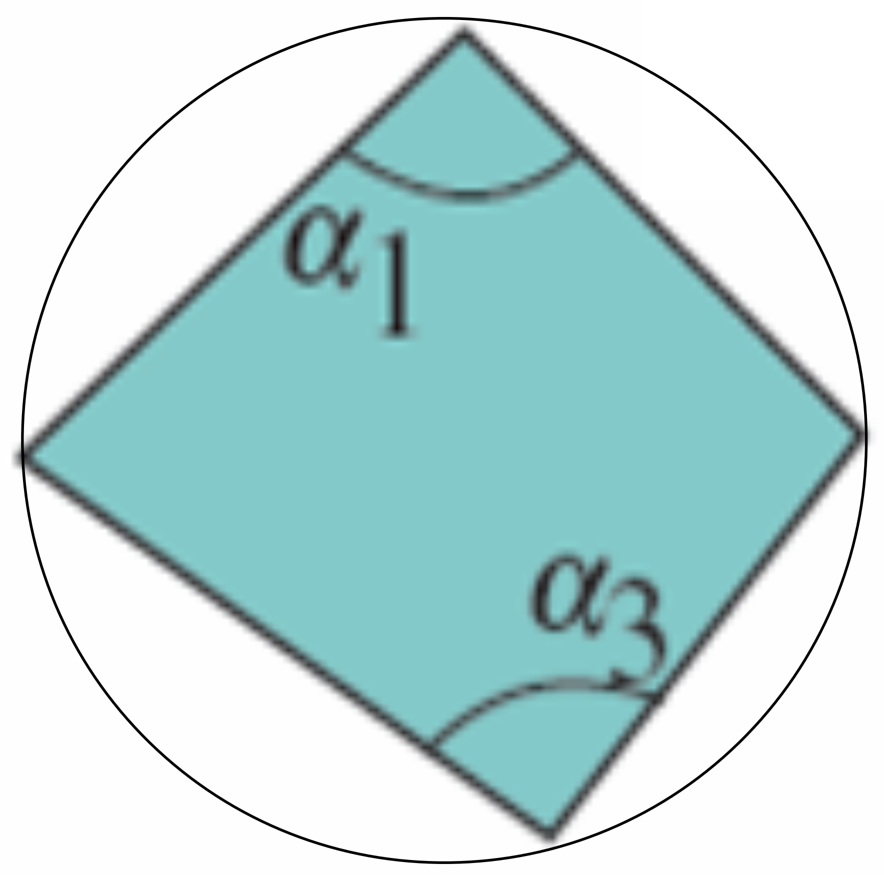


Other Constrained Meshes

Circular Mesh Manifolds

- Circular Meshes
  - ➢ Each face has a circumcircle

$$(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$$
$$\Rightarrow \alpha_1 + \alpha_3 = \pi$$



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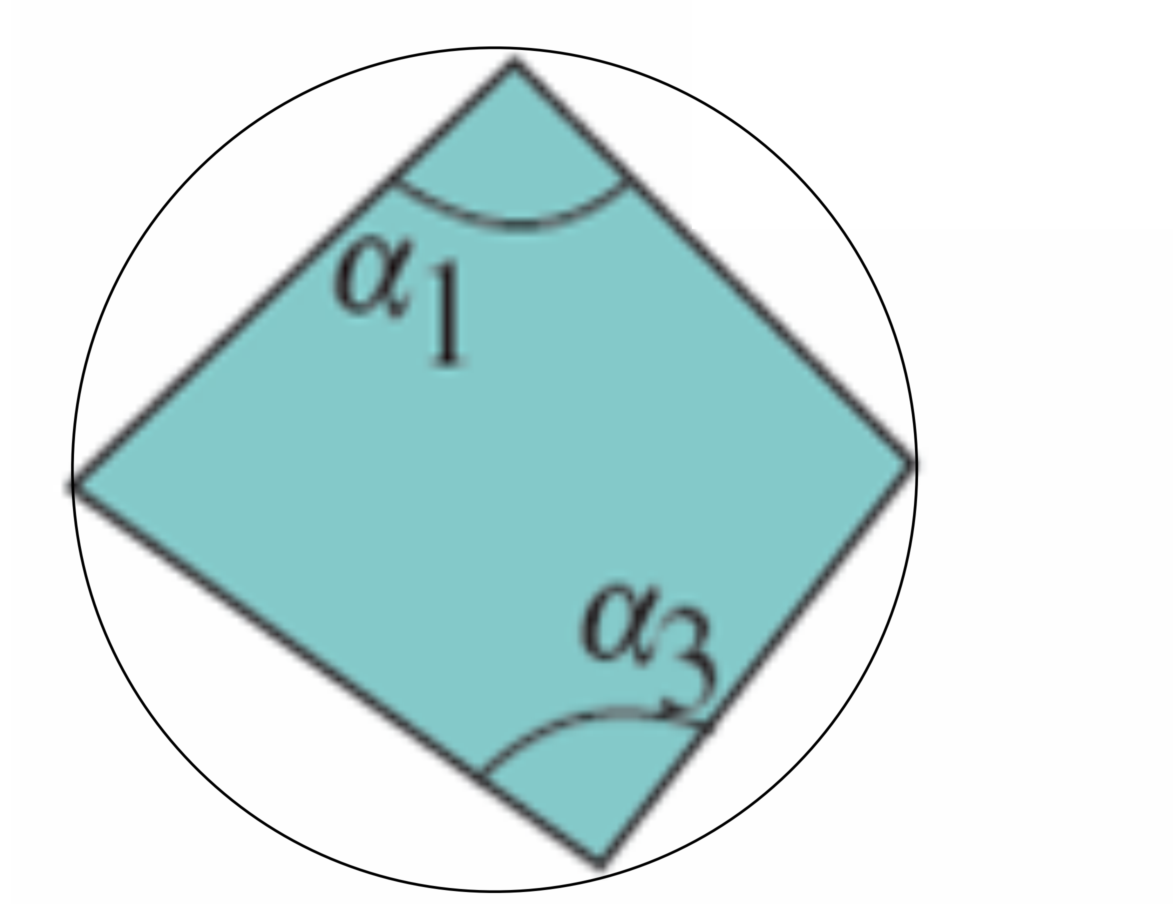
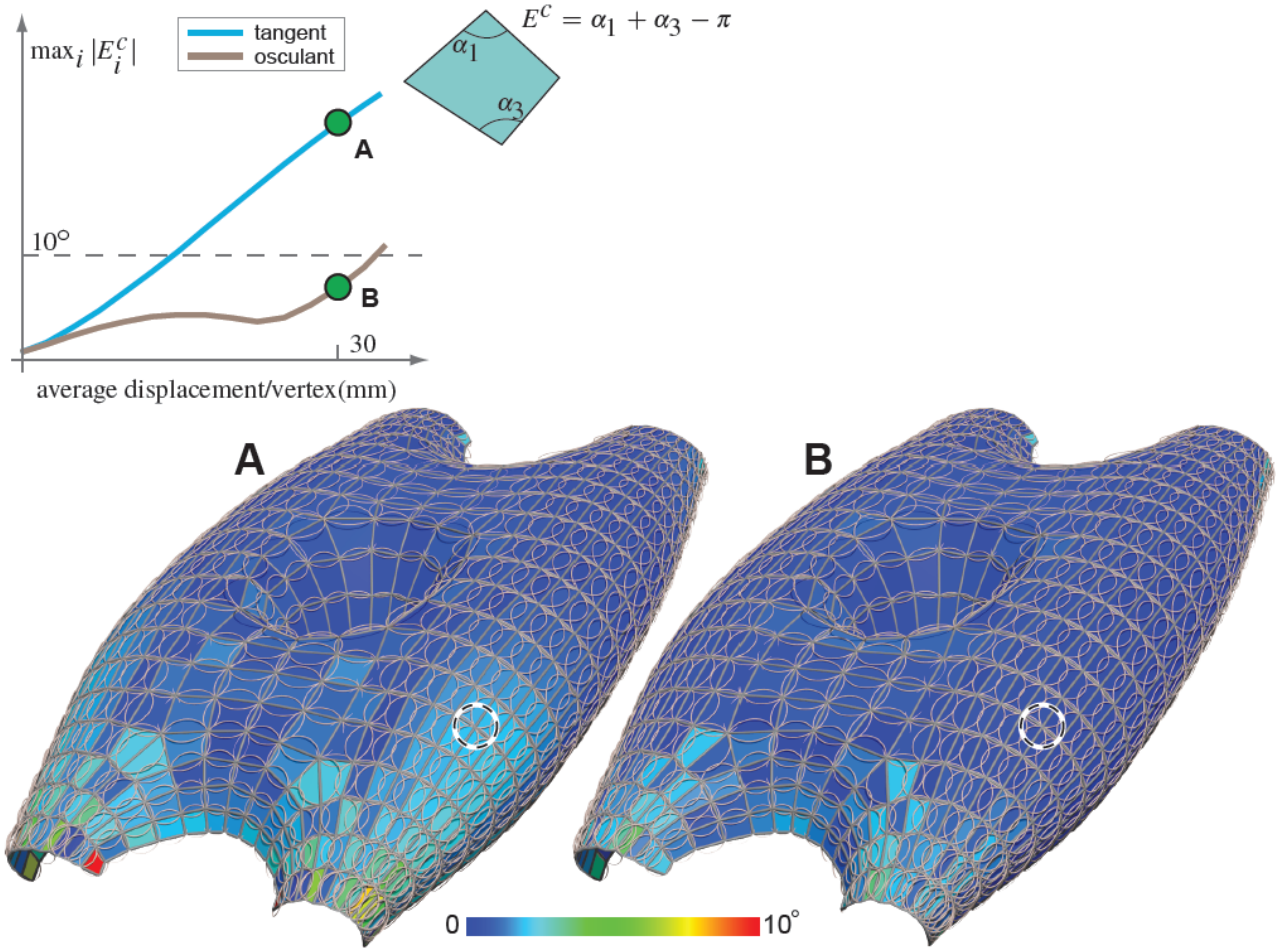
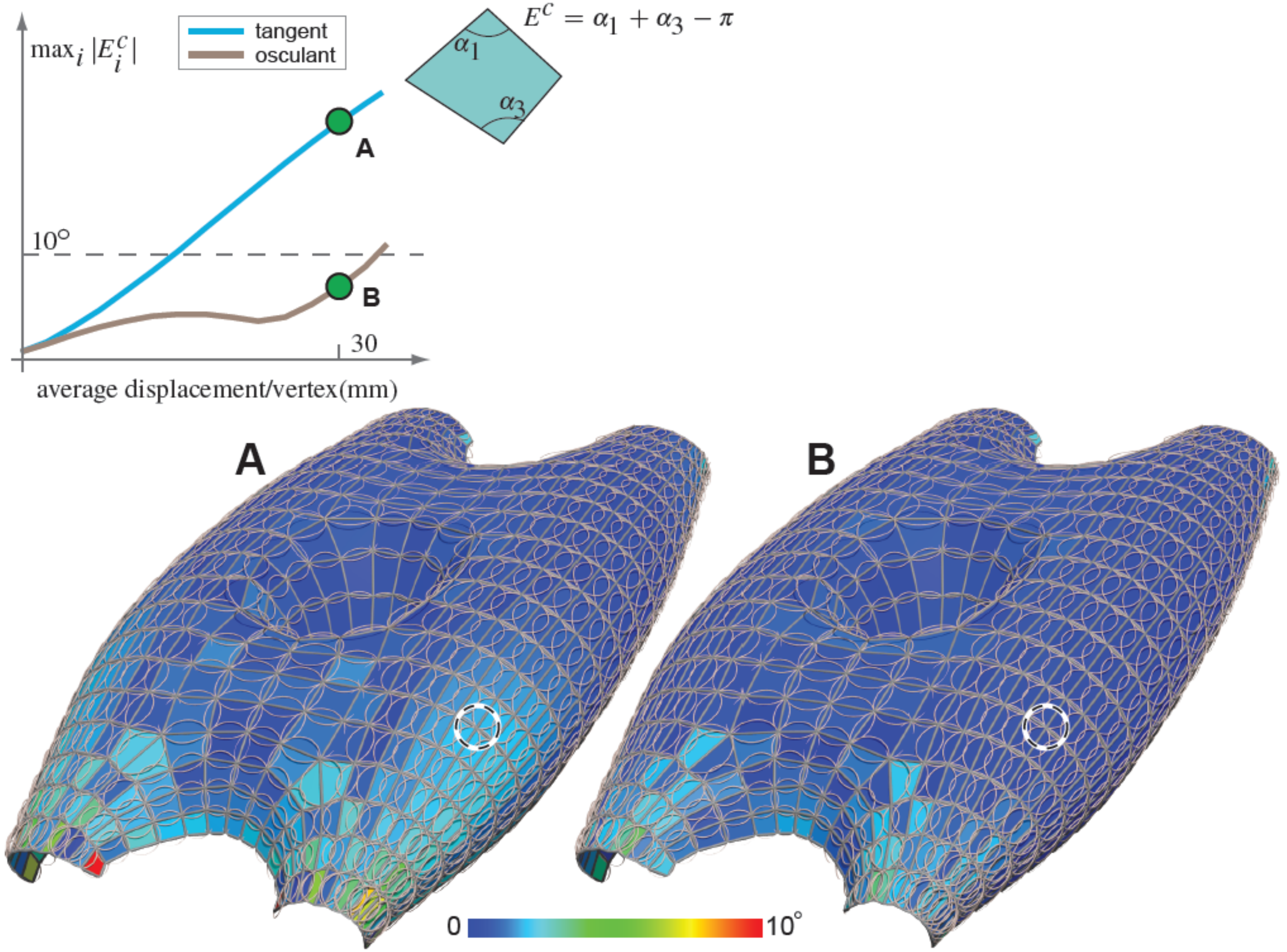


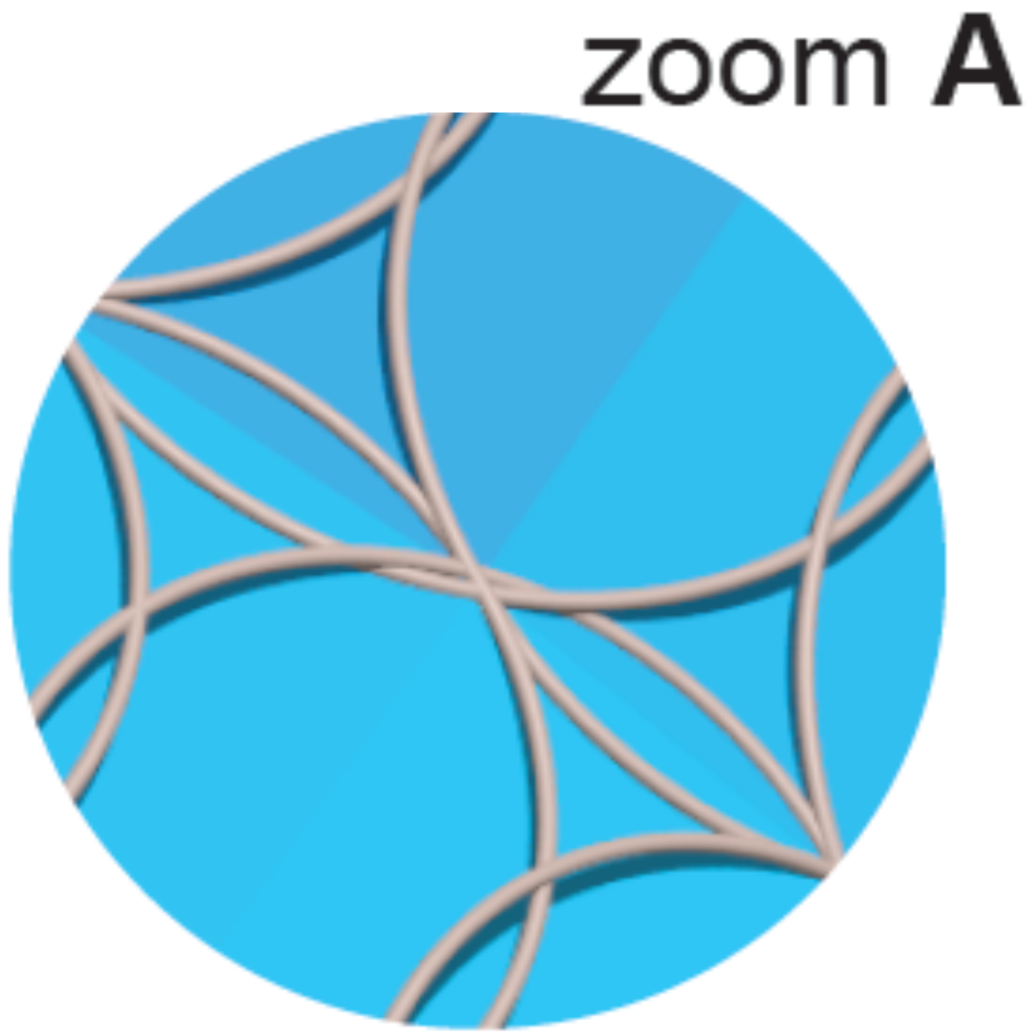
$$E_i^c : \alpha_1 + \alpha_3 - \pi$$

Circular Mesh Shape Space

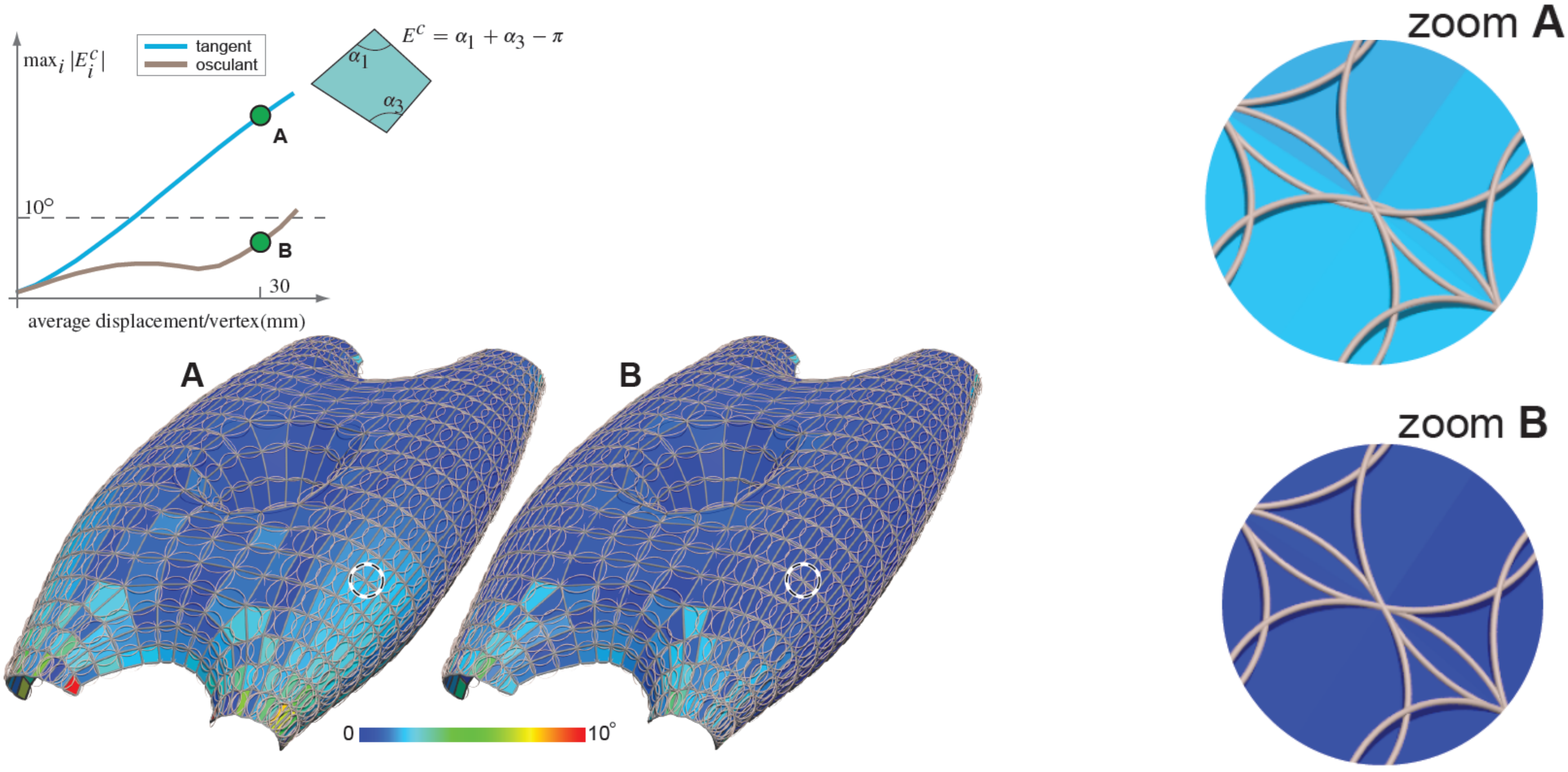


Circular Mesh Shape Space

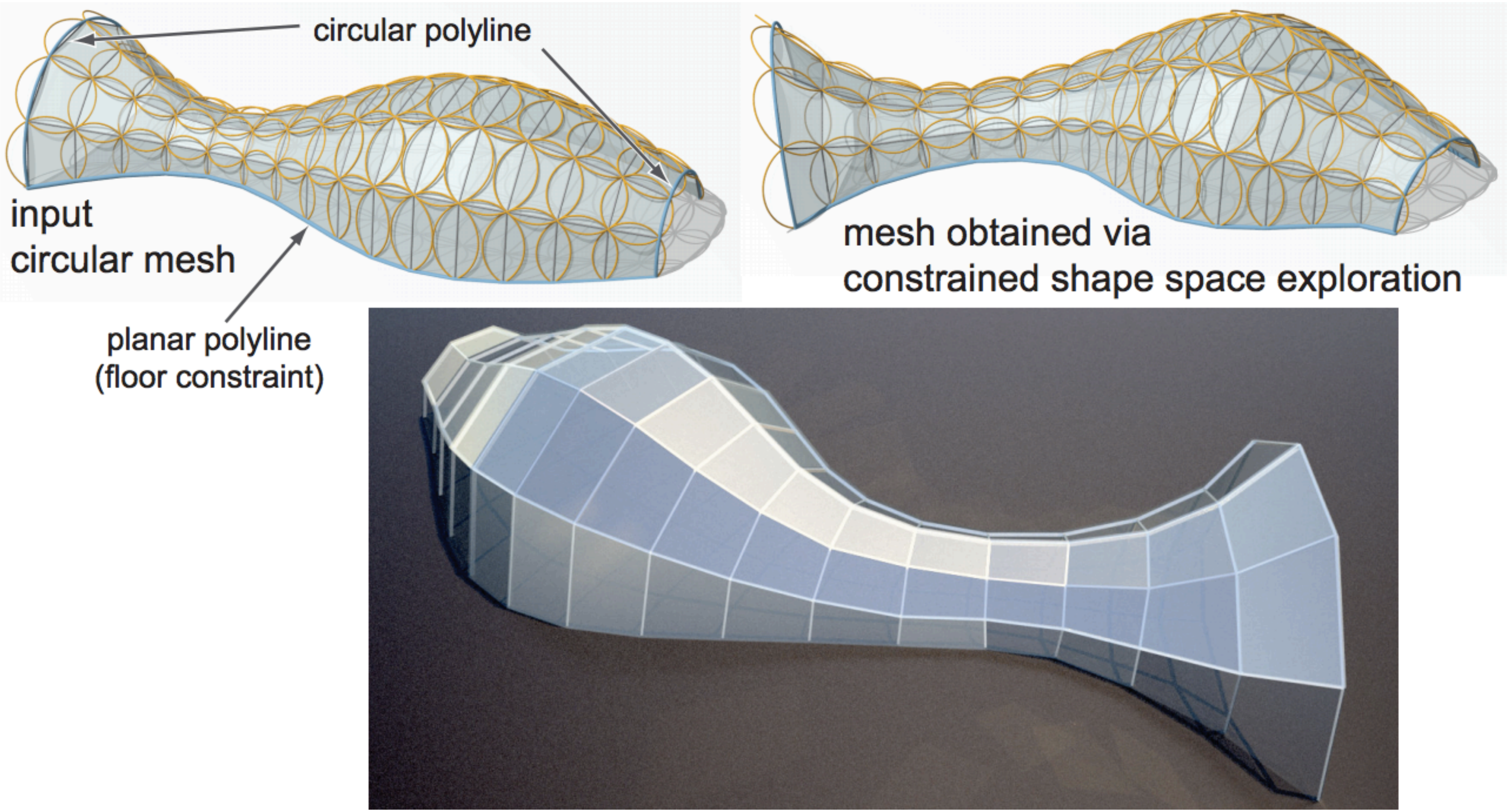




Circular Mesh Shape Space



Combined Constraints Manifolds



Combined Constraints Manifolds

Exploration with floor
and circle constraints
model: Roof
#faces: 75

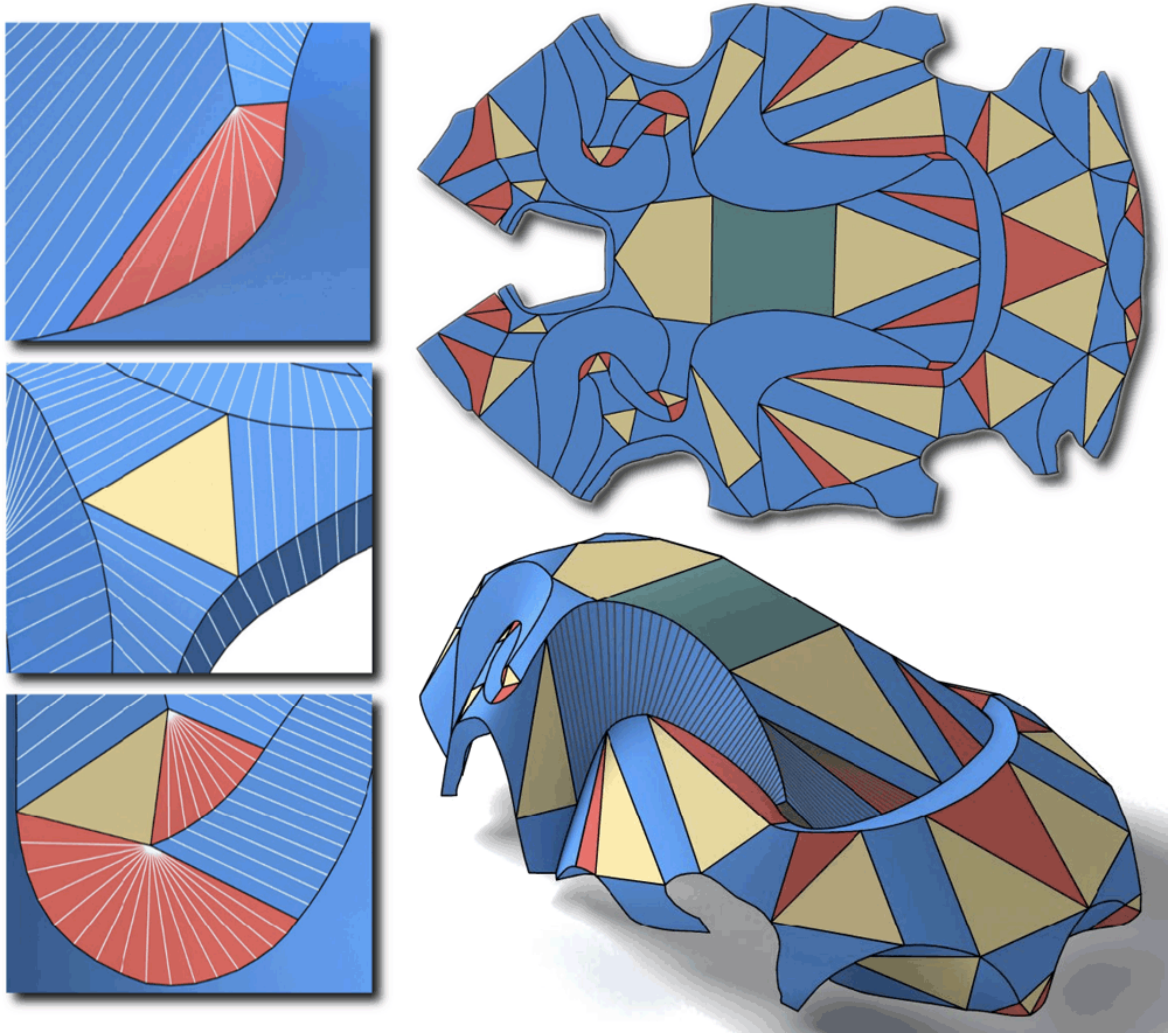
Combined Constraints Manifolds

Exploration with floor
and circle constraints
model: Roof
#faces: 75

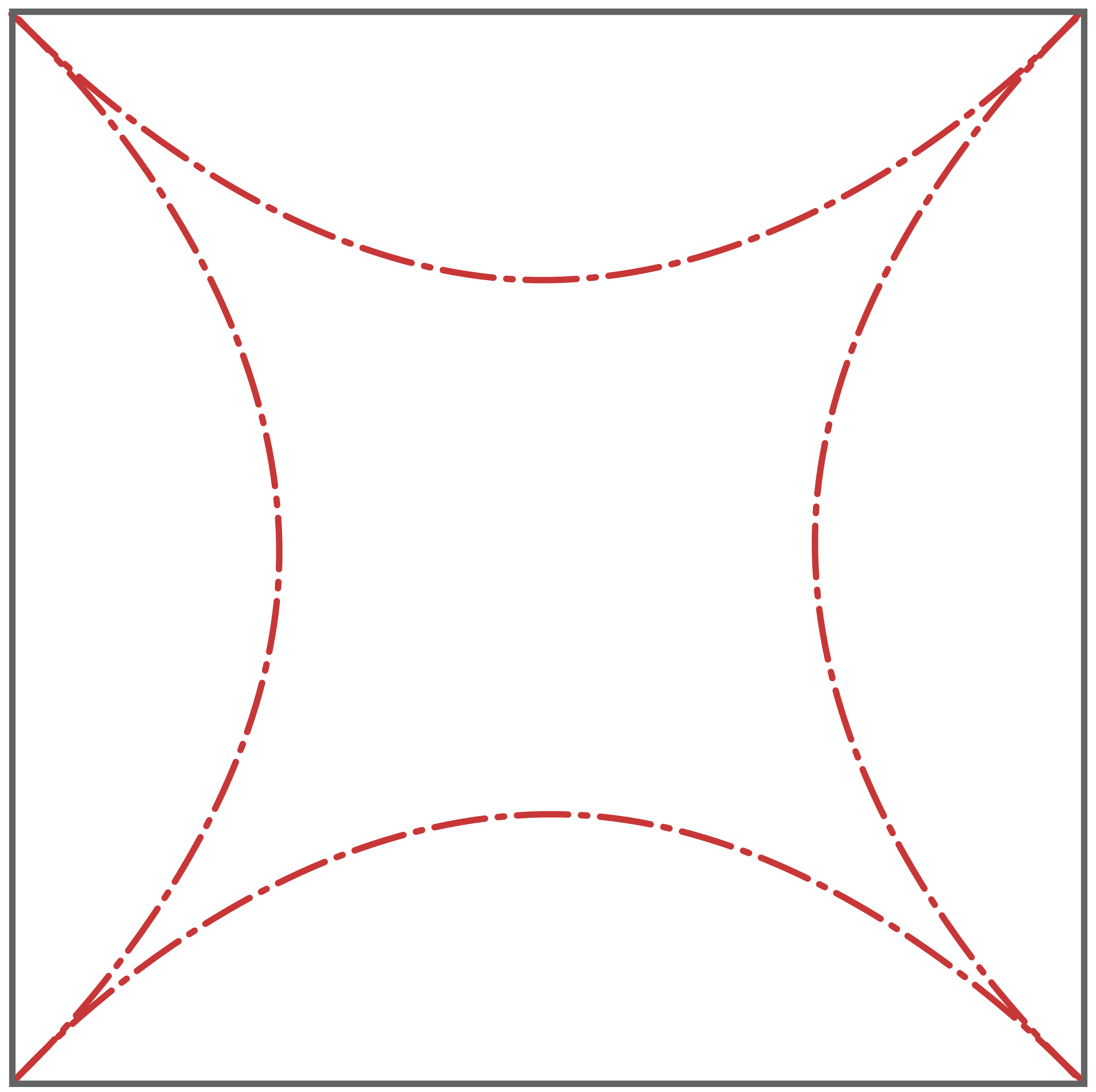
Flat Circular Mesh Exploration

Flat Circular Mesh Exploration

What can be Folded?

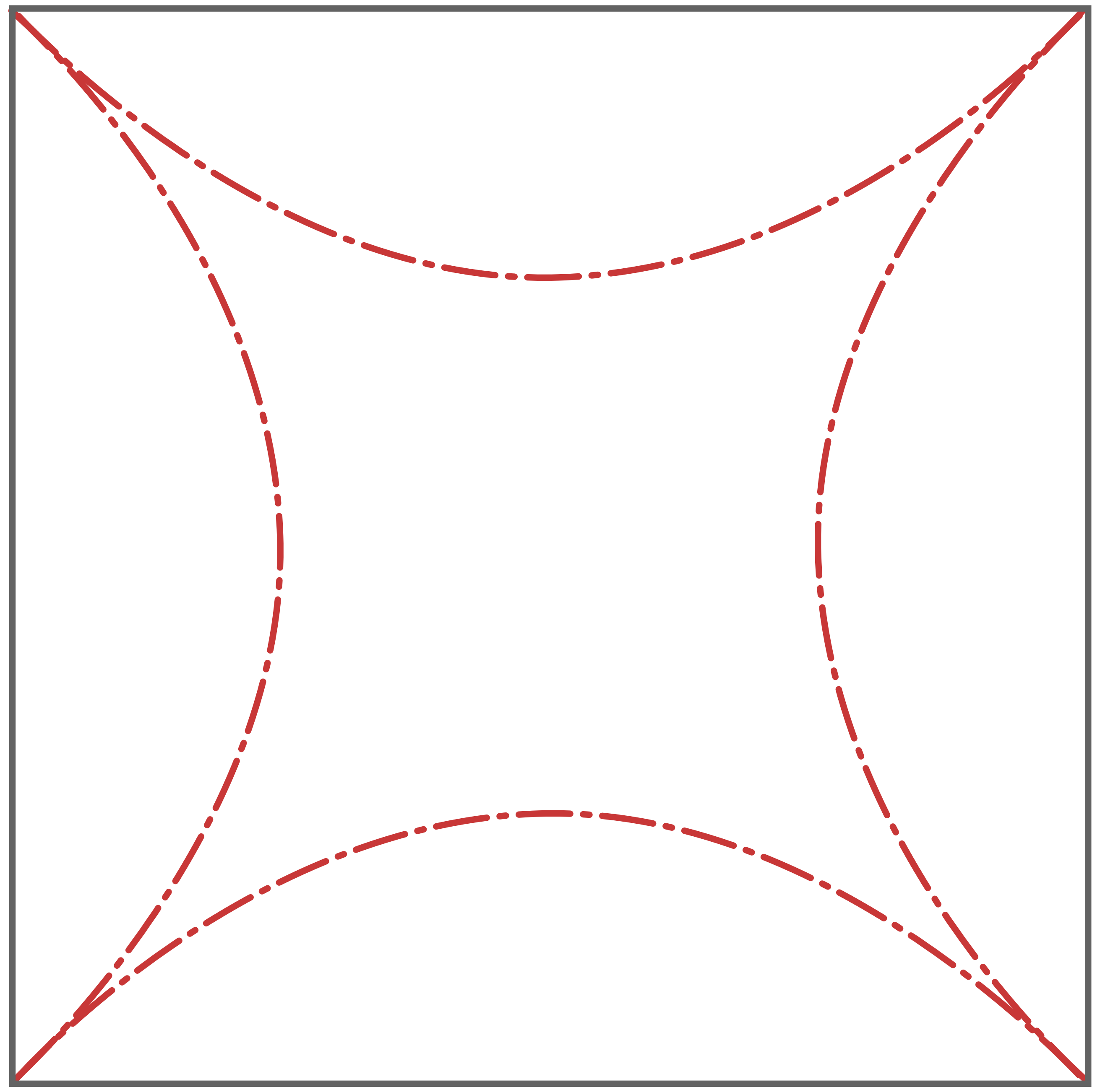


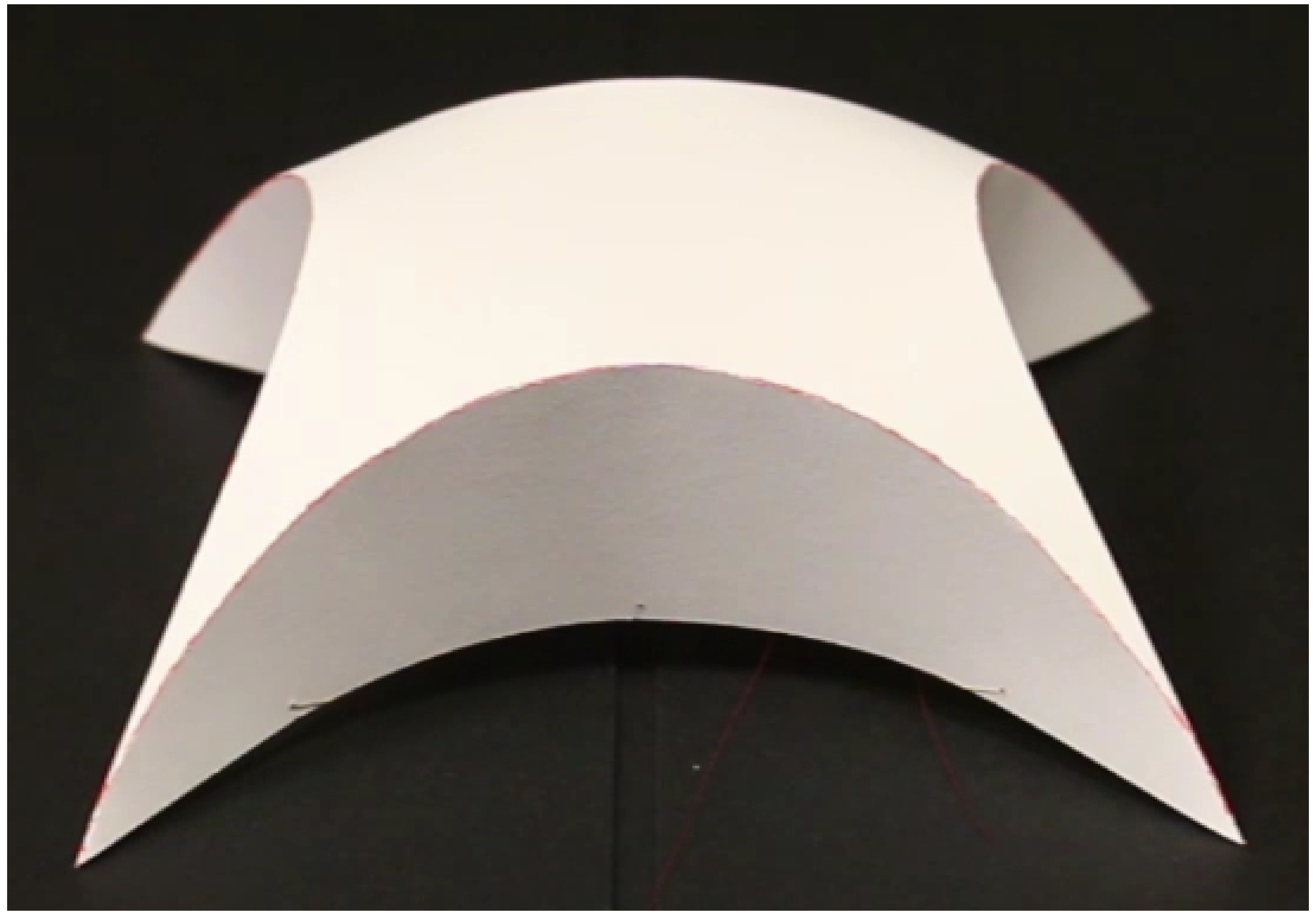
Curved Folds



[w/ Kilian, Monszpart]

Curved Folds



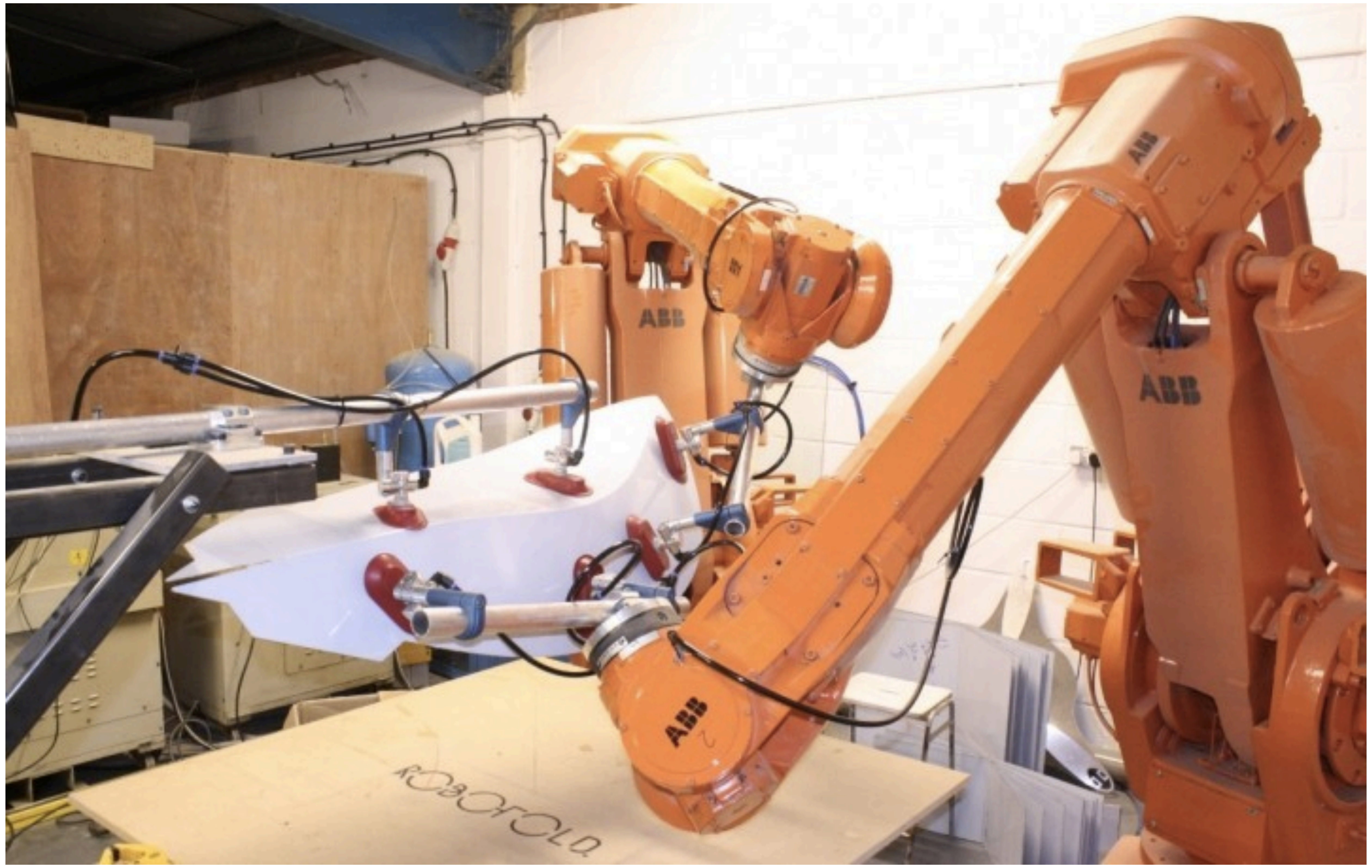


[w/ Kilian, Monszpart]

How to Fold?

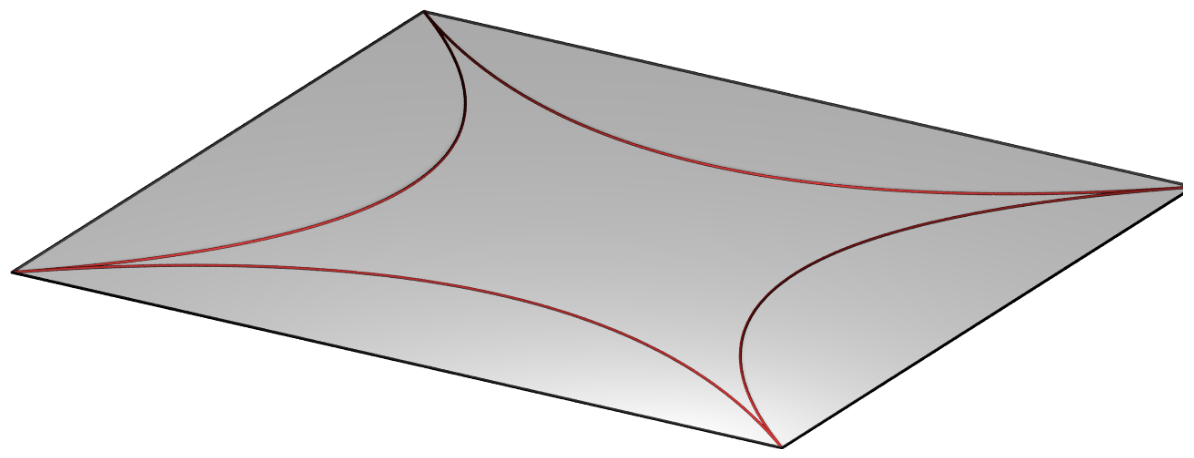
[© RoboFold]

How to Fold?

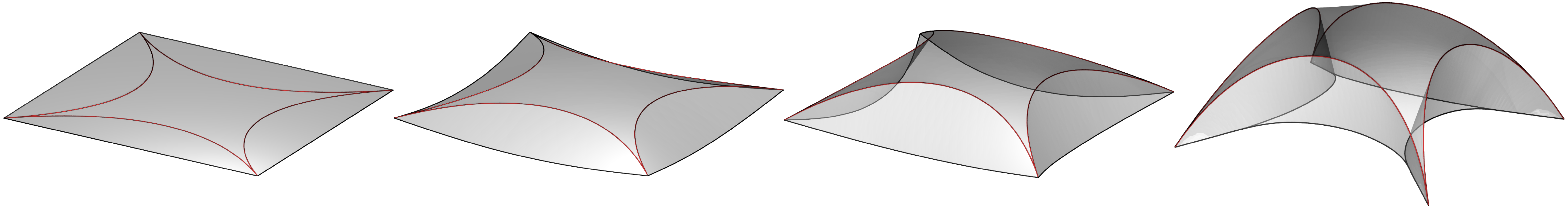


[© RoboFold]

String Actuated Folding

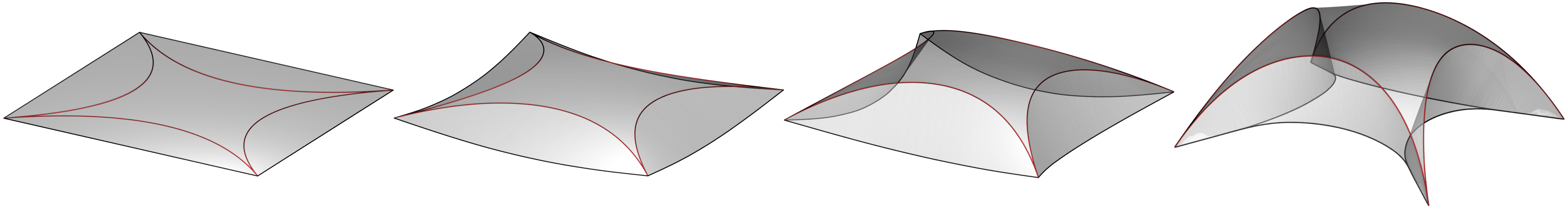


String Actuated Folding



String Actuated Folding

$$S : [0, 1] \times U \to \mathbb{R}^3$$



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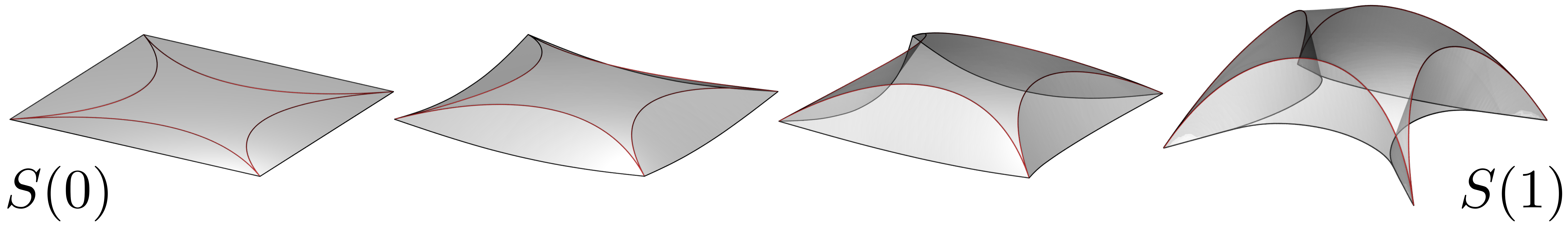

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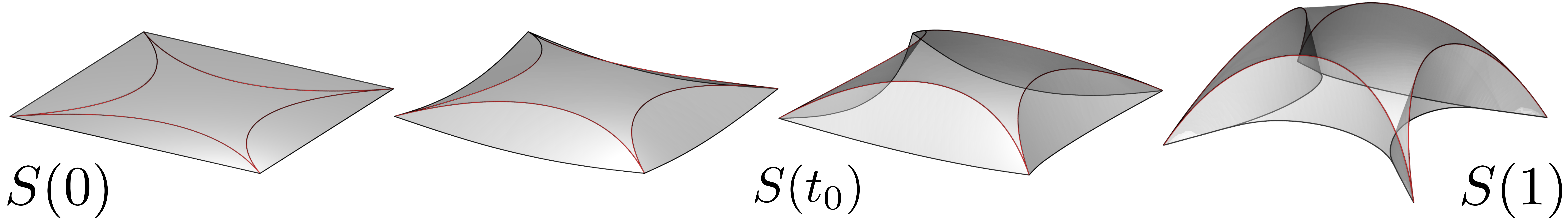

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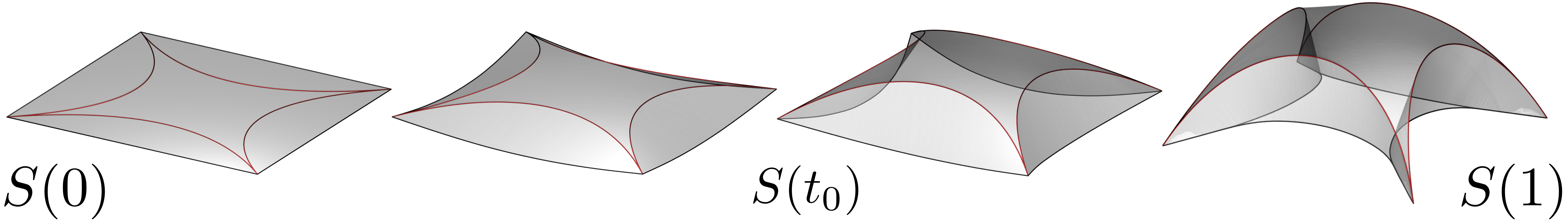

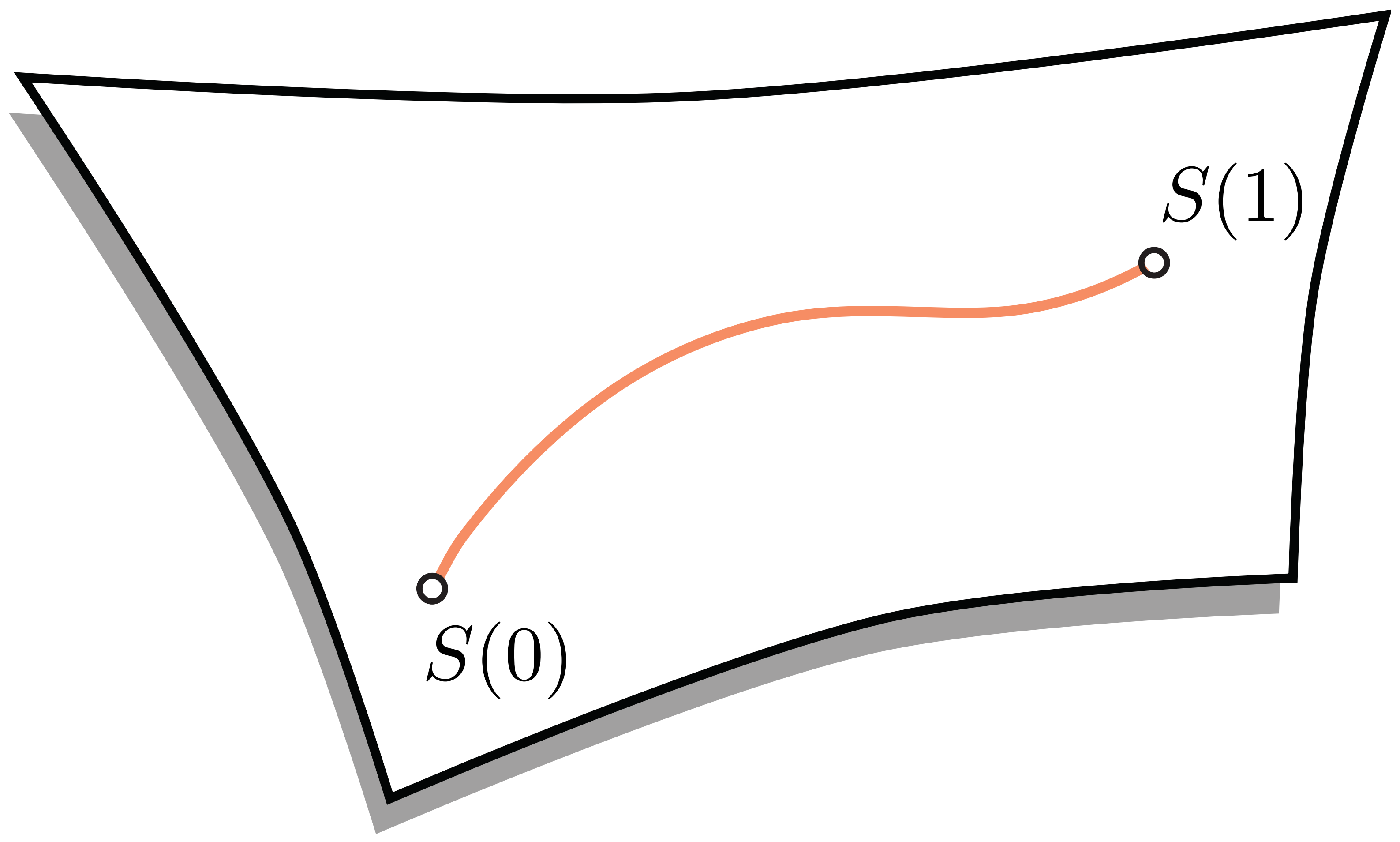

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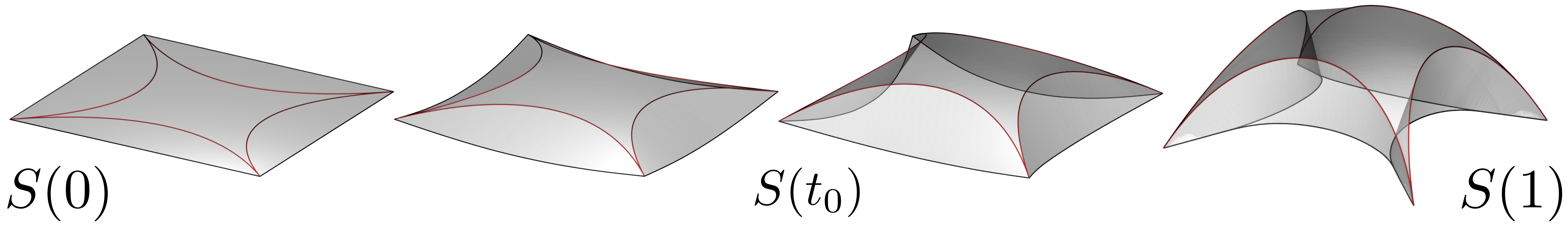
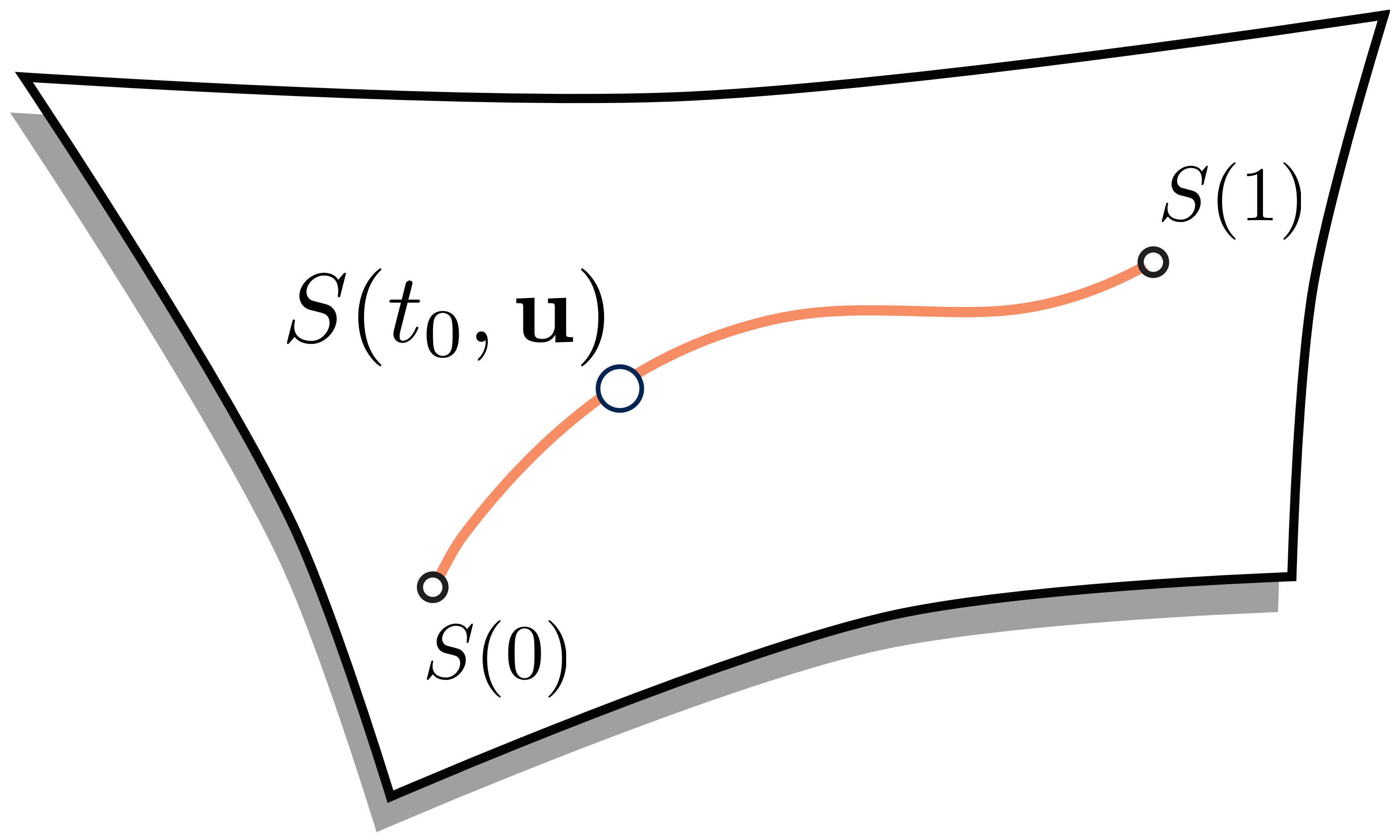




Questions

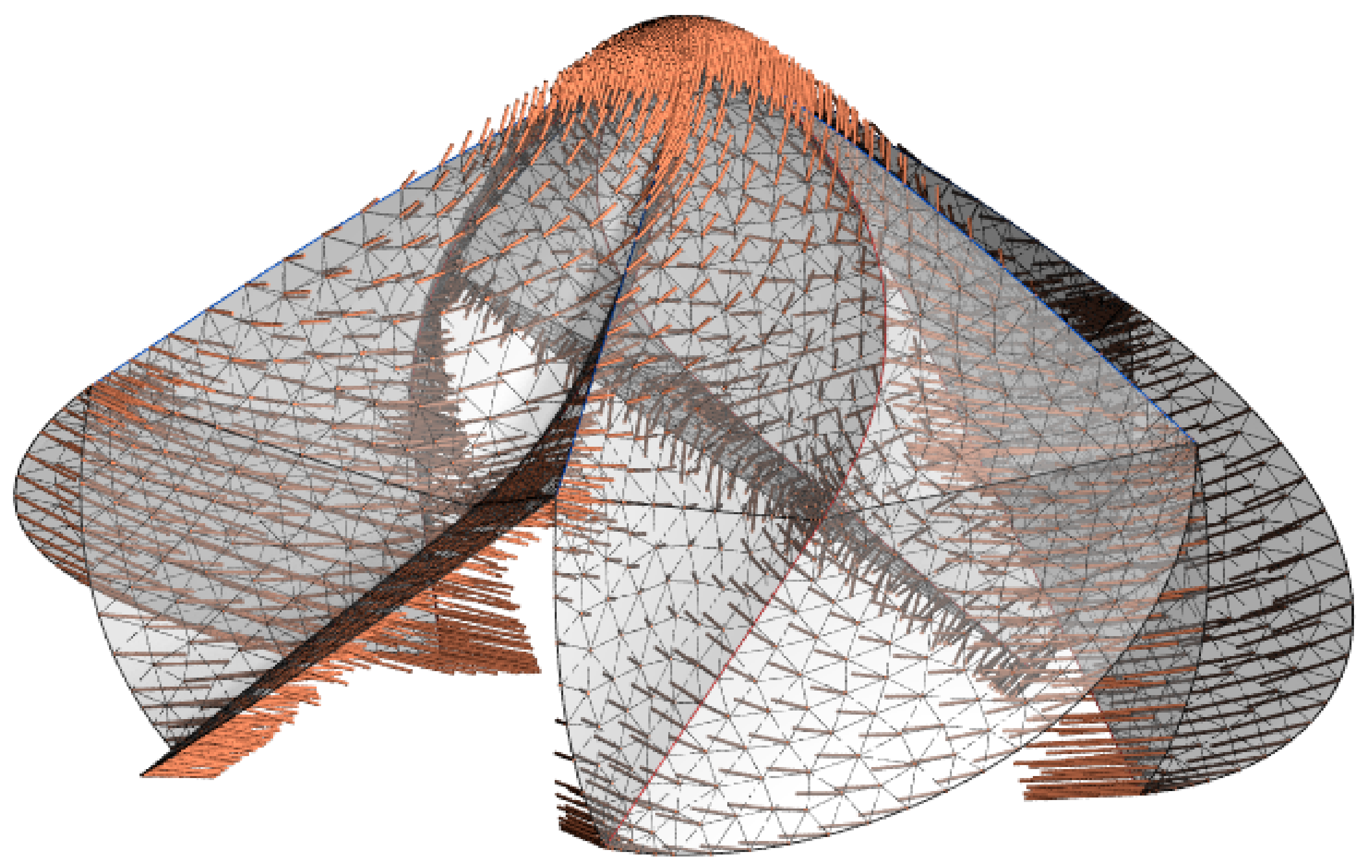
Questions

- **Which** points to connect?

Questions

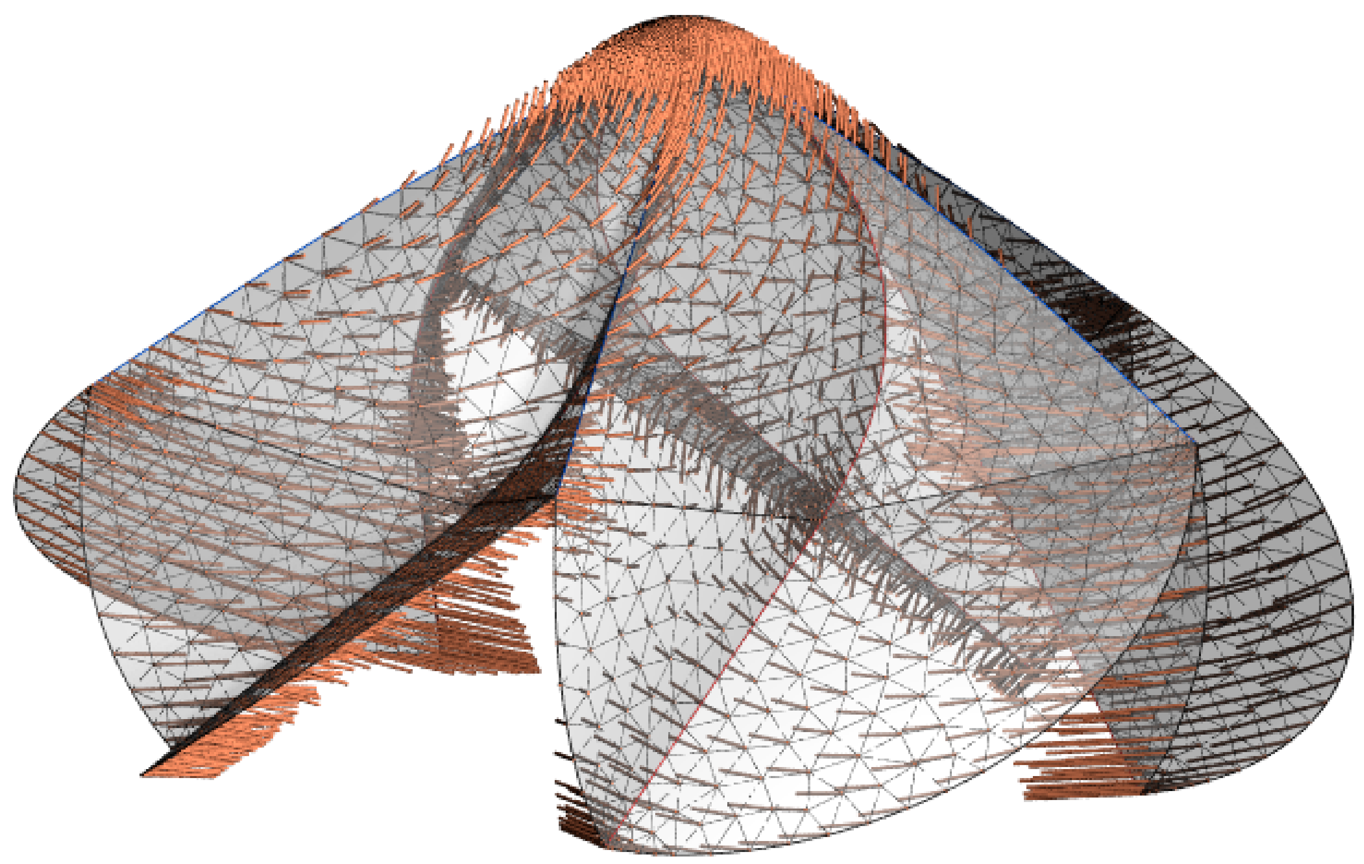
- **Which** points to connect?
- **How** to connect the points?

Deformation in Shape Space



Deformation in Shape Space

$$X(t_0, \mathbf{u}) := \frac{\partial}{\partial t} S(t_0, \mathbf{u})$$



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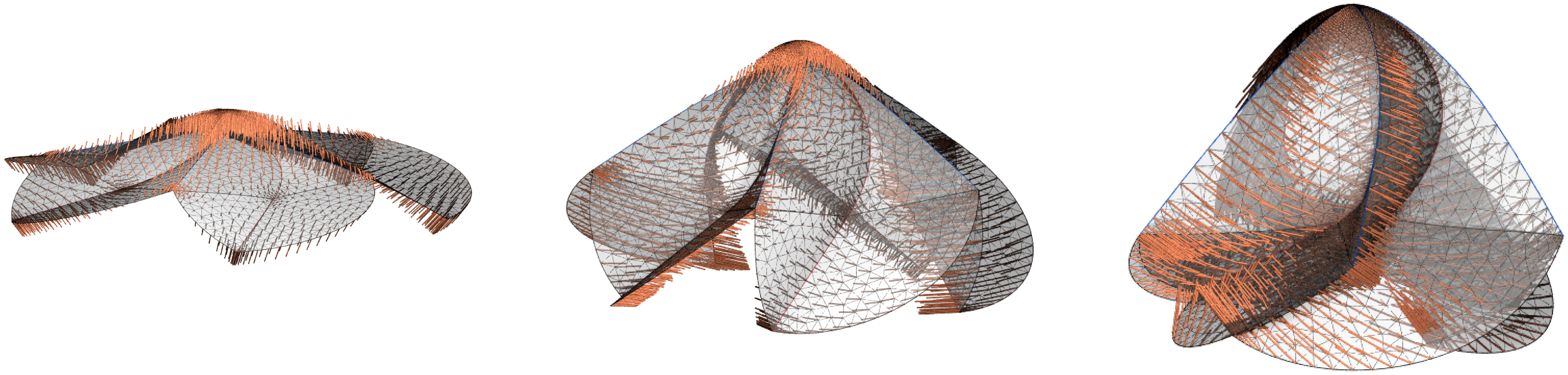
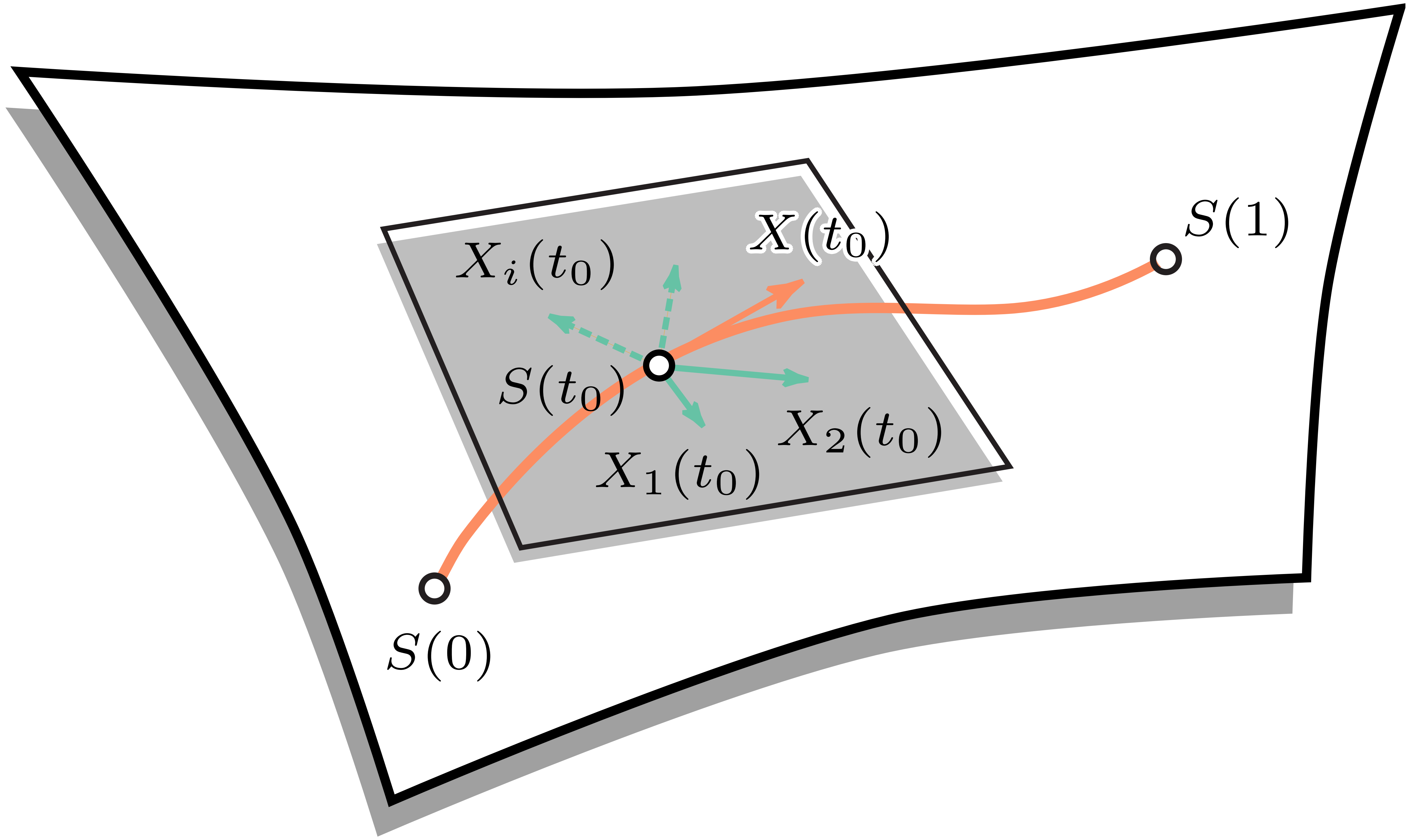


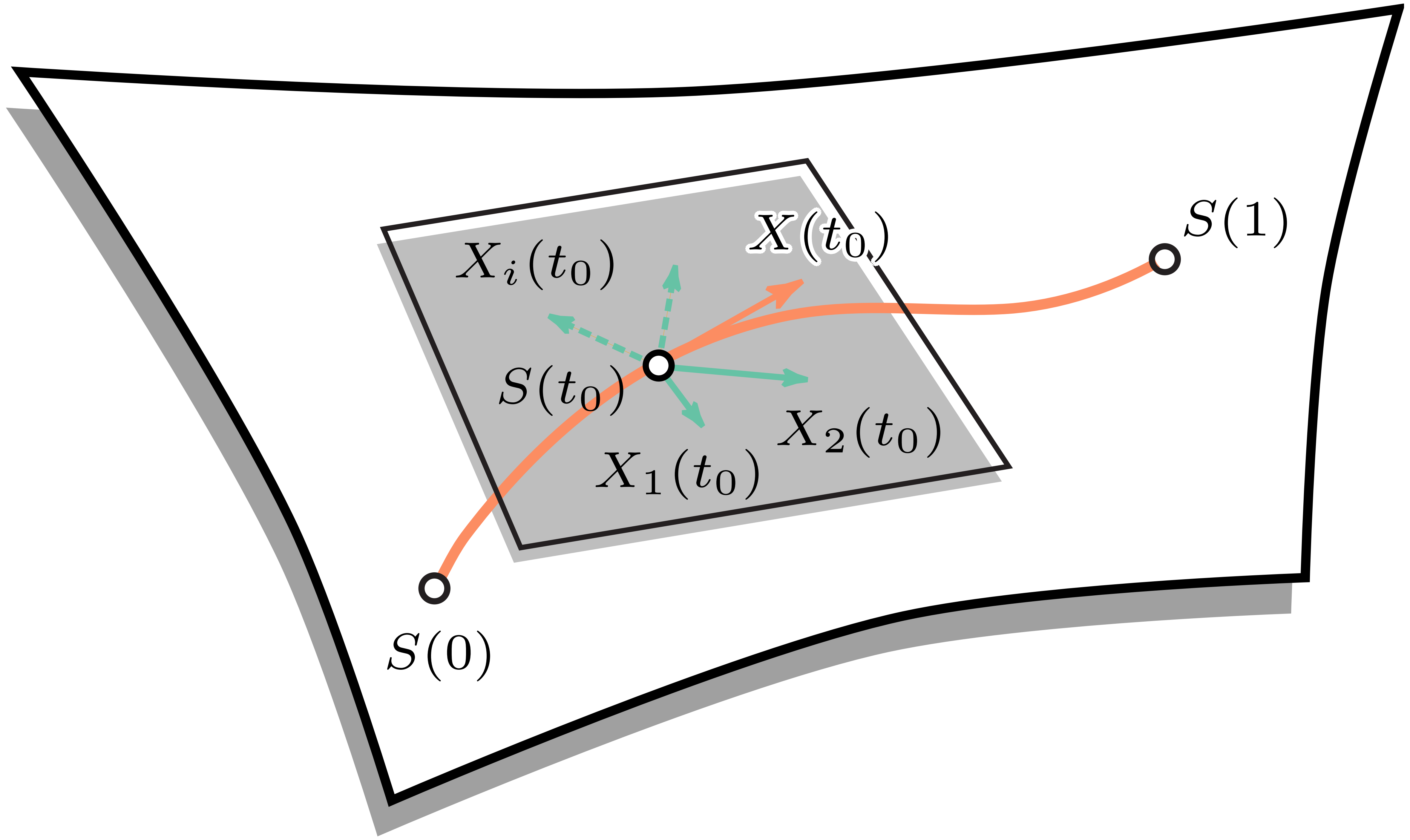
Main Idea

- Express deformation in terms of **actuation modes**

$$X_i(t_0, \mathbf{u}) := \frac{\partial}{\partial s} S_i(t_0)(0, \mathbf{u})$$

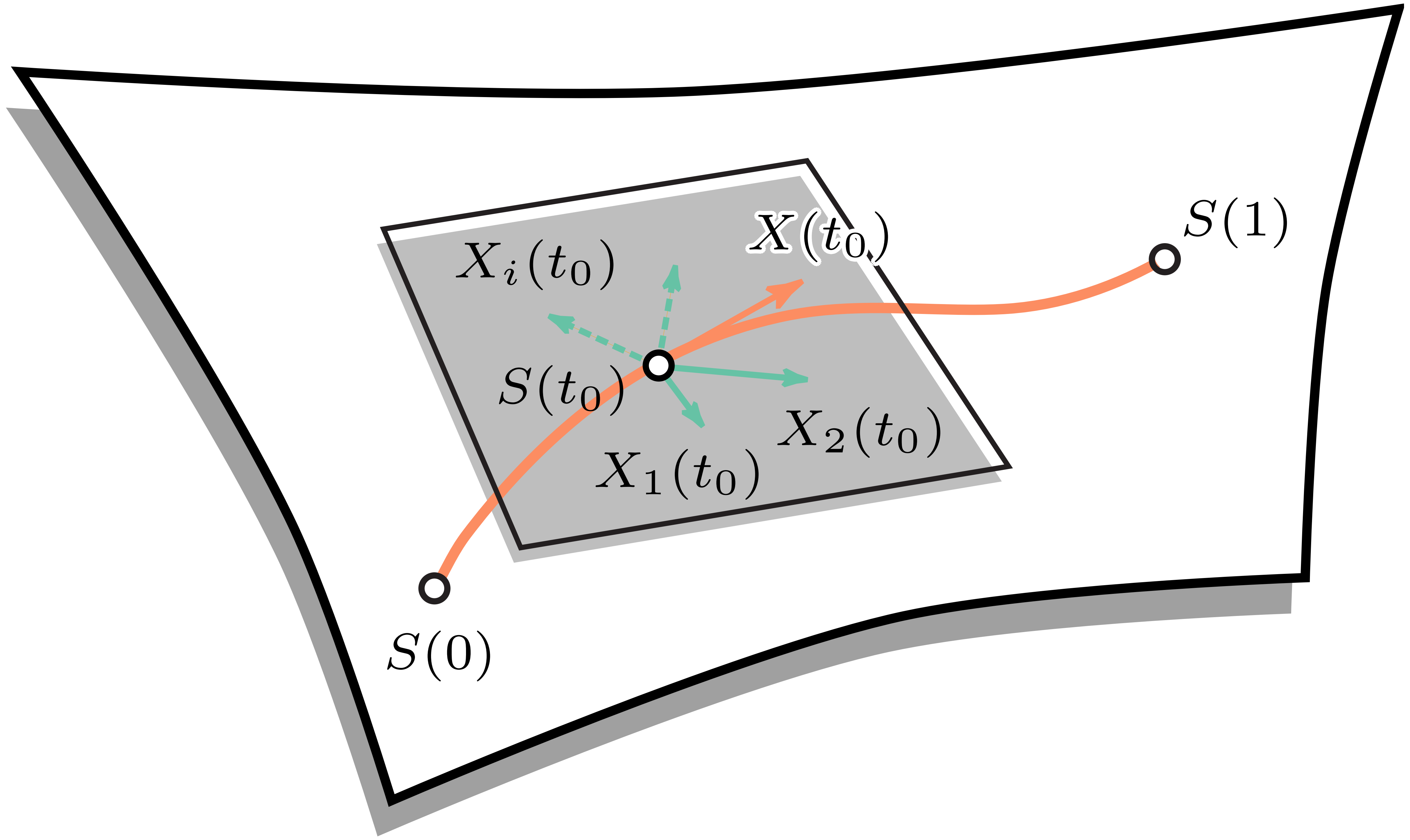


Deformations using 'Actuation Modes'



Deformations using 'Actuation Modes'

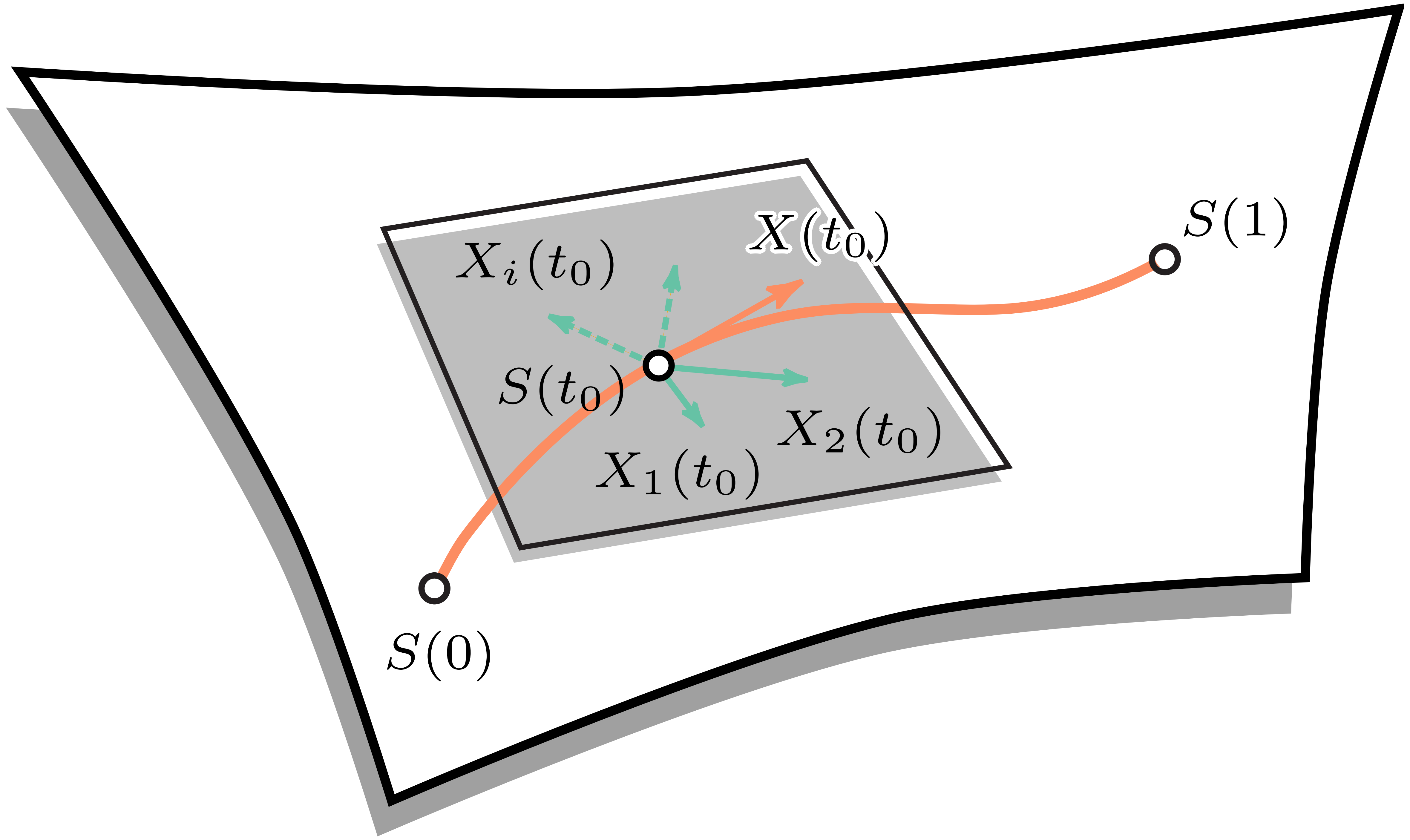
$$X(t_0) \approx \sum_i \lambda_i(t_0) X_i(t_0)$$



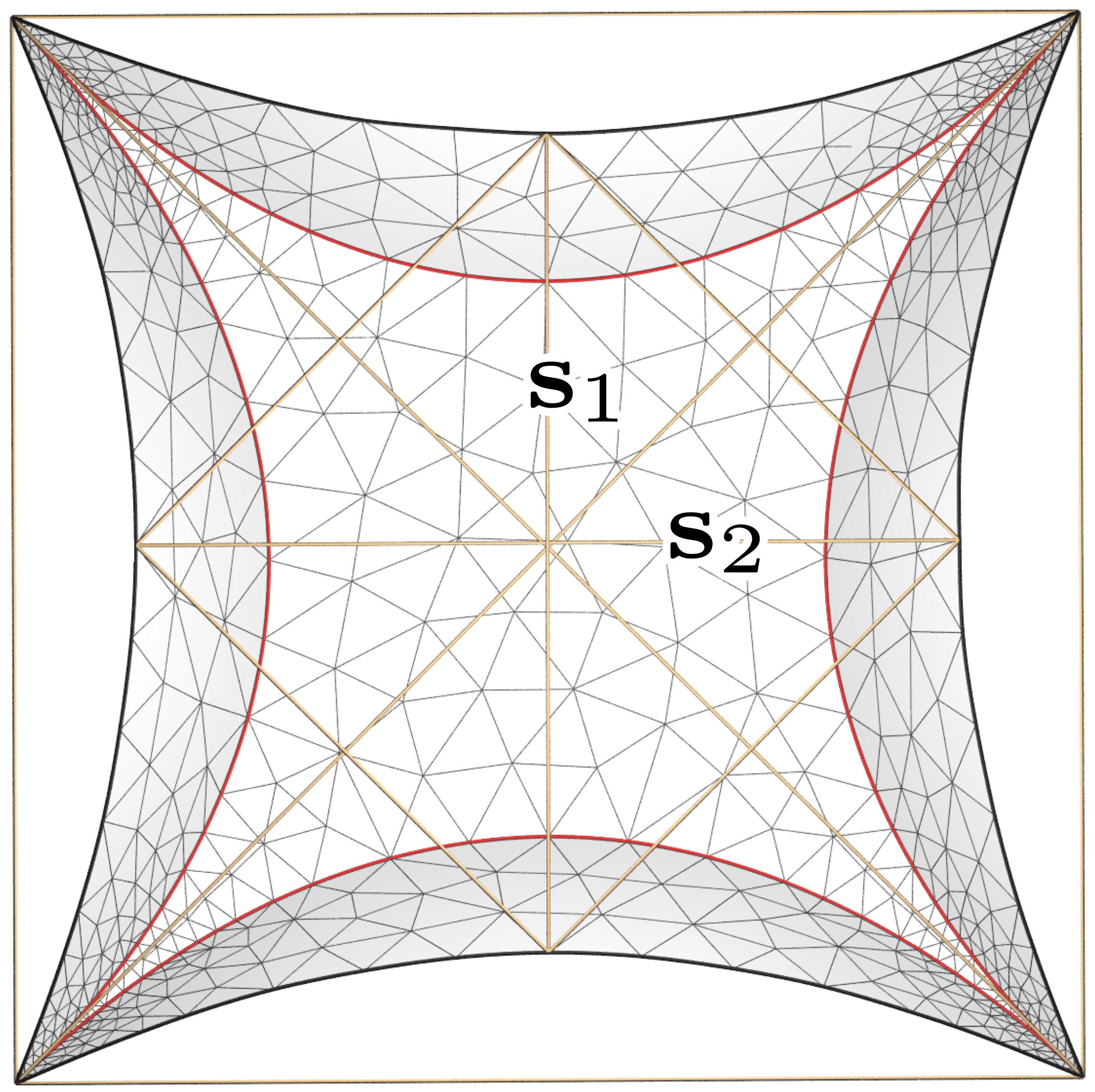
Deformations using 'Actuation Modes'

$$X(t_0) \approx \sum_i \lambda_i(t_0) X_i(t_0)$$

$$\min_{\Lambda} \|X(t_0) - \sum_i \lambda_i(t_0) X_i(t_0)\|^2$$



Example: Actuation Modes



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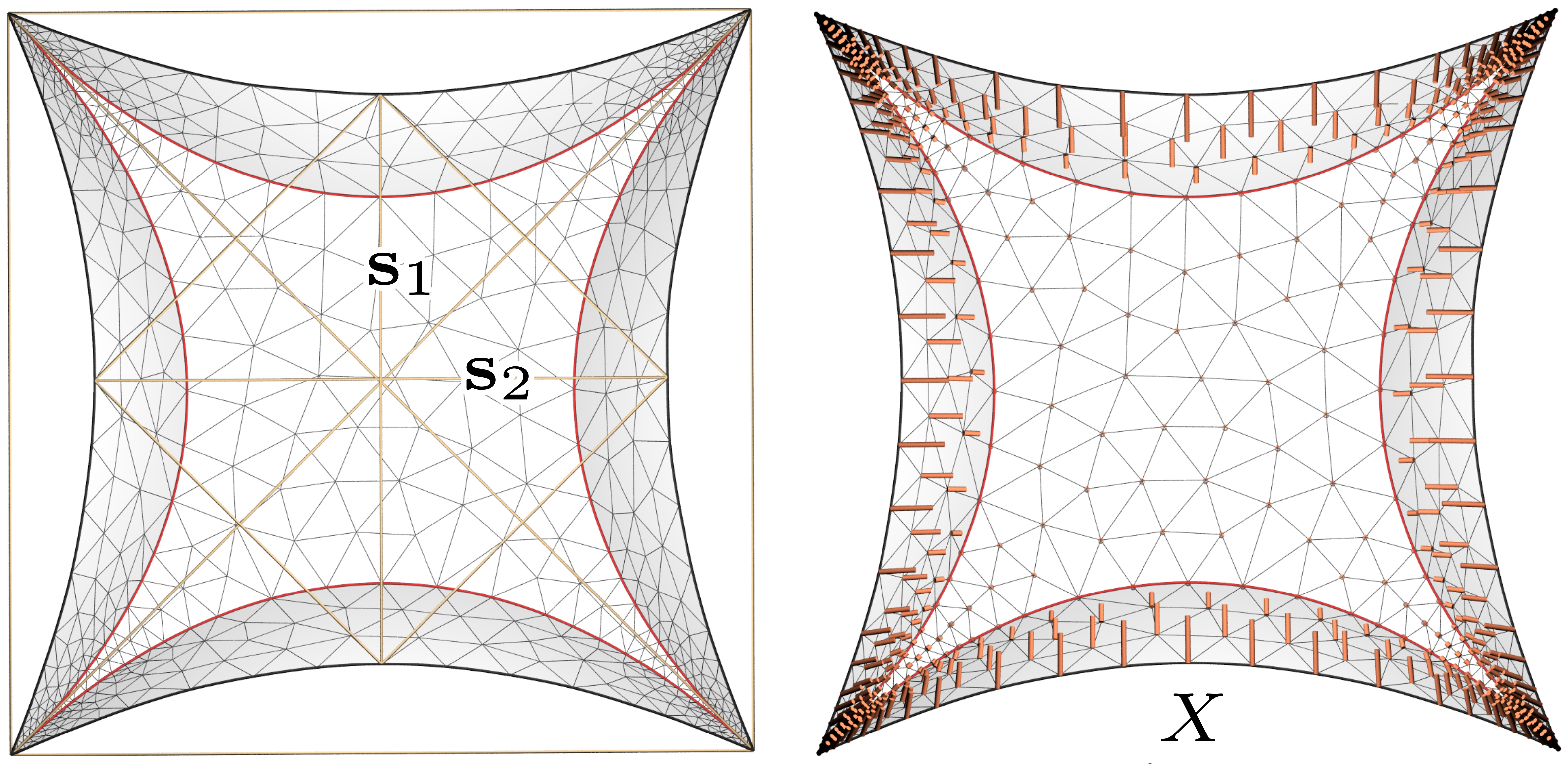

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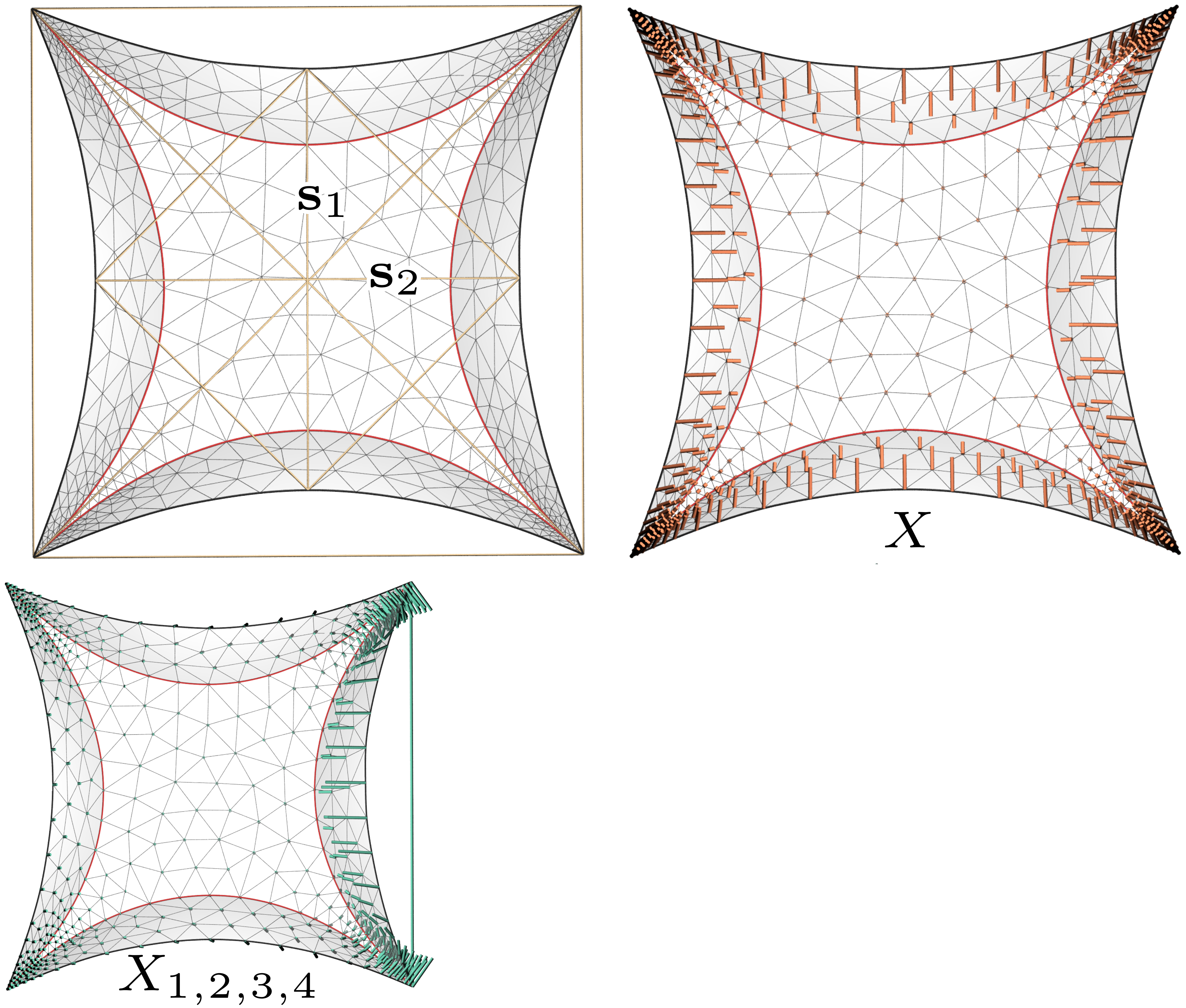

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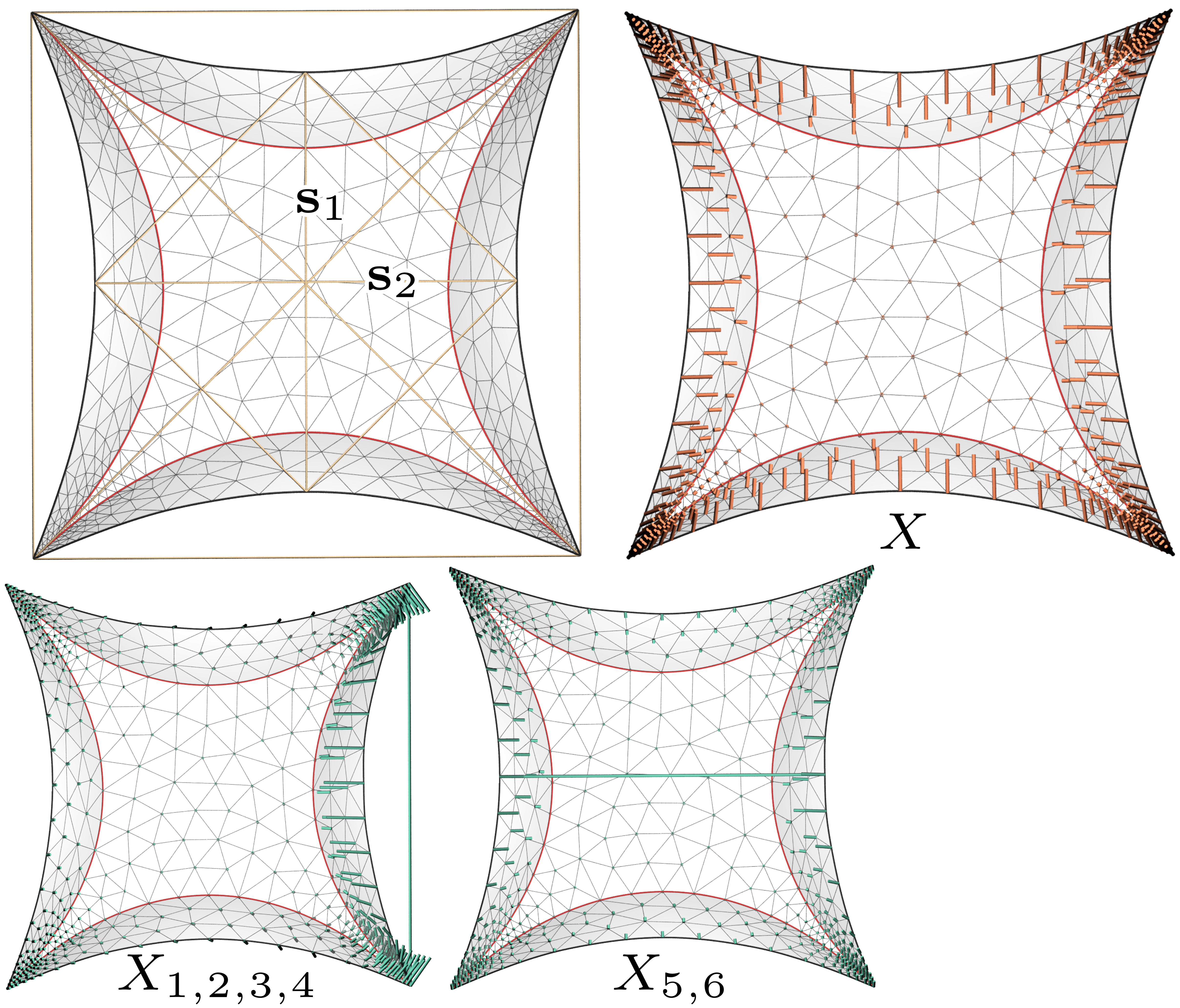

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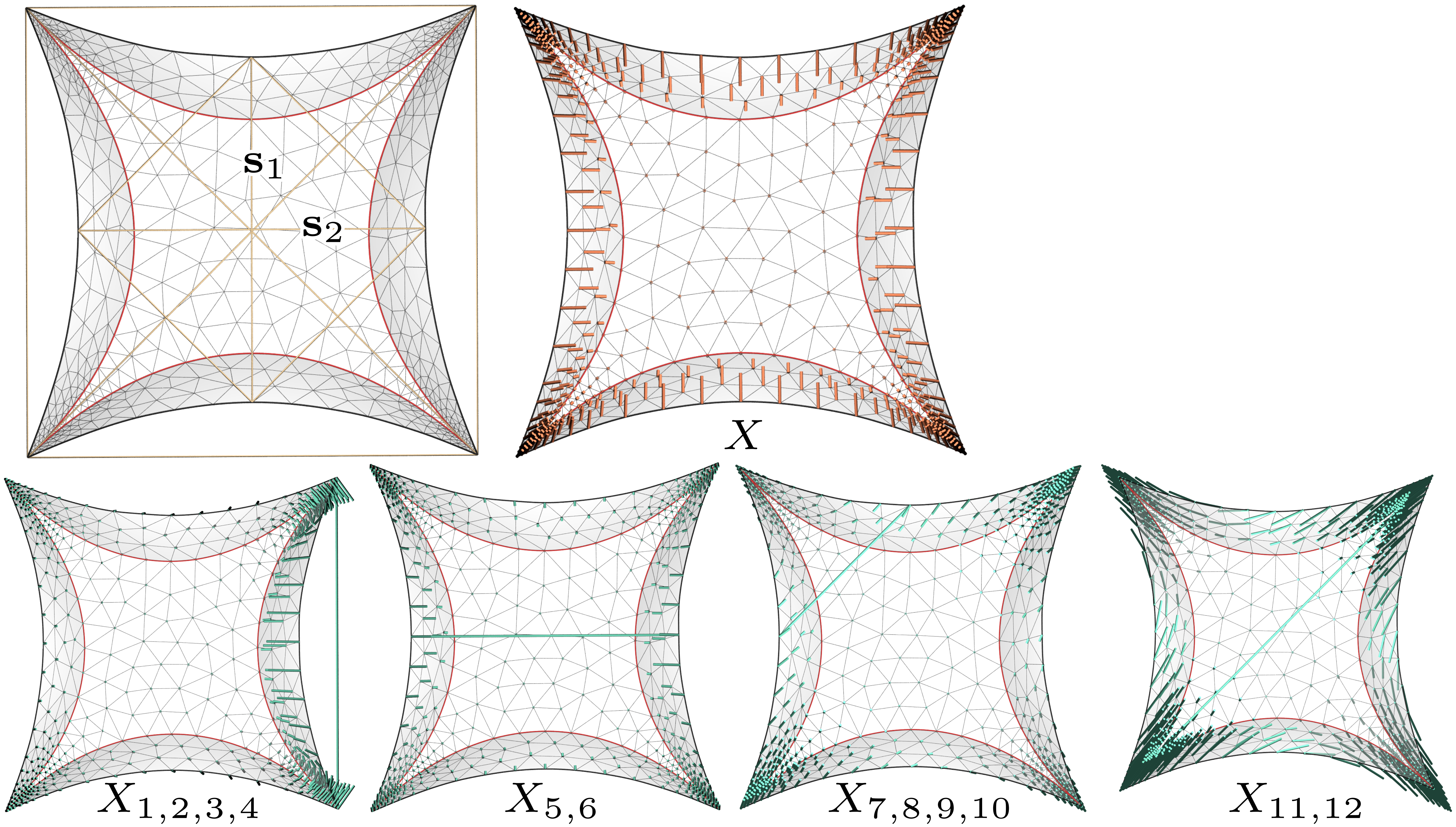

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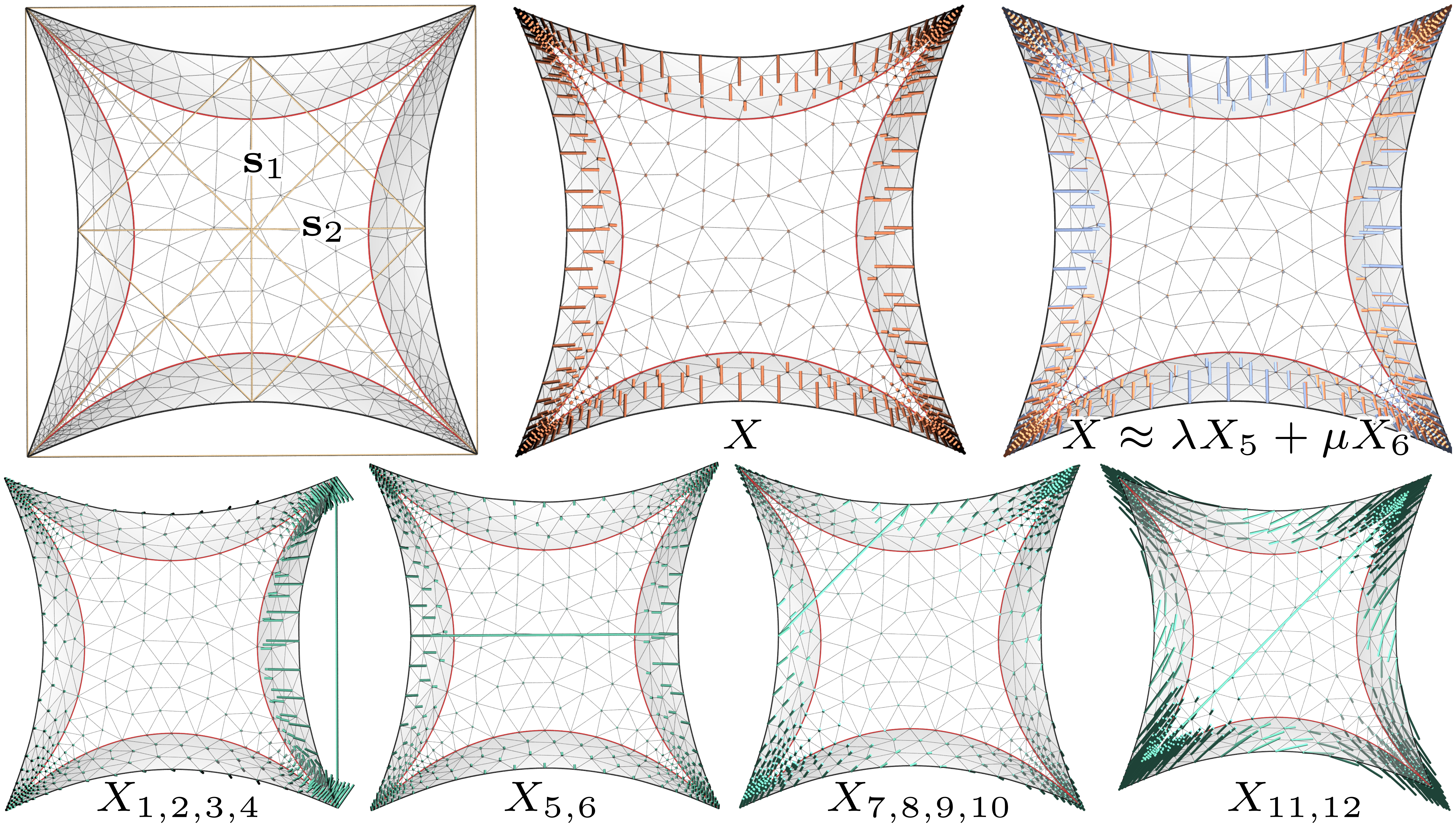
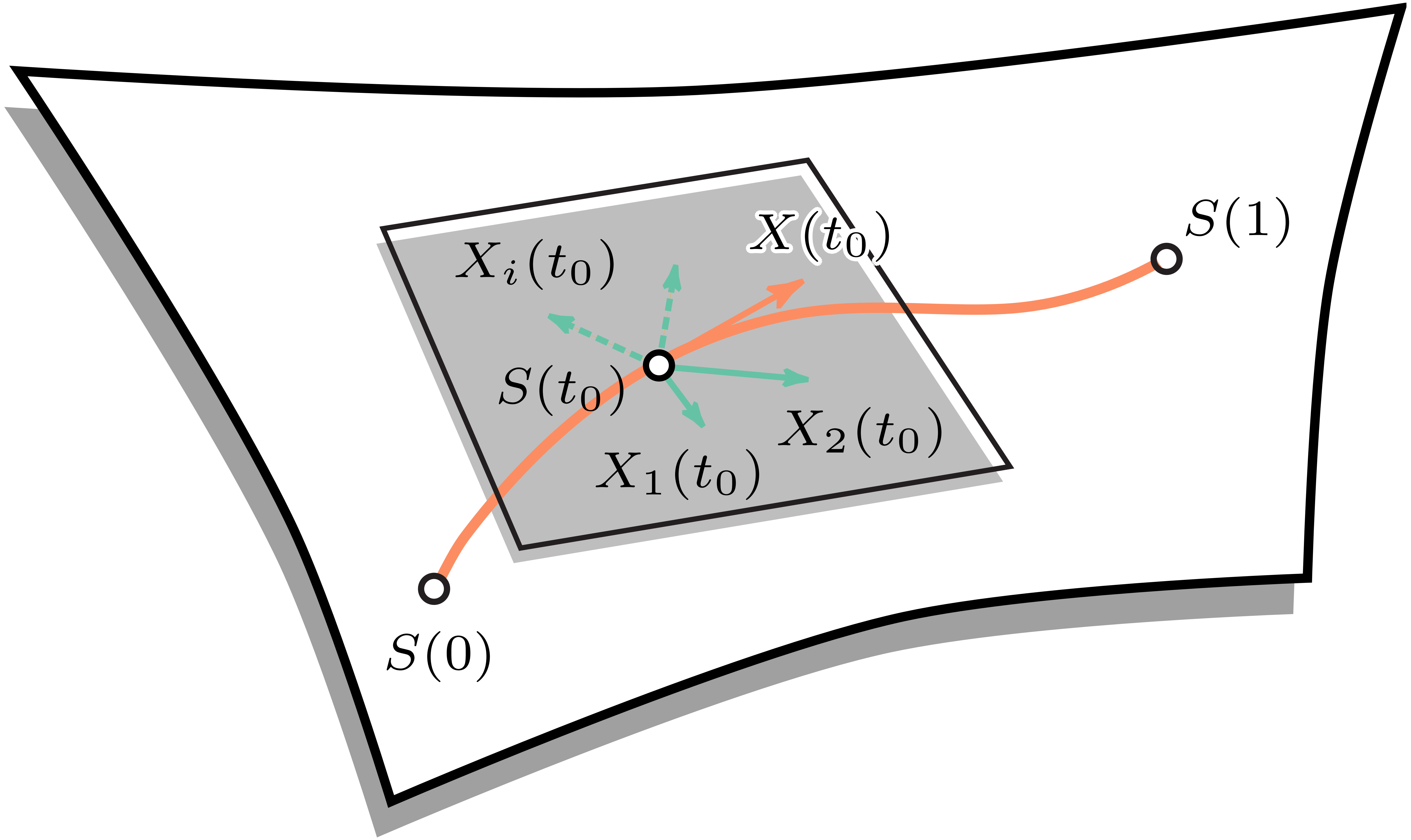
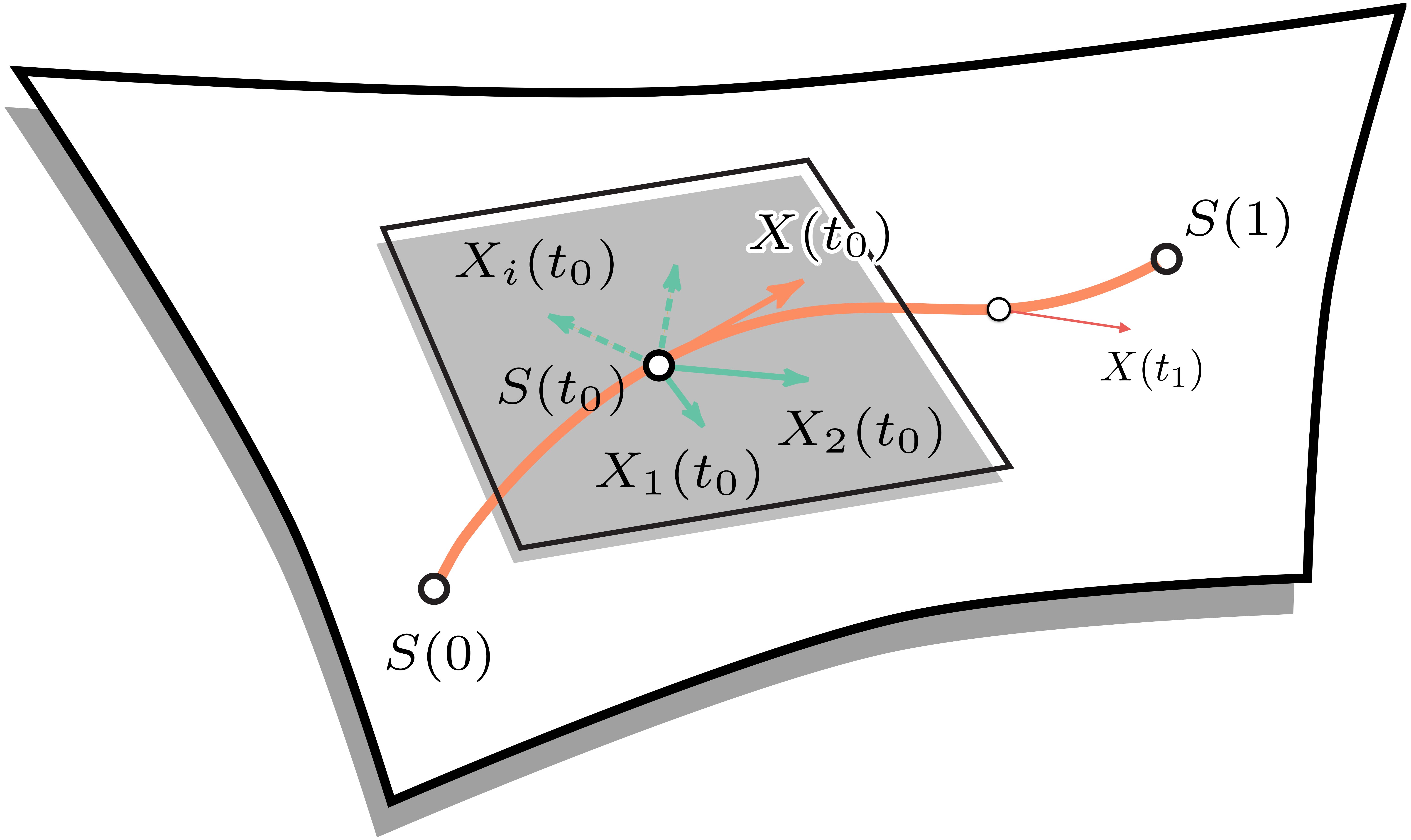


Globally Consistent Solution



Globally Consistent Solution



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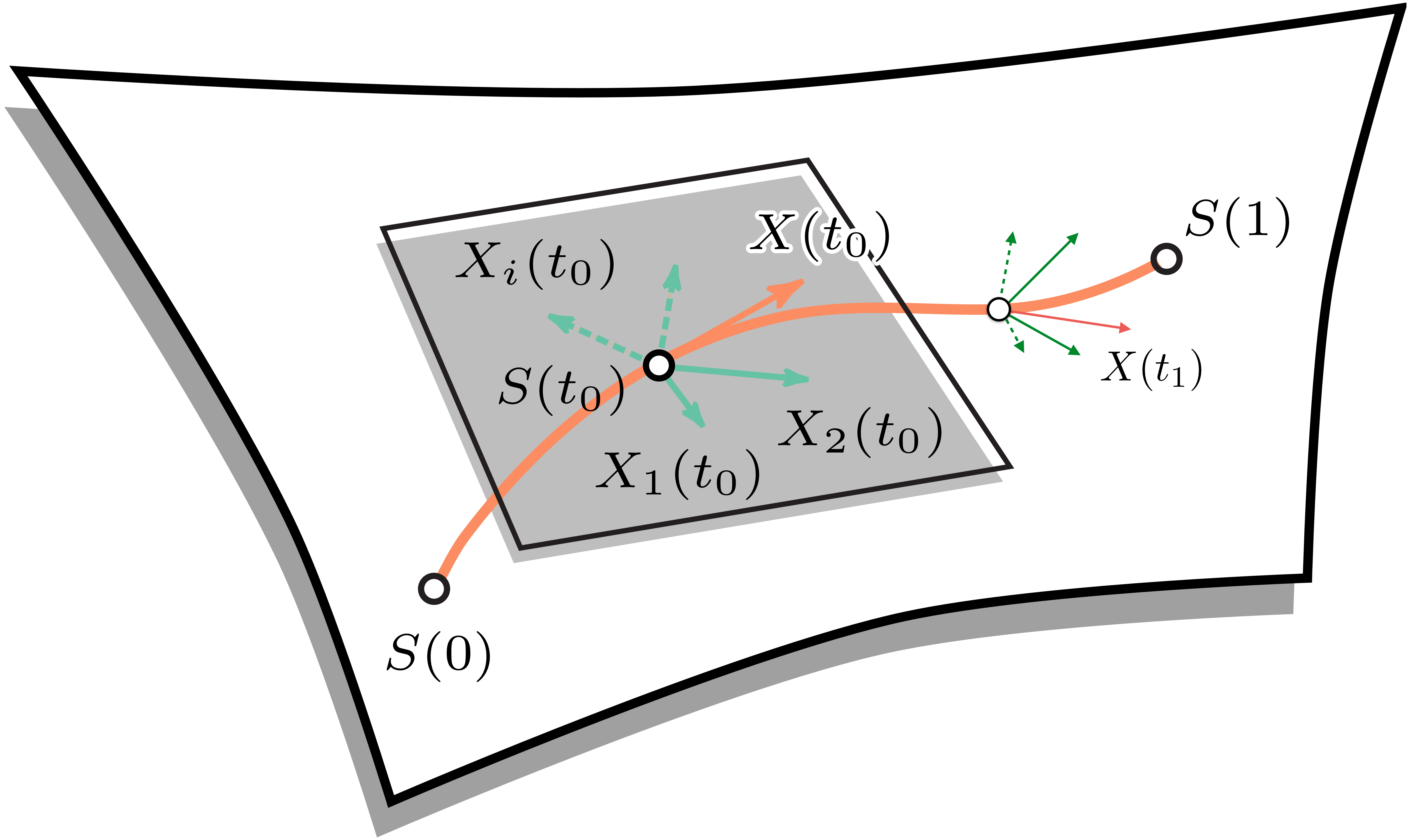
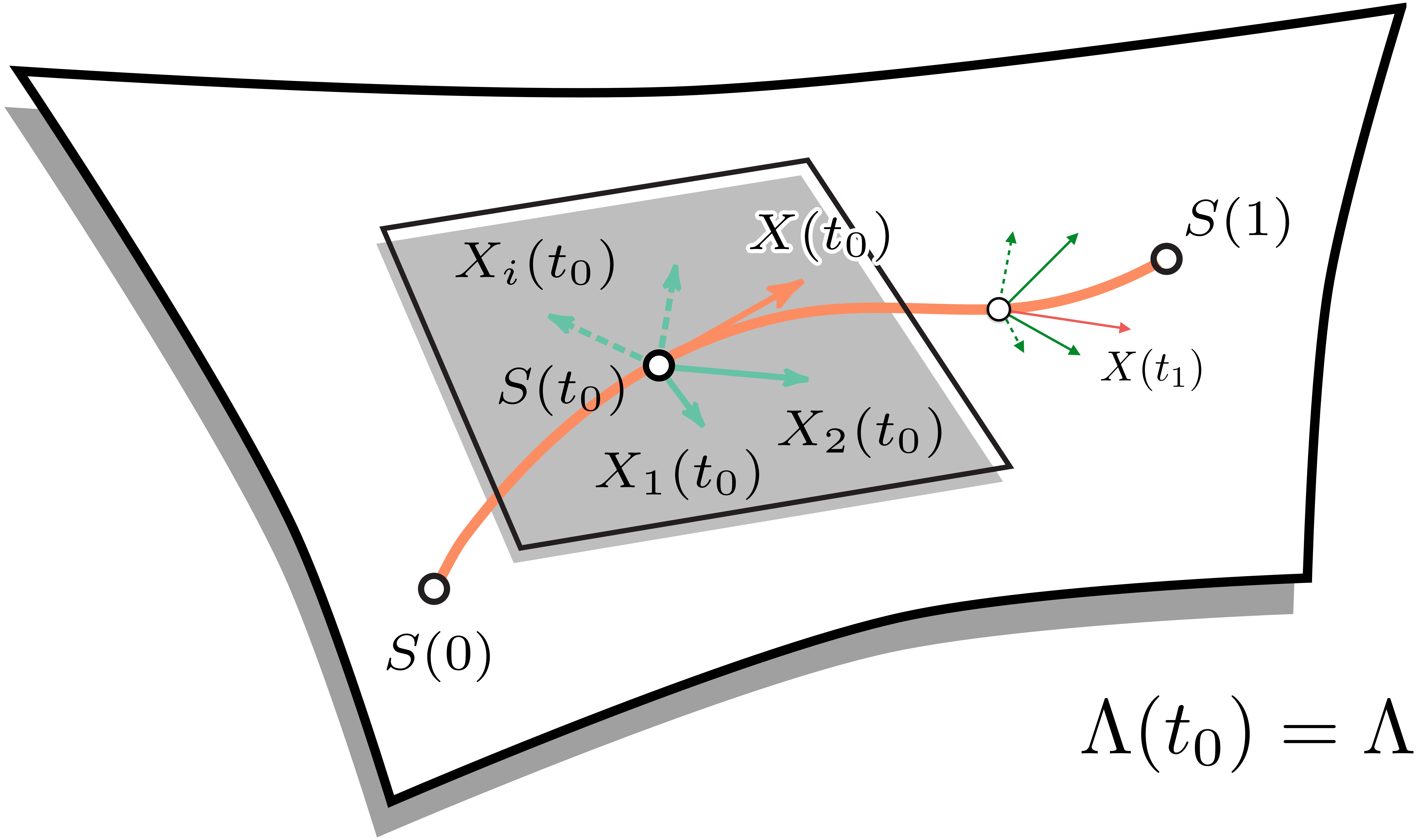


Globally Consistent Solution



$$\Lambda(t_0) = \Lambda(t_1) = \dots$$

Global Solution

$$g_{ij}(\xi, \lambda) := \boxed{\xi_i}\lambda_{ij}, \quad i = 1, \ldots, m, \quad j = 1, \ldots, n,$$

$$g_{ij}(\xi, \lambda) = 0$$

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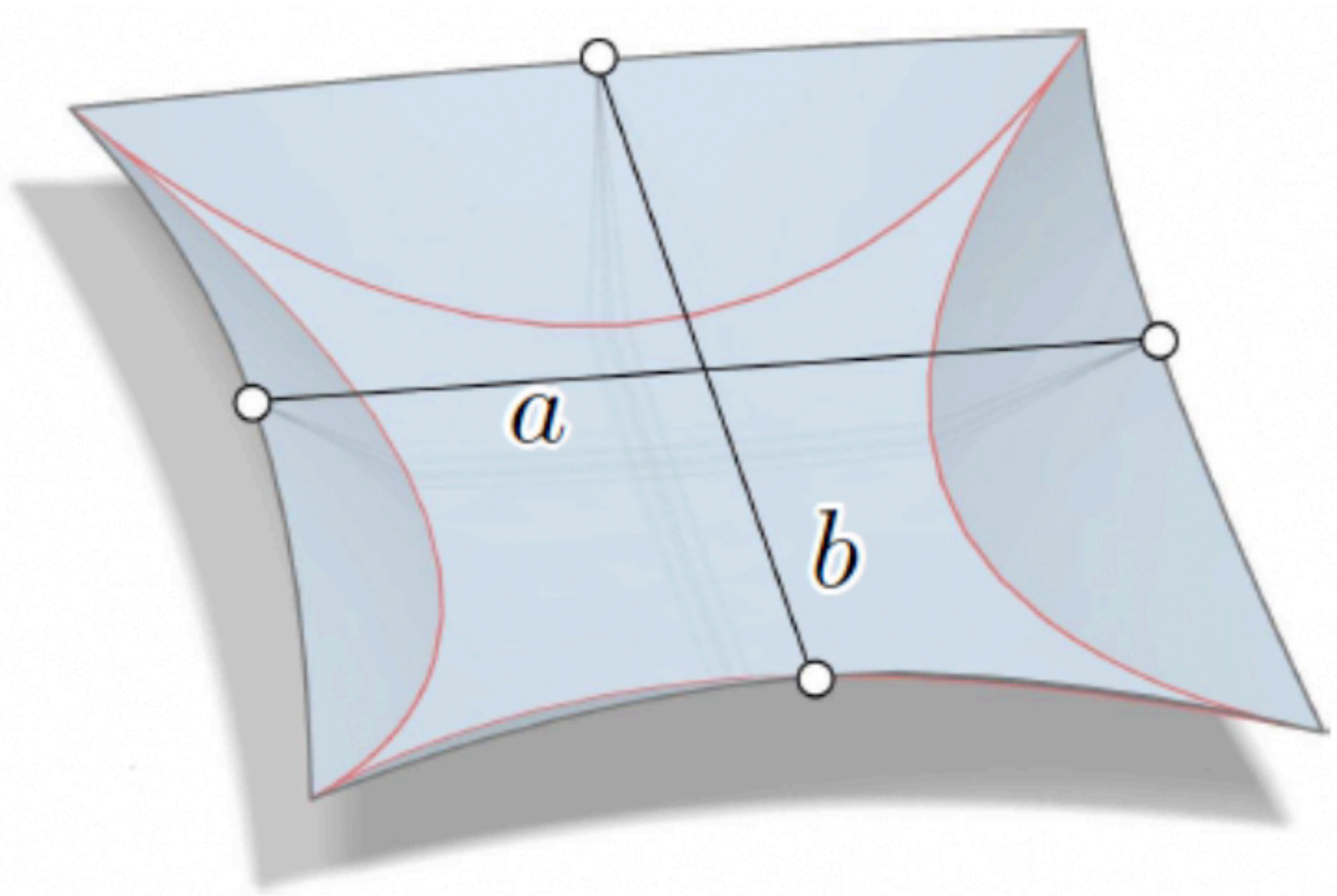
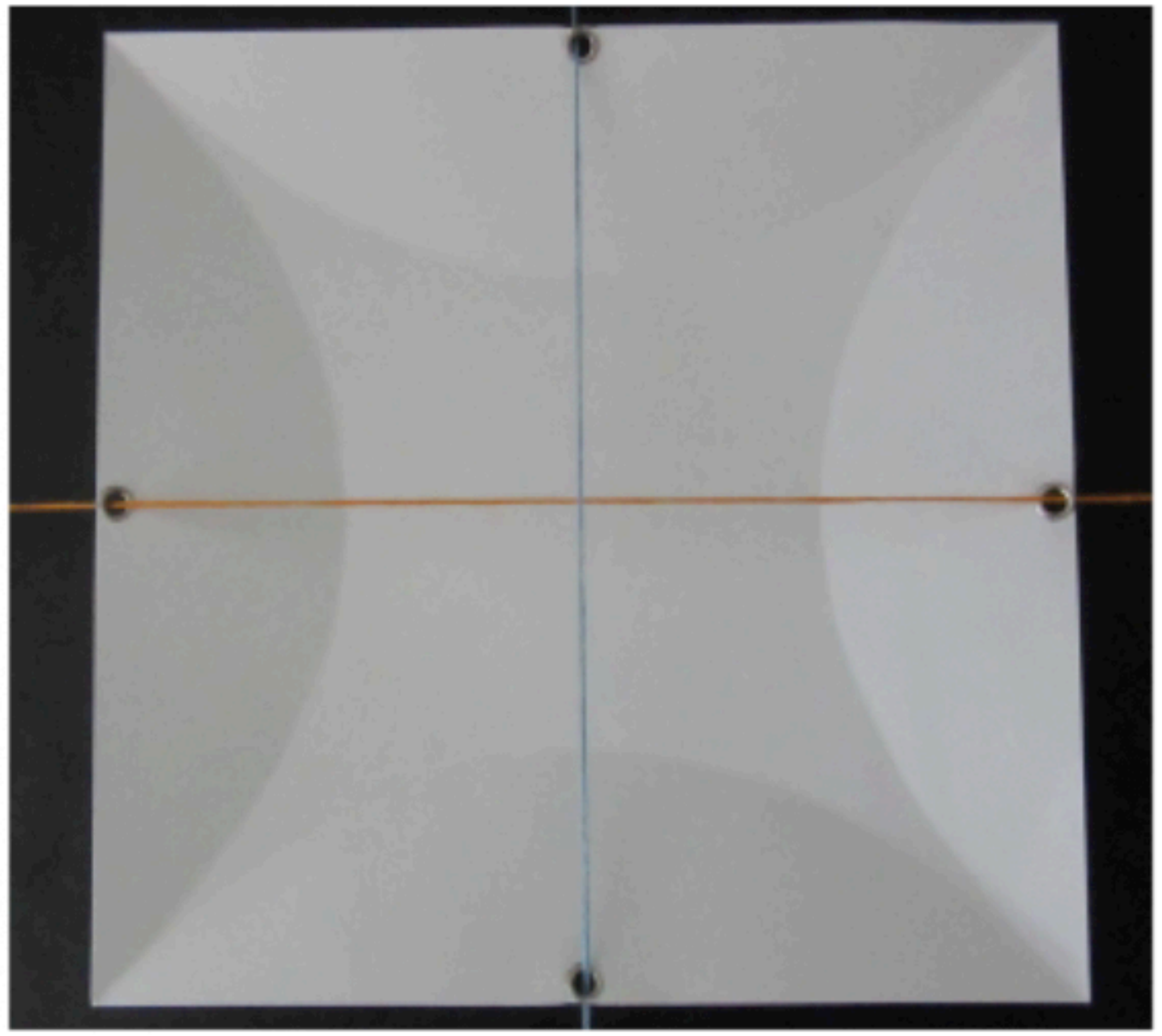
$$g_{ij}(\xi, \lambda) = 0$$

$$\min \left[\boxed{w \sum_{i=1}^{m} (1 - \xi_i)} + \sum_{j=1}^{n} \|X_j - \sum_{i=1}^{m} \lambda_{ij} X_{ij}\|^2 \right]$$

$$0 \leq \xi_i \leq 1, \quad g_{ij}(\xi, \lambda) = 0, \quad 0 \leq \lambda_{ij}.$$

Sequence: QUAD





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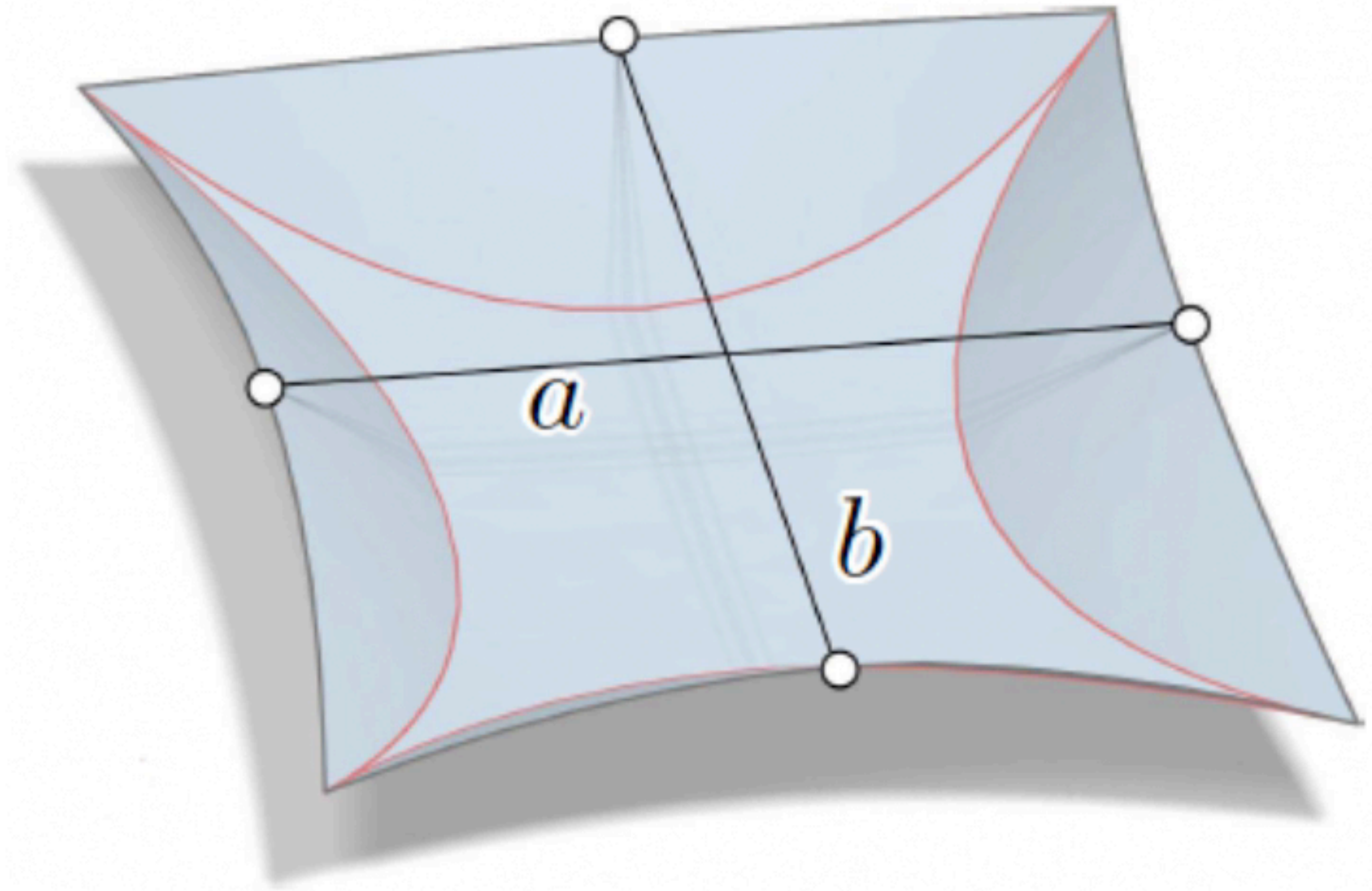

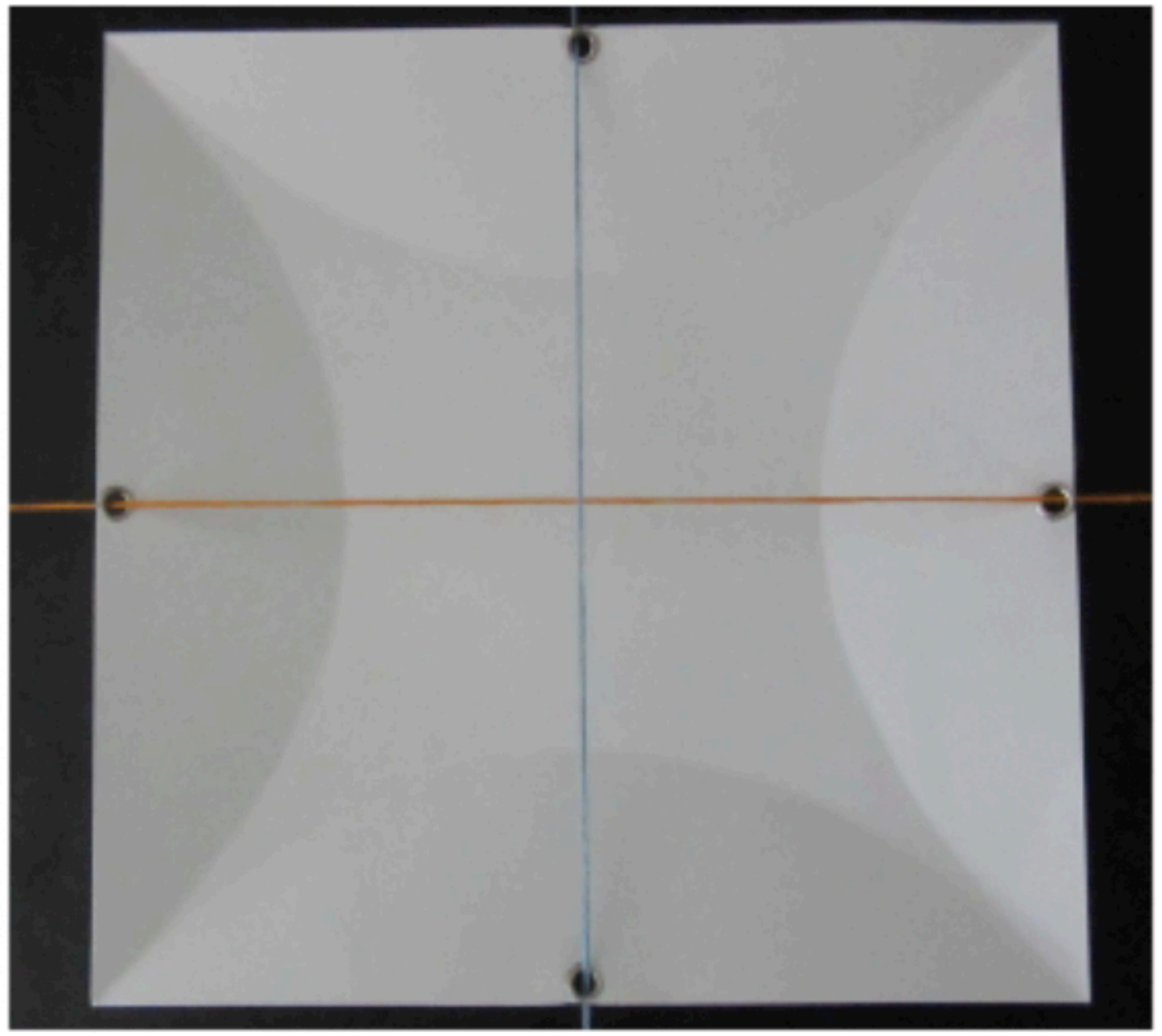

Implementation Details

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- Isometric deformation framework
  - driven by string lengths
  - searching for fold angles

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Implementation Details

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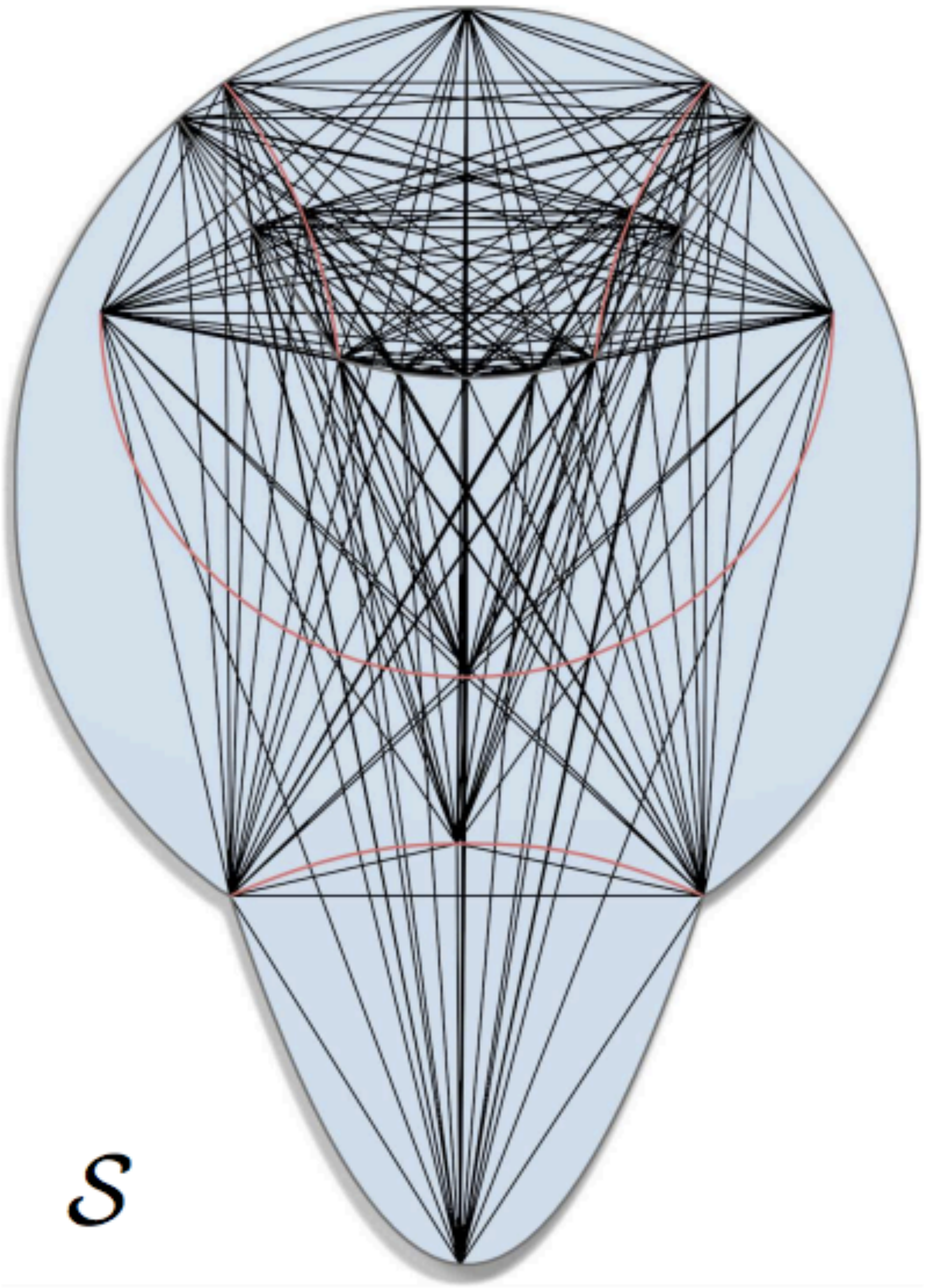
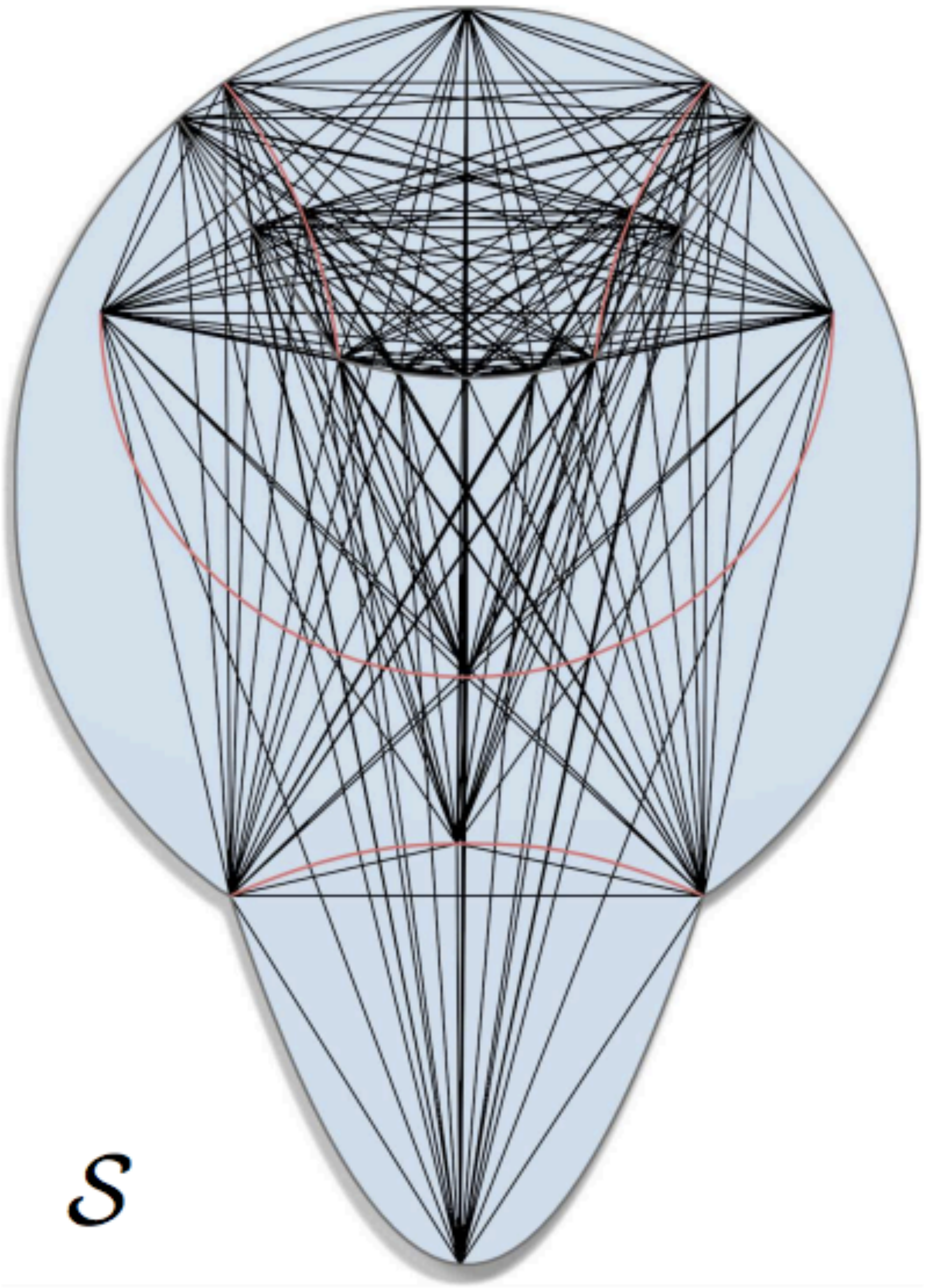
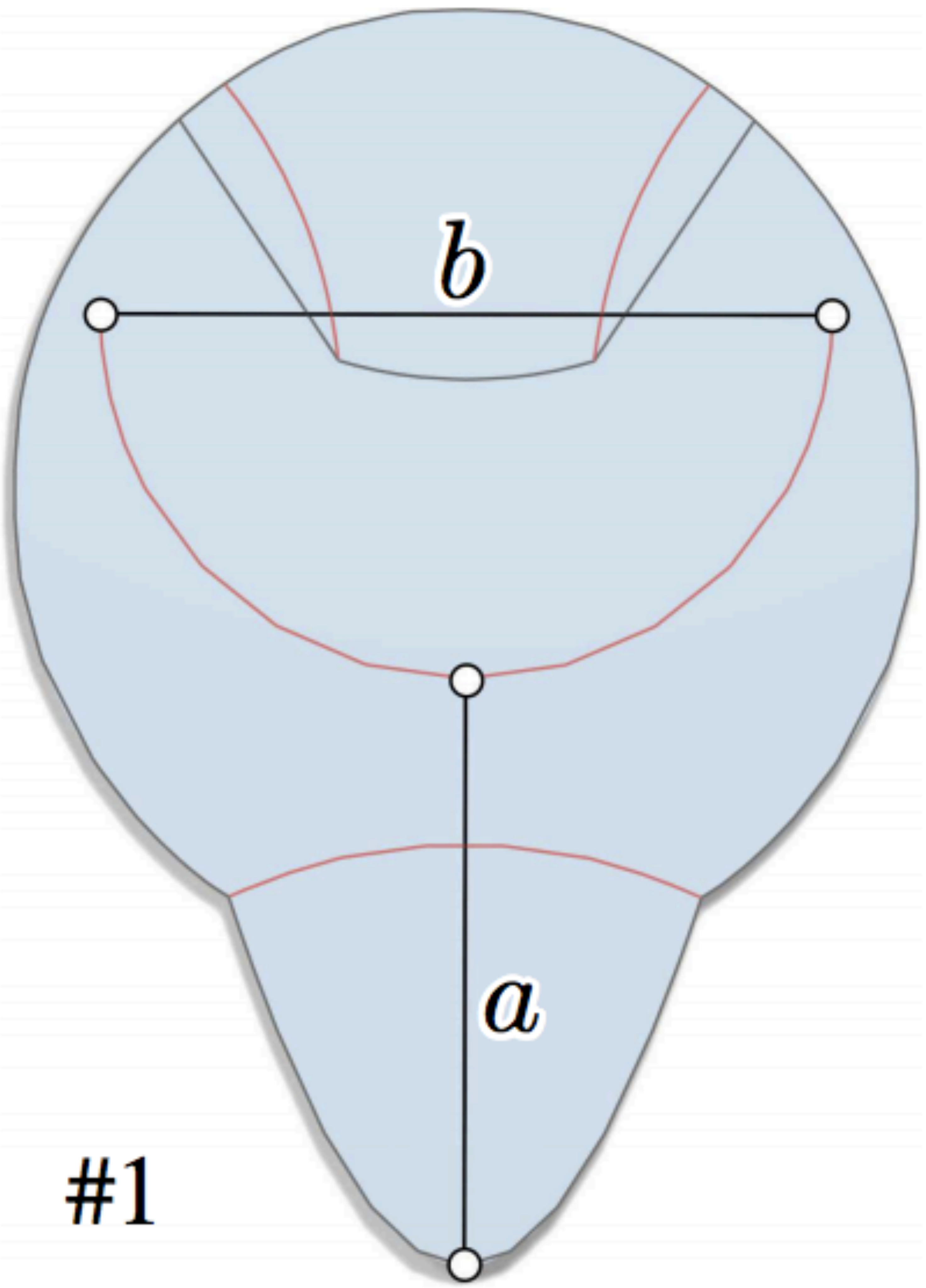
  - driven by string lengths
  - searching for fold angles
- Prevent self-intersection
- Pruning initial string candidates
- Dynamic triangulation

Concept Chair

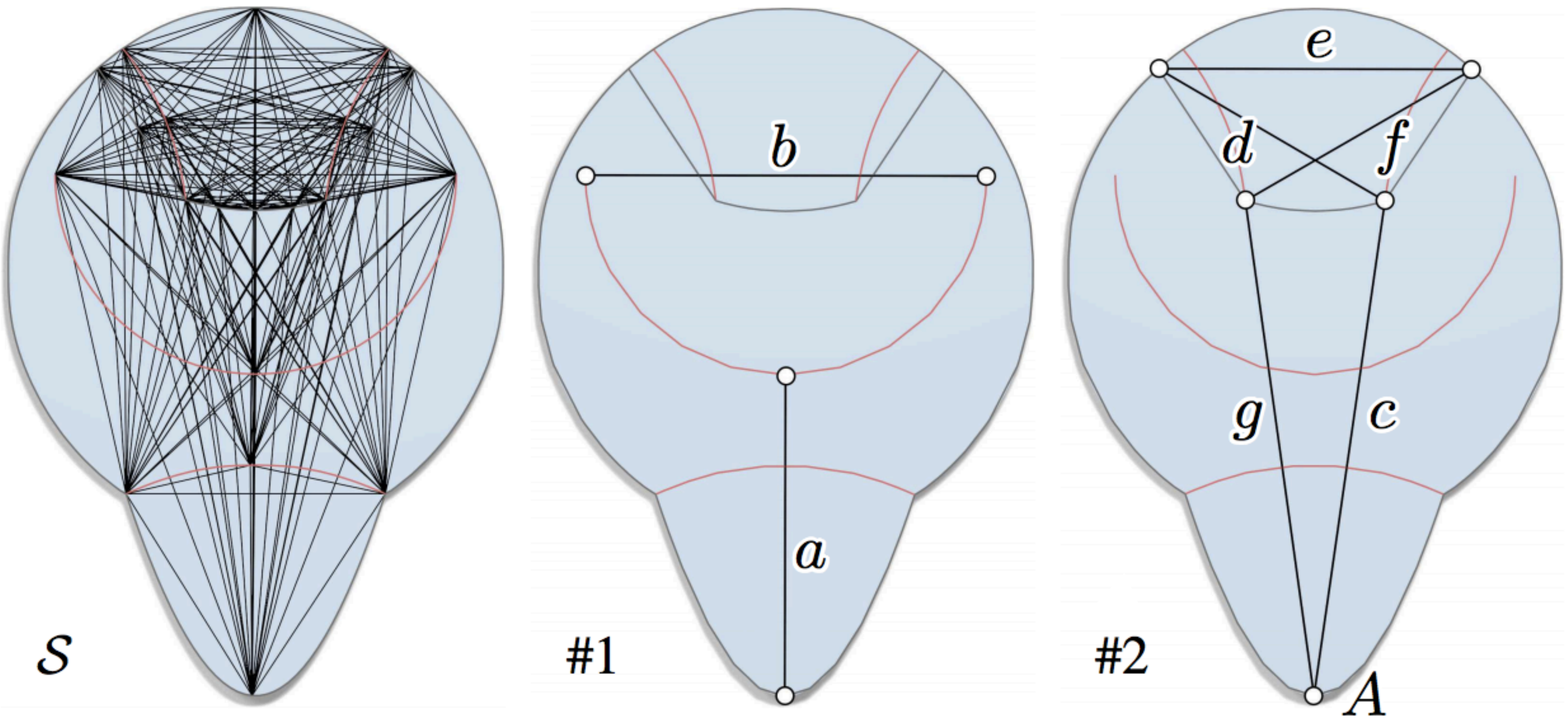


Concept Chair

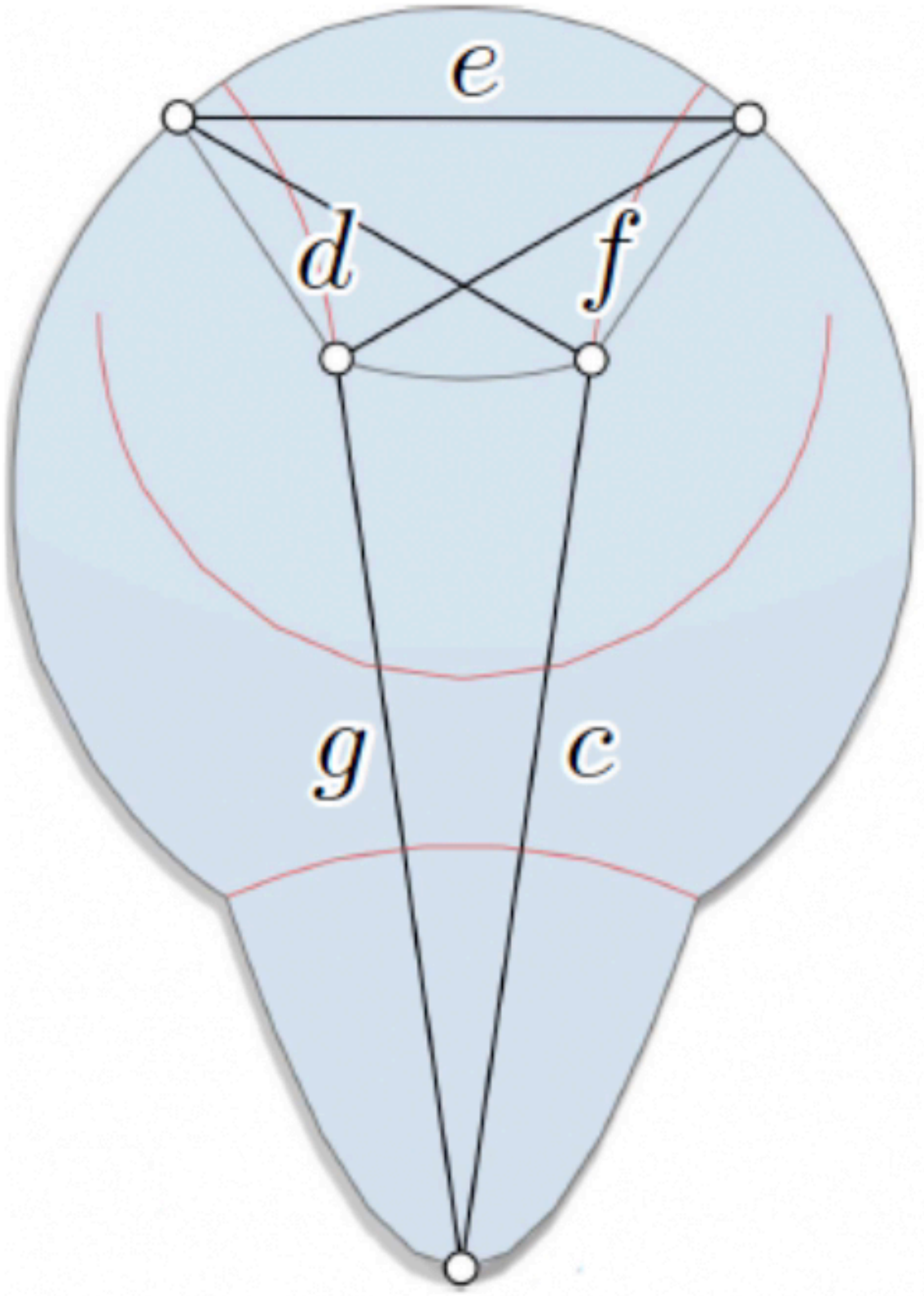


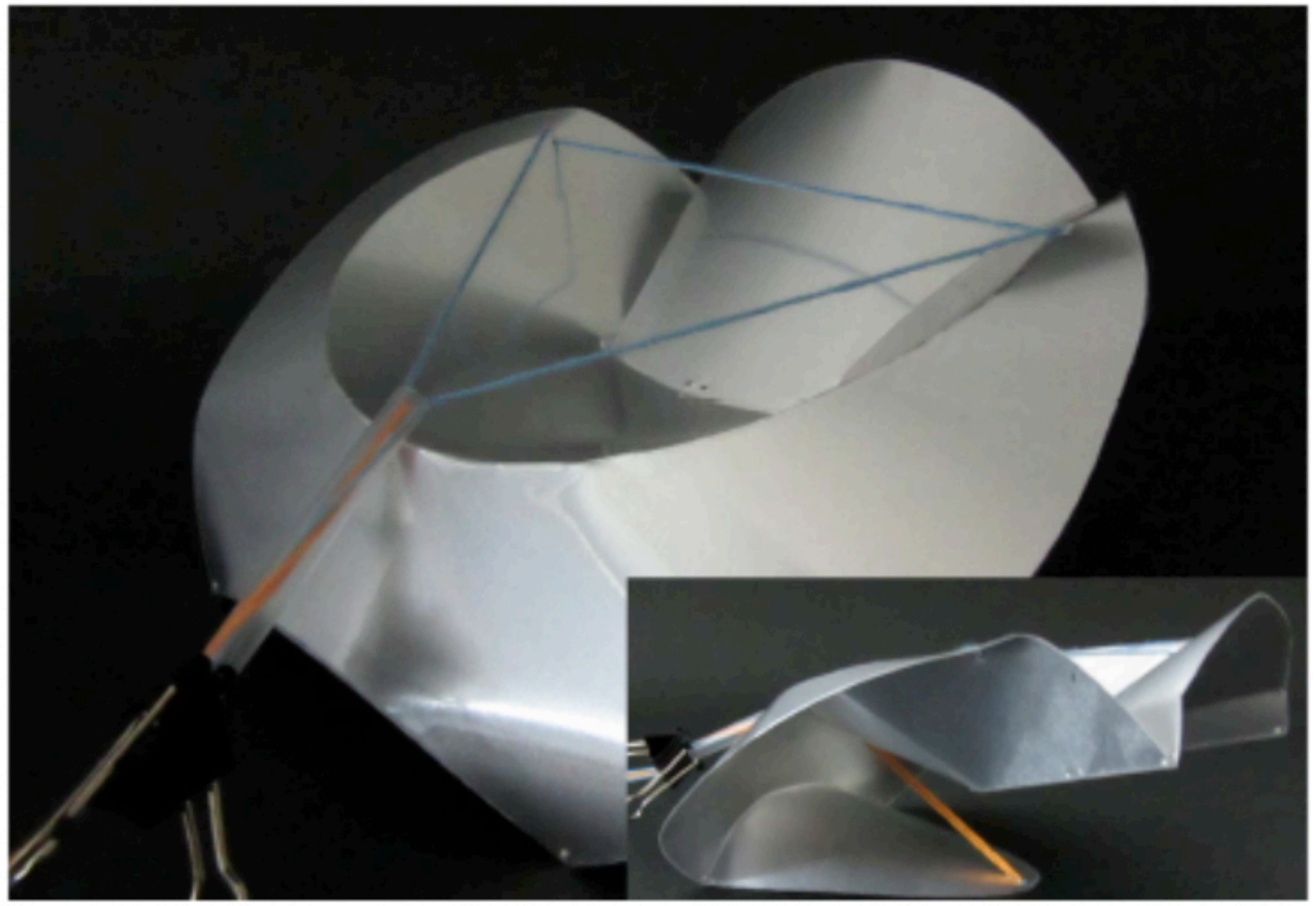


Concept Chair



Sequence: CONCEPT CHAIR





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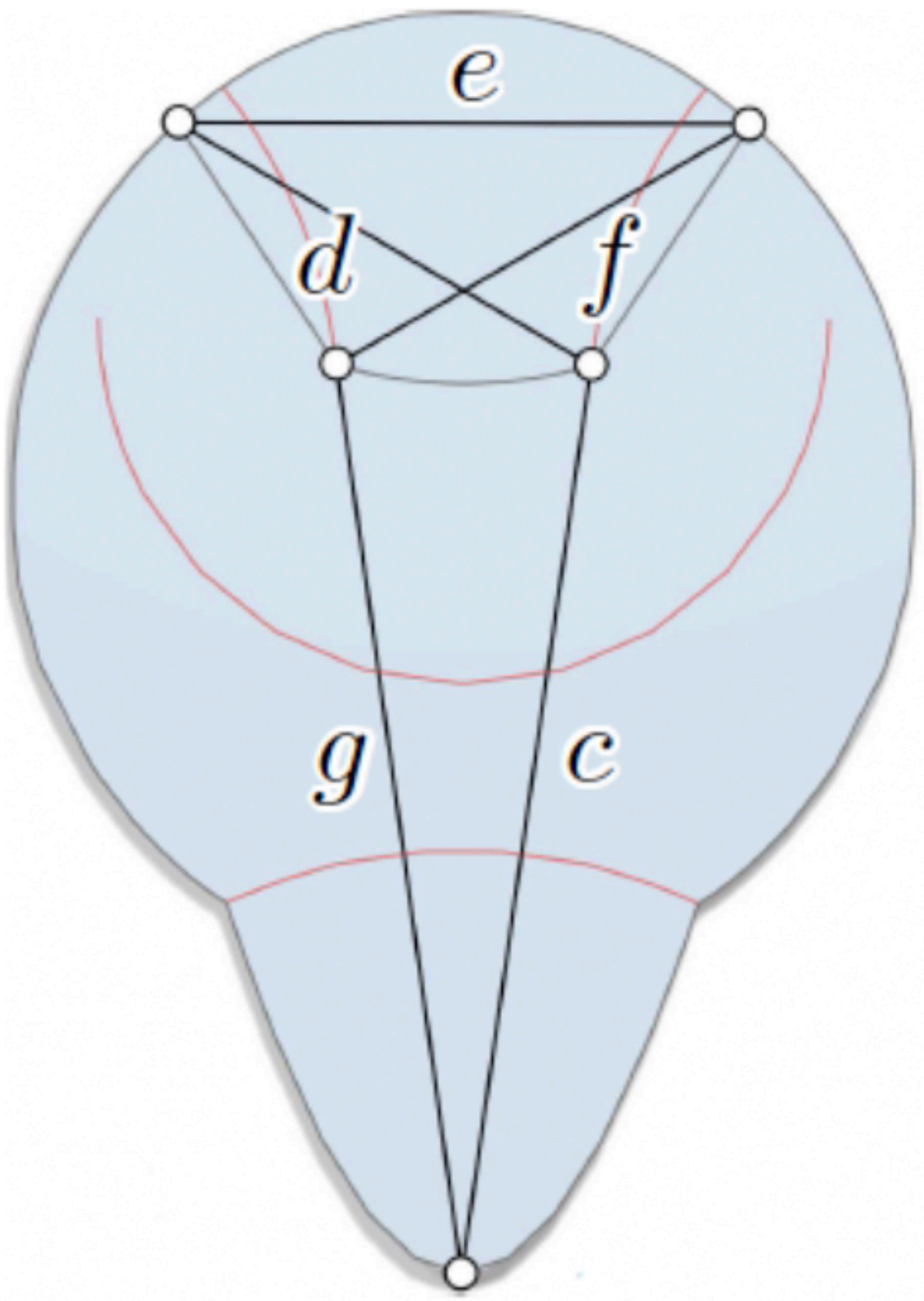

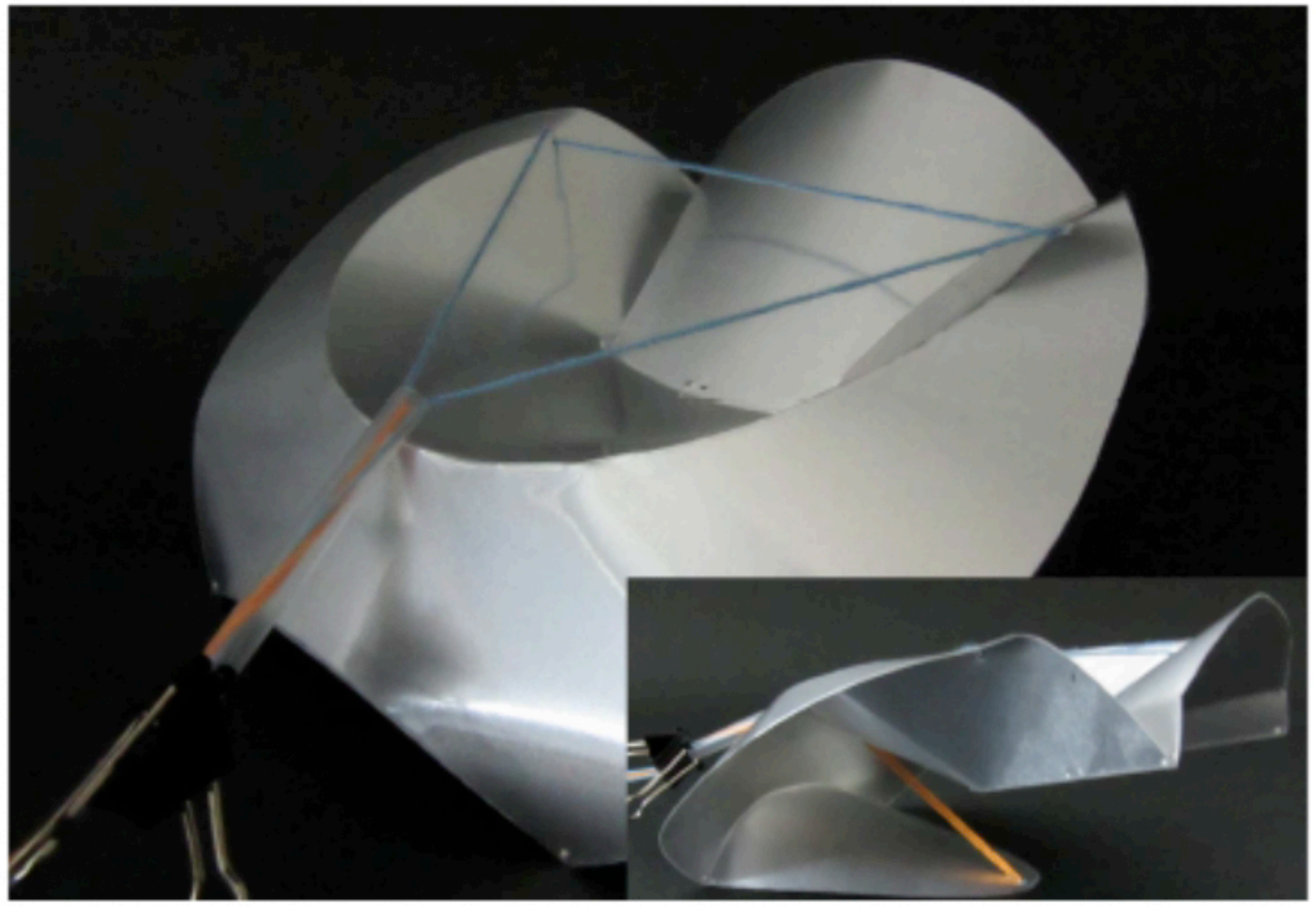

Folded Canopy

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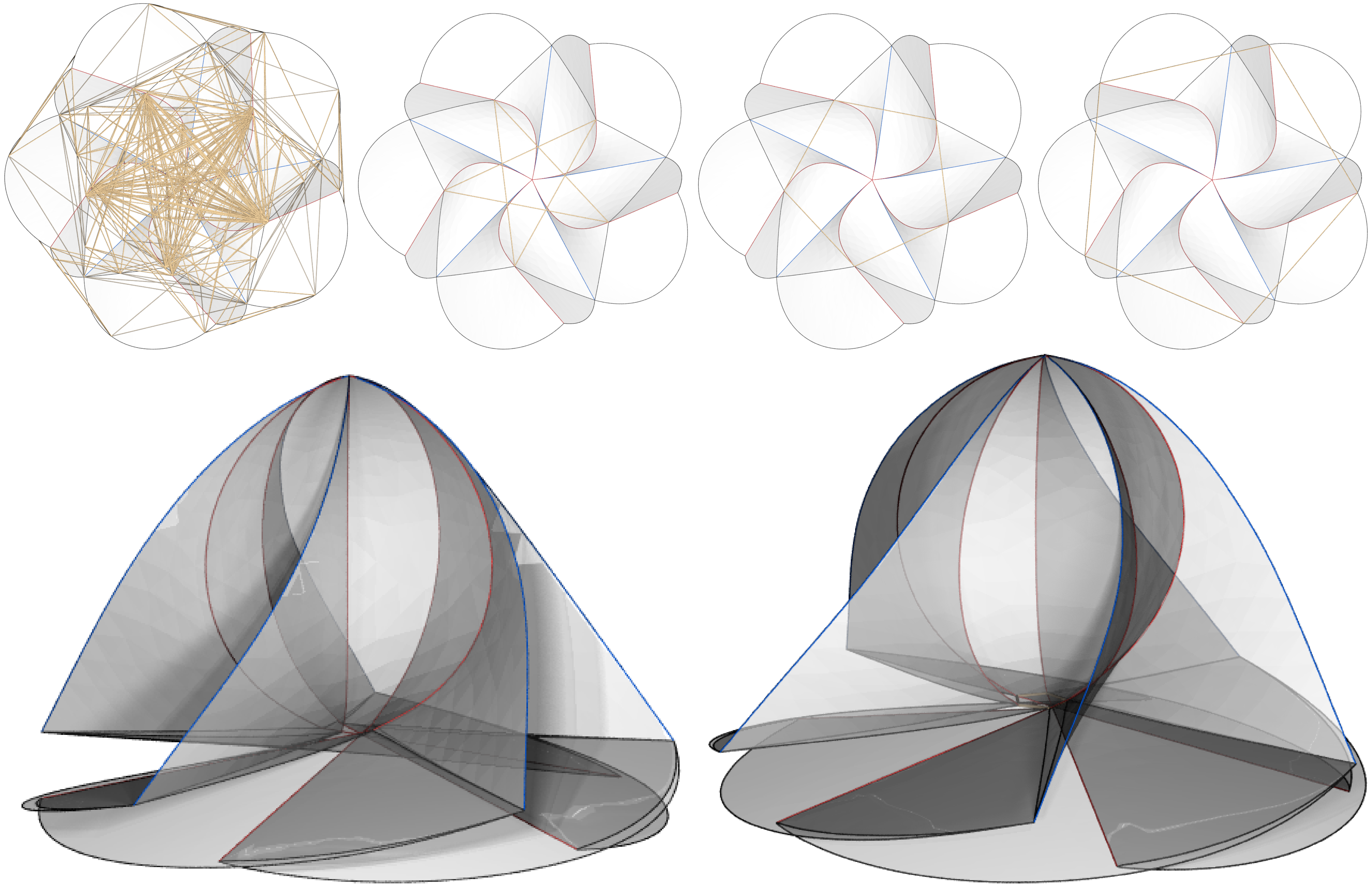
Apricot: Multiple Solutions



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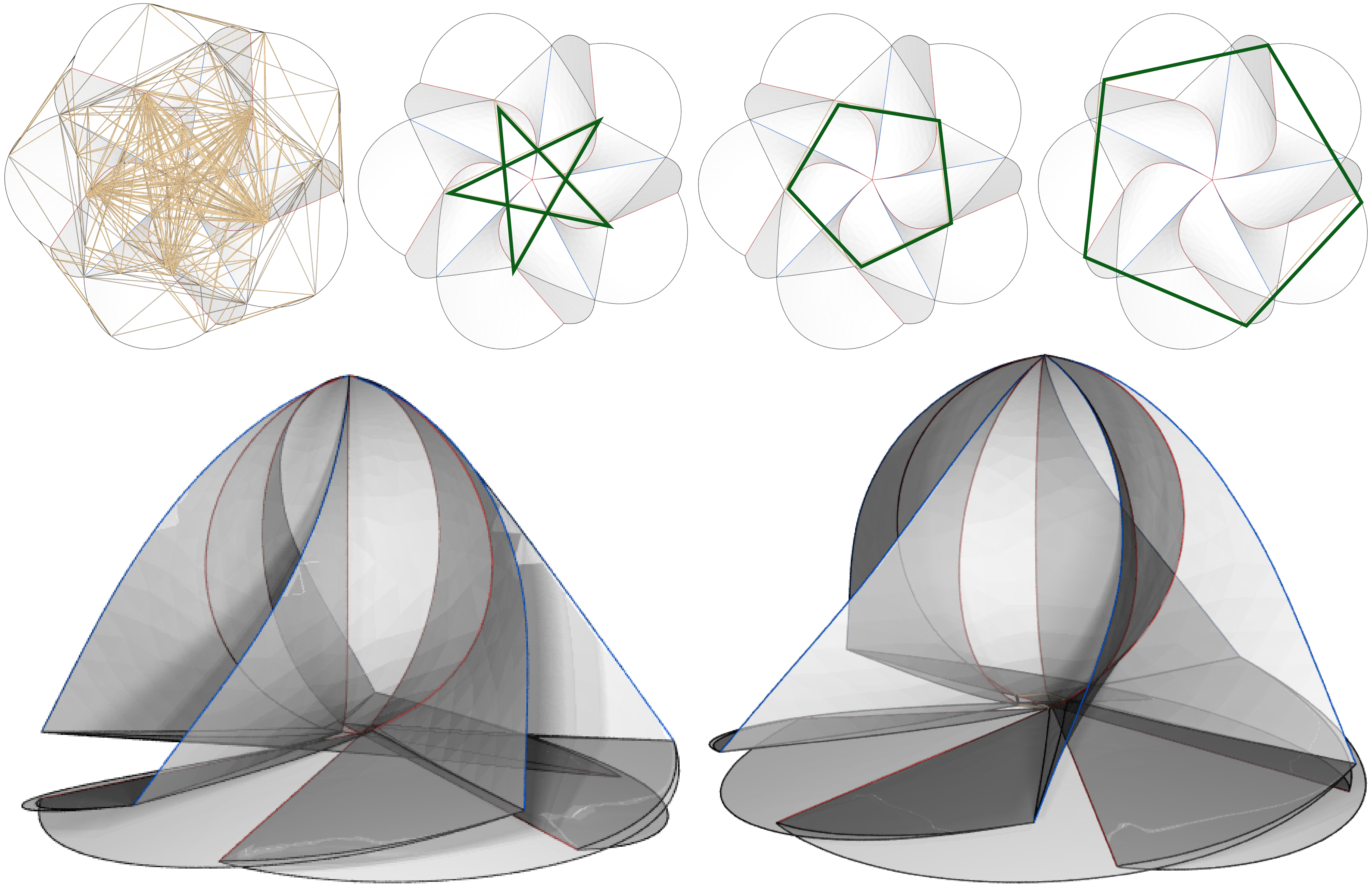
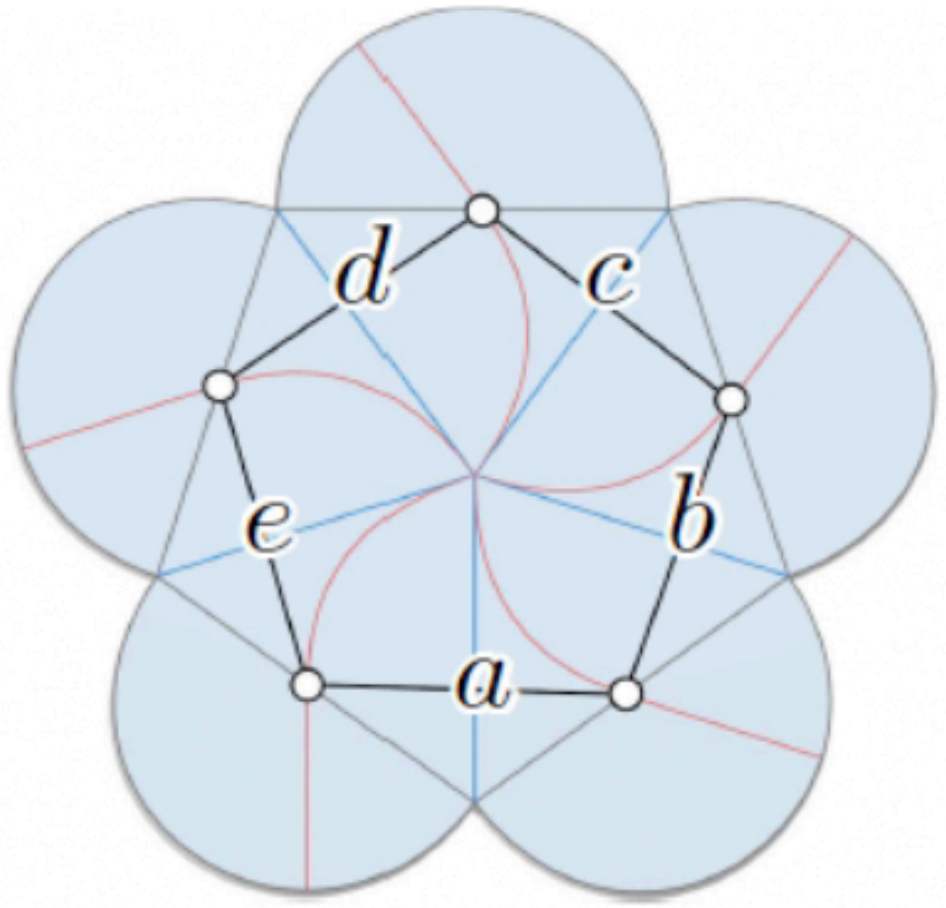
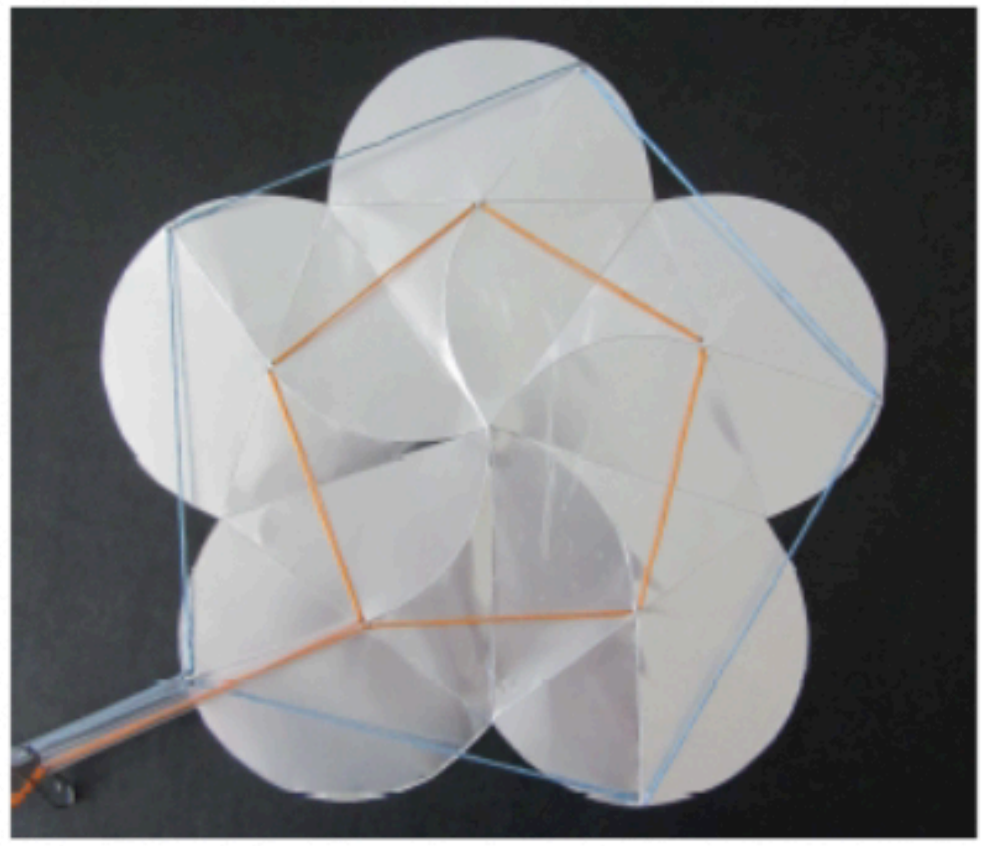
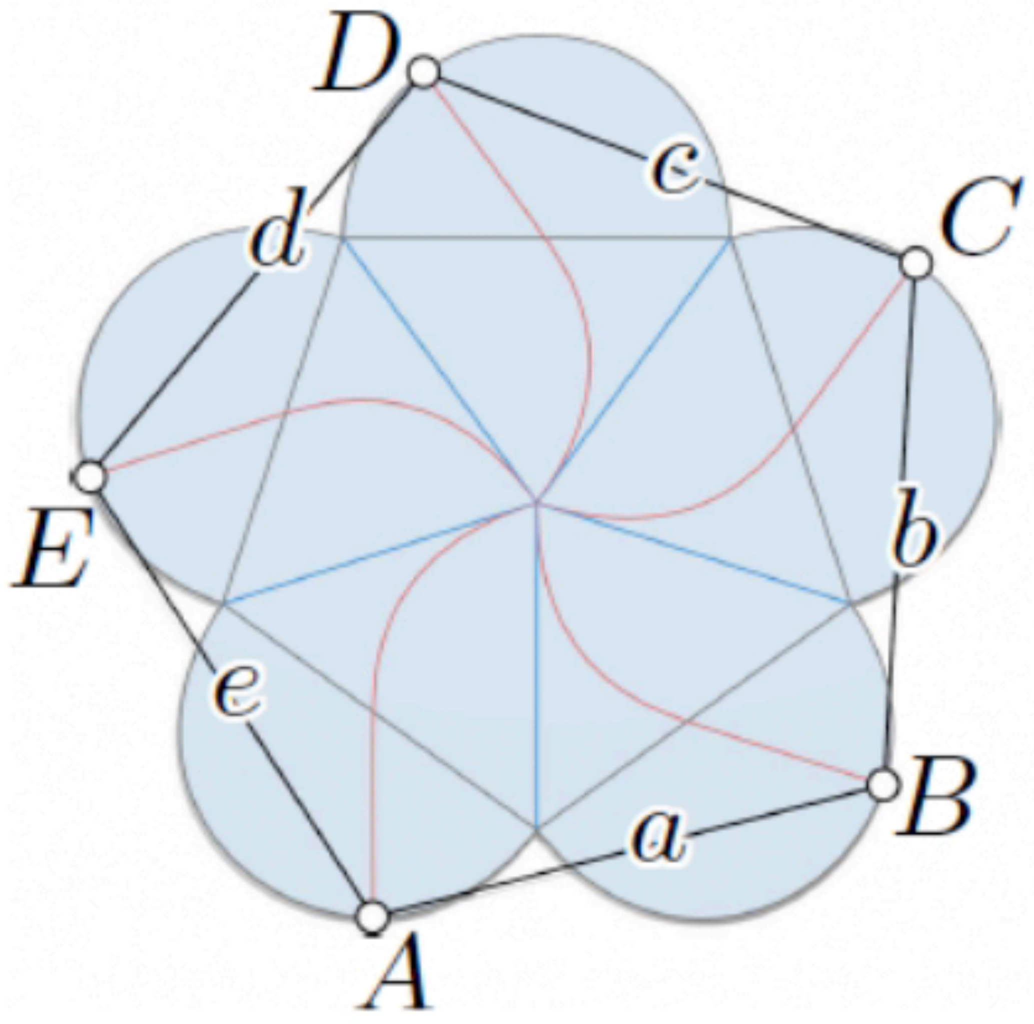
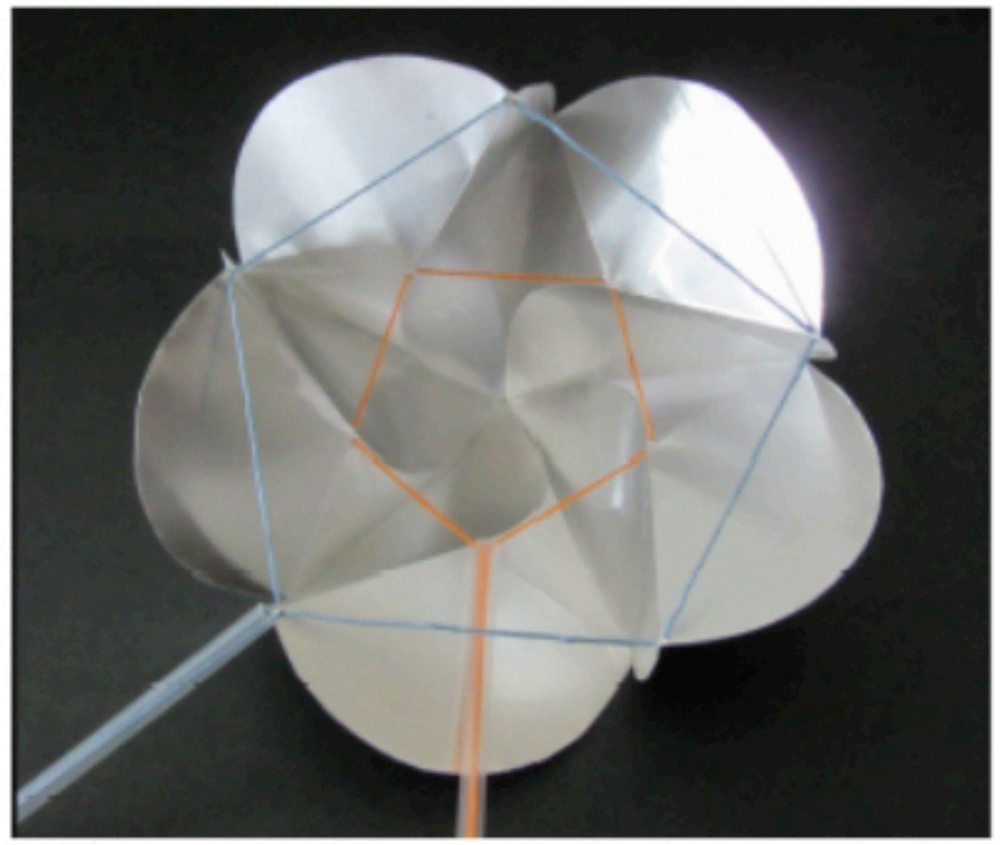


Sequence: APRICOT









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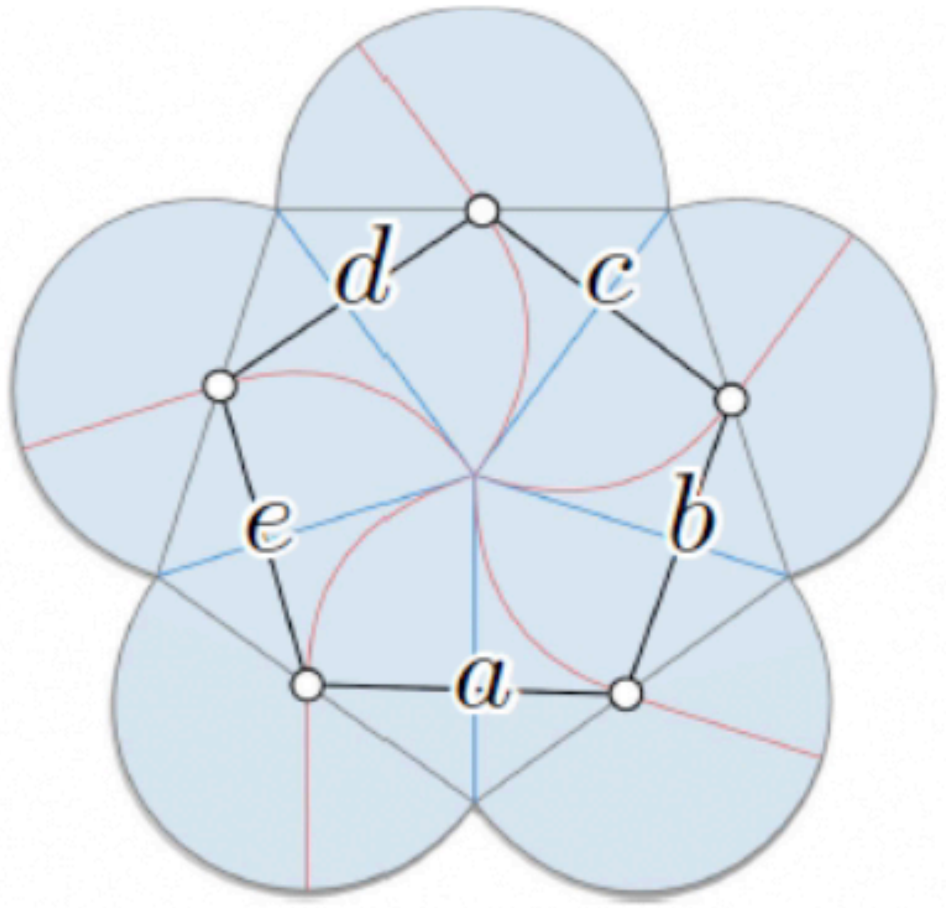
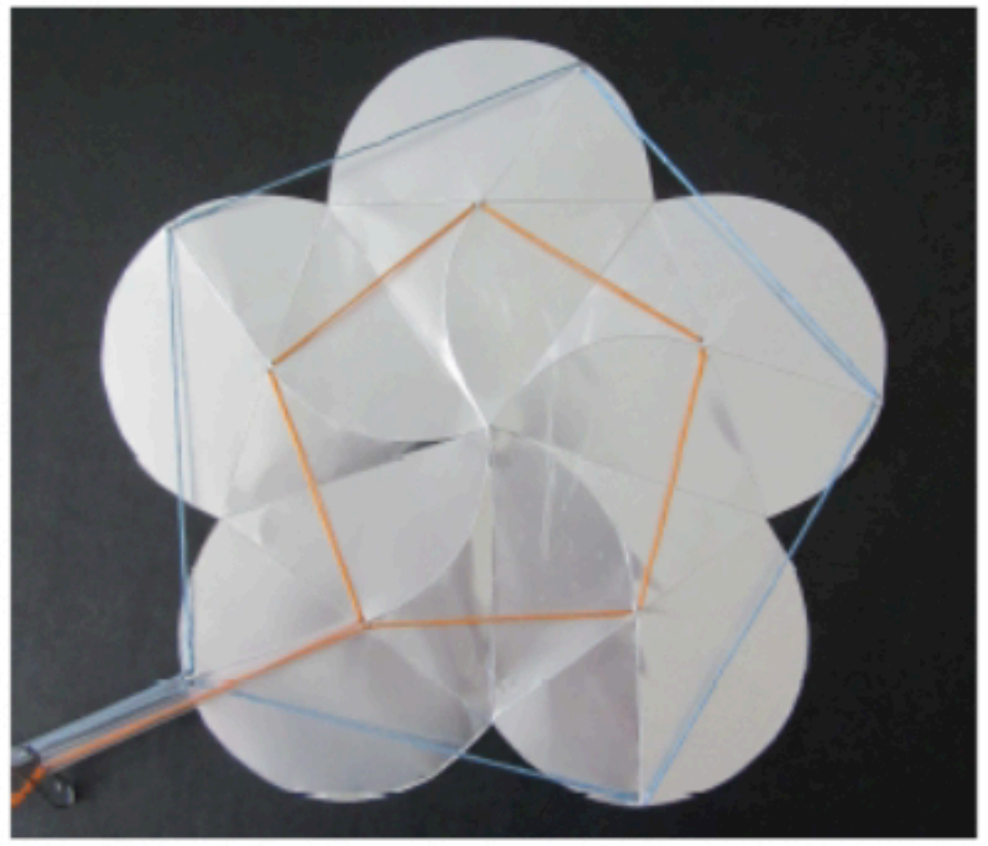
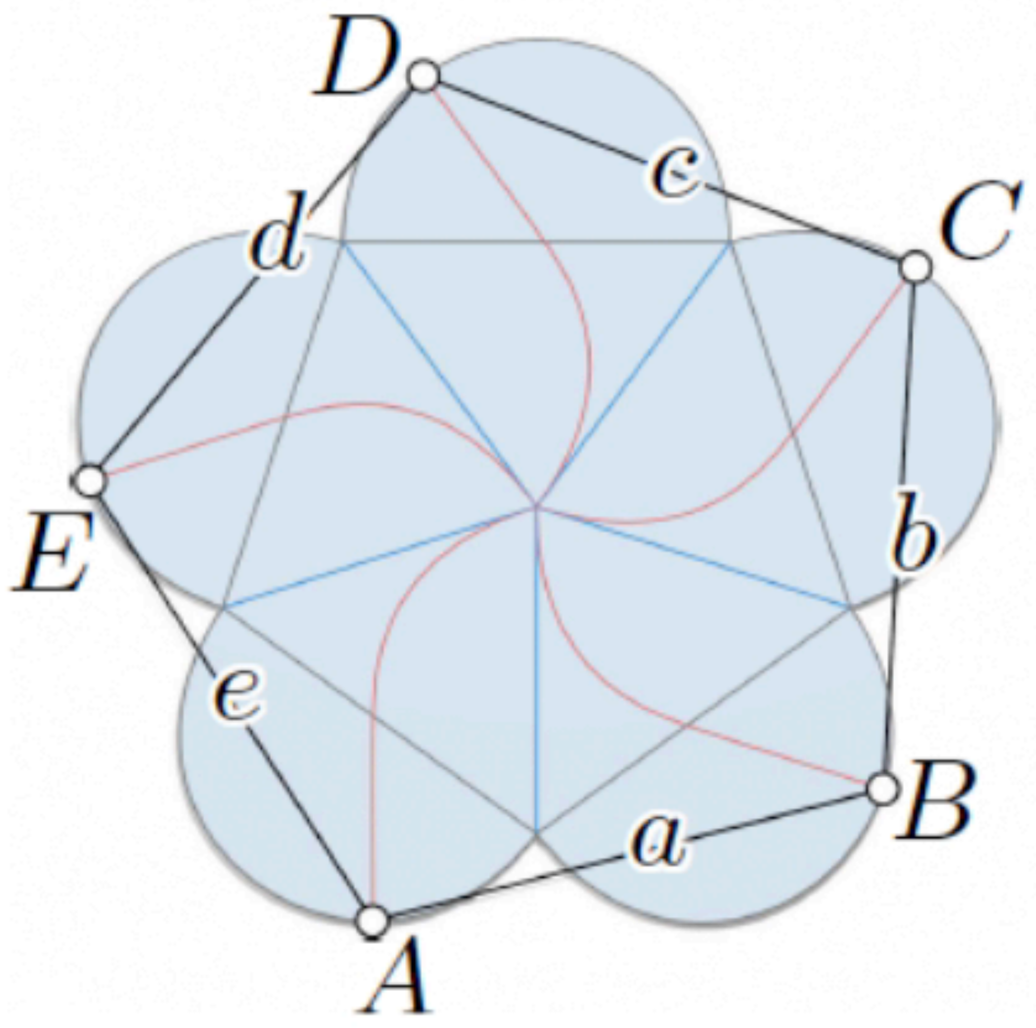
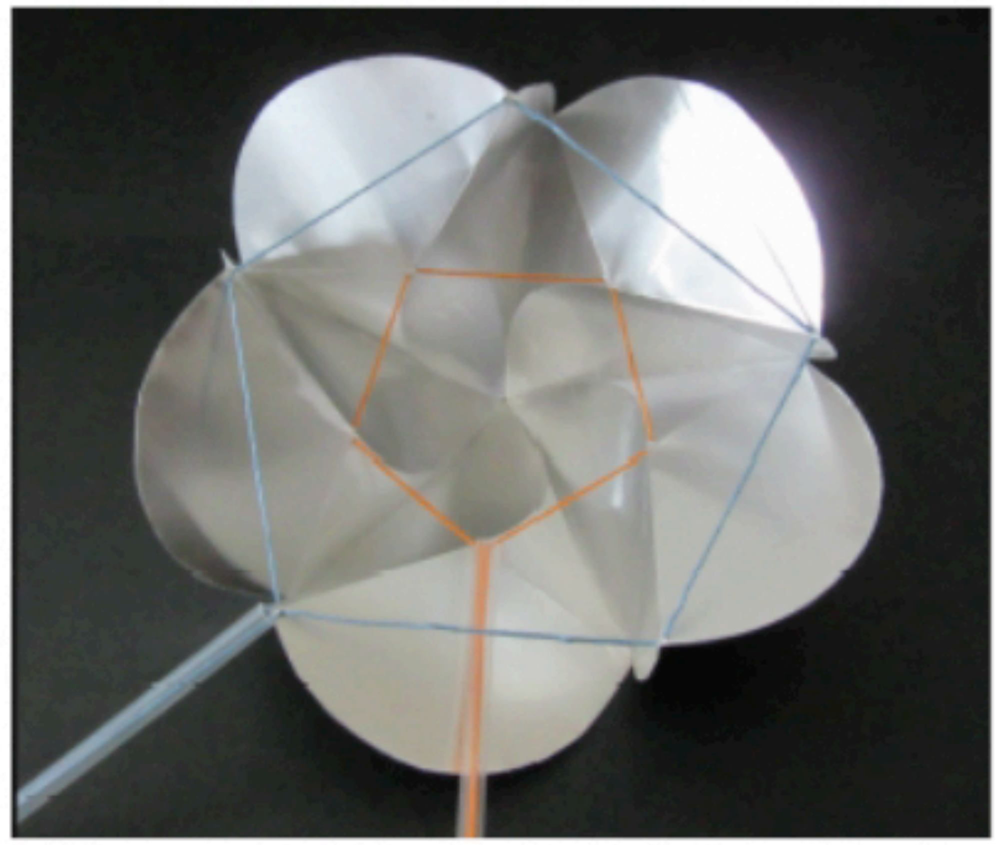
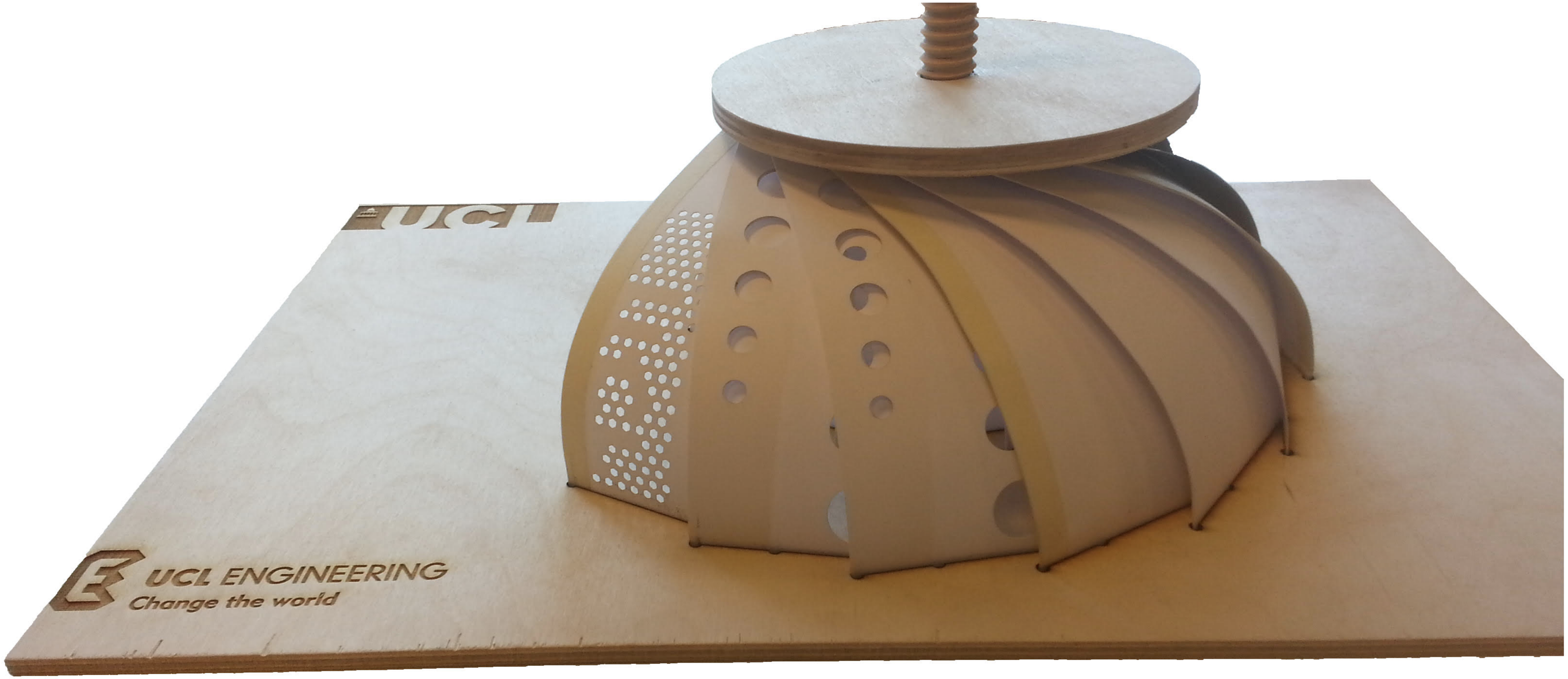








thank you



http://vecg.cs.ucl.ac.uk/Projects/SmartGeometry/

Open Challenges

Bernd Bickel
Niloy Mitra

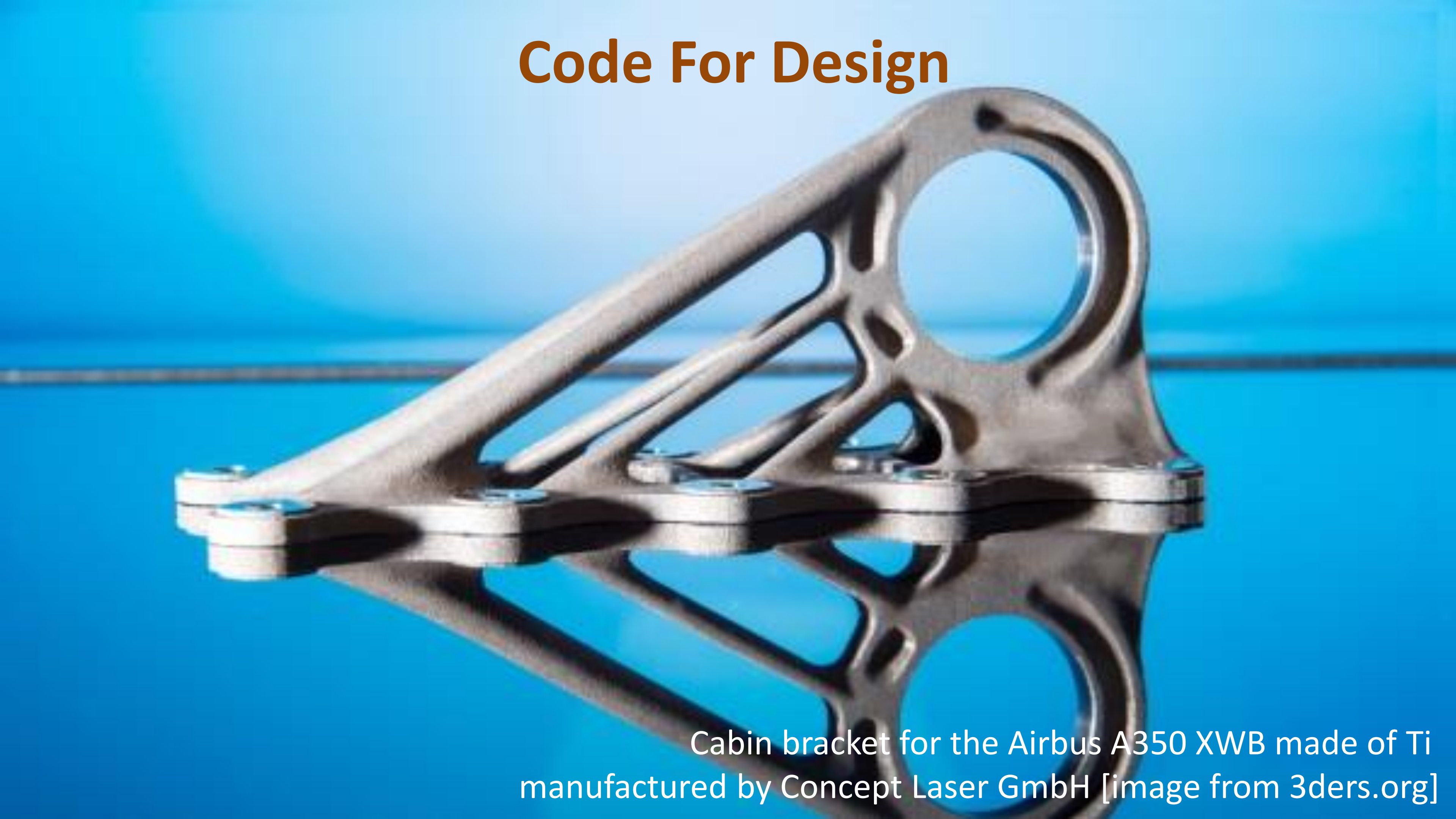
IST AUSTRIA
Institute of Science and Technology

UCL

Code For Machines

- Giga voxels/inch^3, Tera voxels/foot^3
- What are good exchange formats? Standards?
- Smart and reusable material definitions
- Dithering strategies to obtain halftones representations
- Resolution and printer independence
- Simulate printing processes
- Predict qualtiy
- Help users to deal with parameters
- Promoting open-source environment vs. commercial products

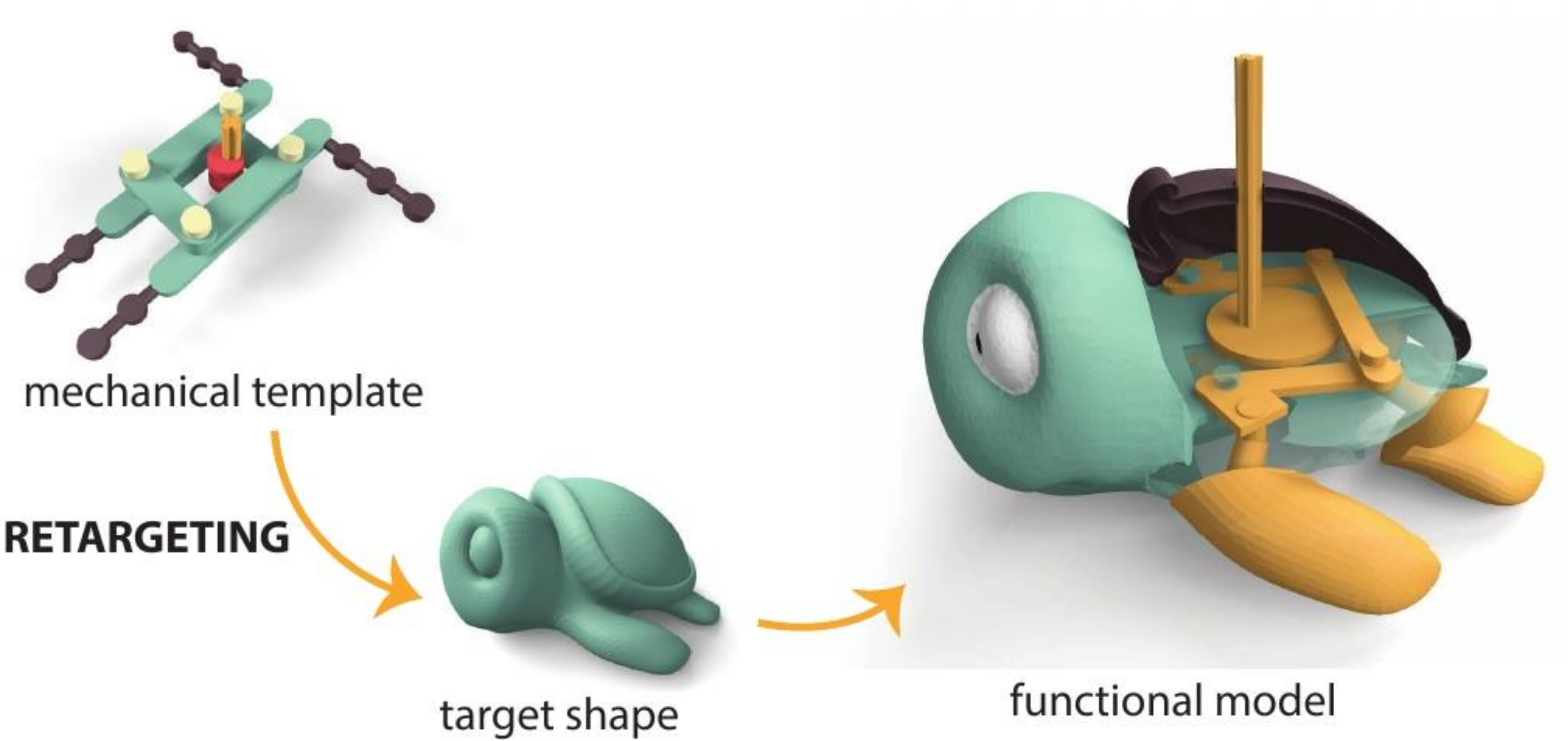
Code For Design



Cabin bracket for the Airbus A350 XWB made of Ti manufactured by Concept Laser GmbH [image from 3ders.org]

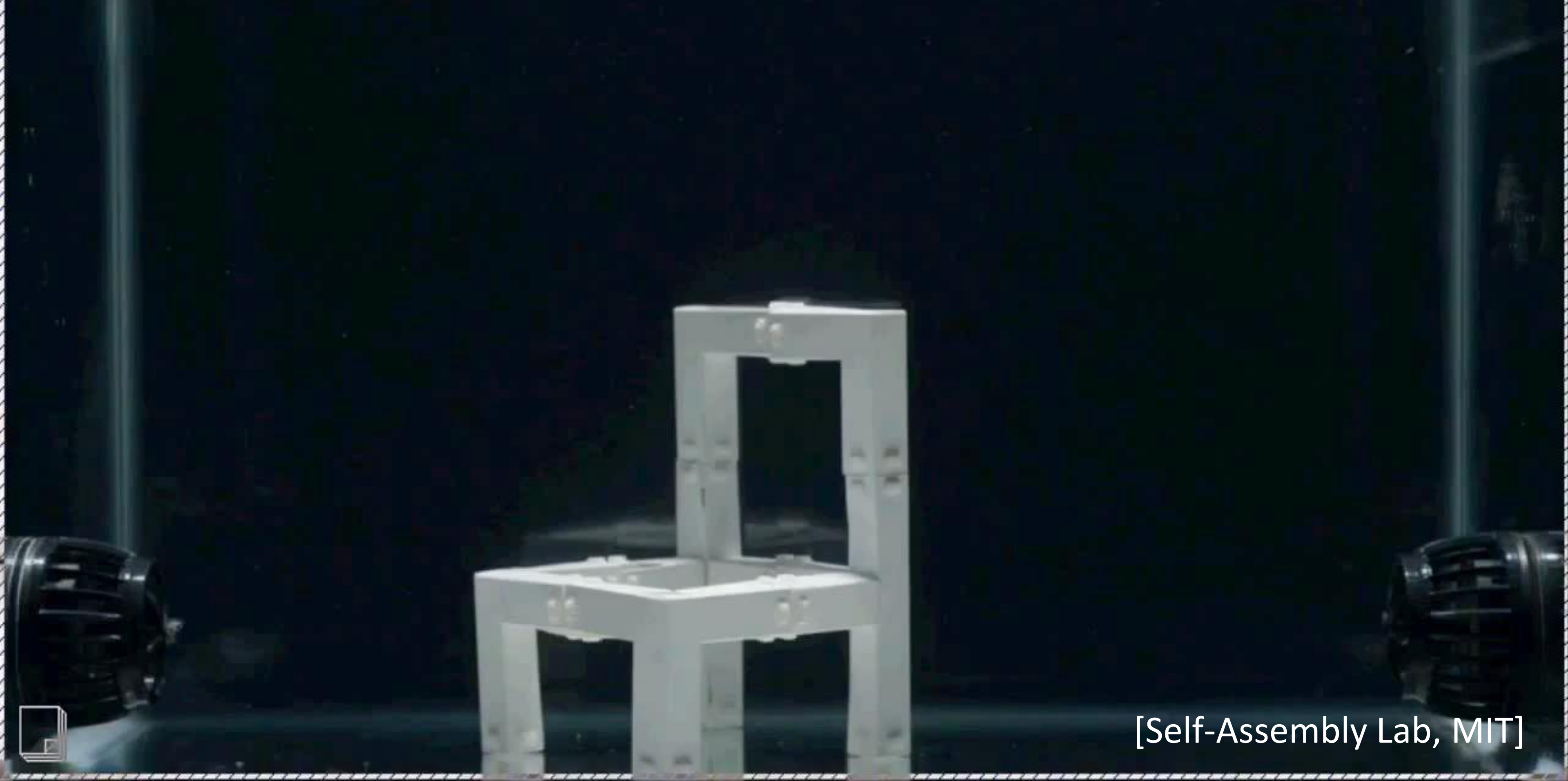
Going Beyond What an Engineer Can Do

Empowering Everyday Users



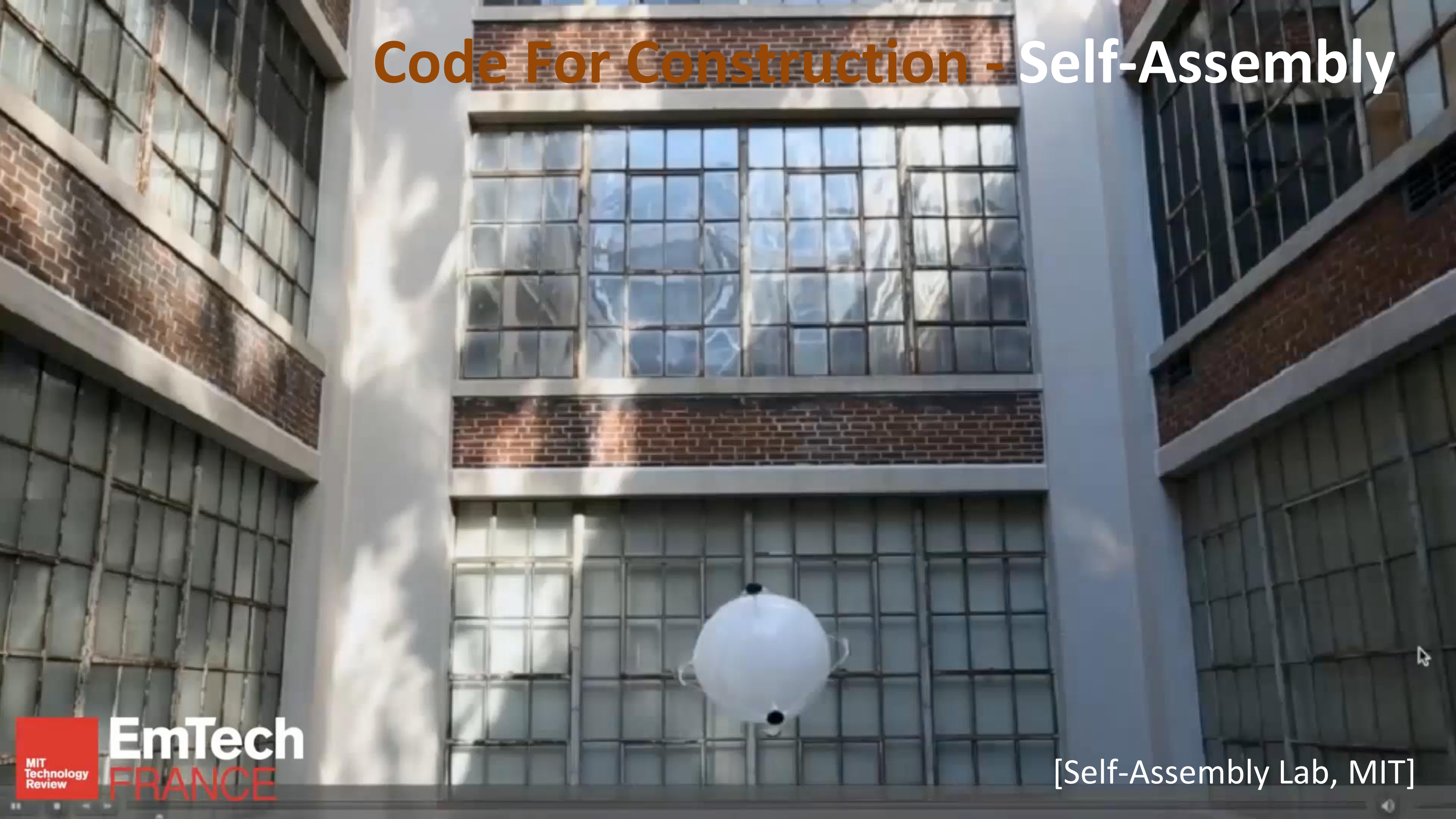
[Zhang et al. 2017]

Code For Construction - Self-Assembly



[Self-Assembly Lab, MIT]

Code For Construction - Self-Assembly



[Self-Assembly Lab, MIT]

Granular Materials

[Gramazio Kohler Research, ETHZ, and Self-Assembly Lab, MIT]