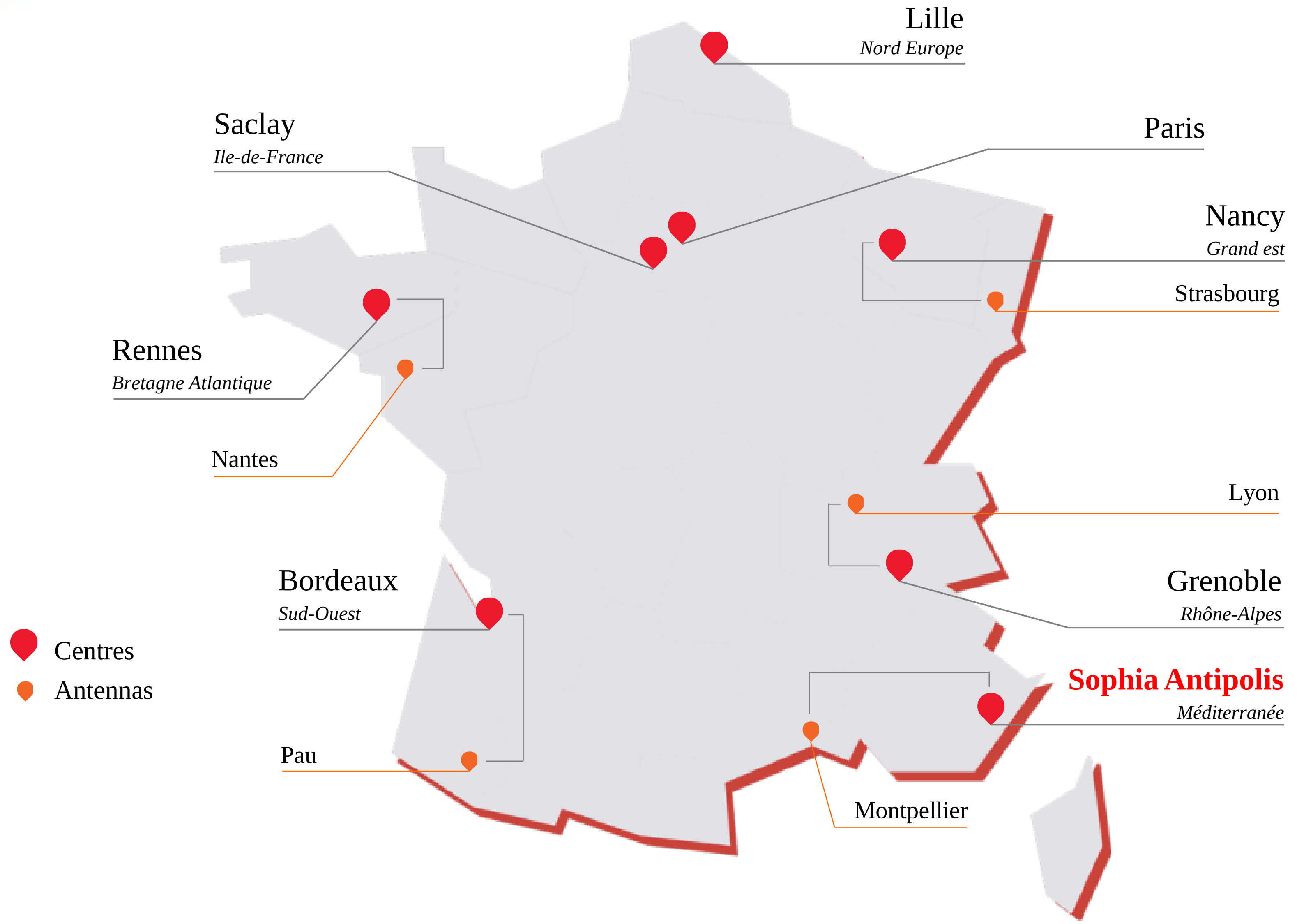
Surface Reconstruction

Pierre Alliez

Inria Sophia Antipolis – Méditerranée

TITANE team: https://team.inria.fr/titane/

pierre.alliez@inria.fr


Lille
Nord Europe
Saclay
Ile-de-France
Paris
Nancy
Grand est
Strasbourg
Rennes
Bretagne Atlantique
Nantes
Lyon
Bordeaux
Sud-Ouest
Grenoble
Rhône-Alpes
Centres
Antennas
Sophia Antipolis
Méditerranée
Pau
Montpellier

Outline

- Context
  - Sensors
  - Applications
- Problem statement
- Main approaches
- Quest for robustness
- What next

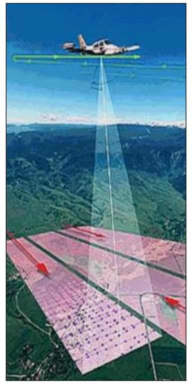
Context

Sensors

- Contact -> contact-free
- Short -> long range sensing

Contact

Laser



Aerial

Remote Sensing

Context

Sensors

- Structured-light (infrared, active)
- Passive stereo vision
- Digital cameras

Depth sensing

Photo-modeling

Context

Instrumented sensors

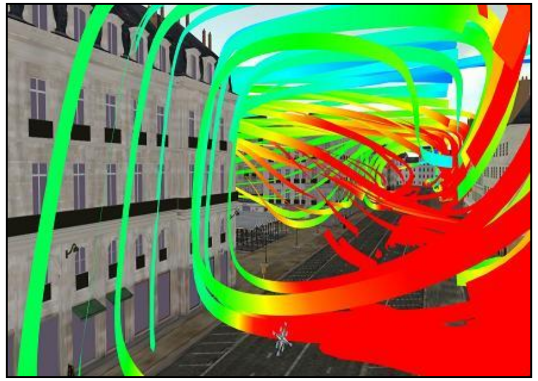
- Accelerometer
- Gyroscope
- GPS
- Compass / magnetometer
- Robotized platforms

Photo Phoenix Aerial Systems

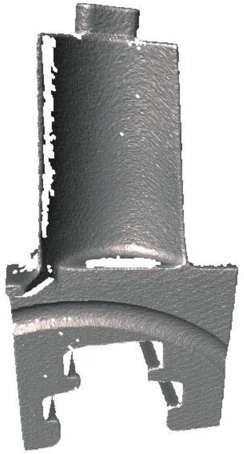
Digitizing the Physical World

Data from Navteq

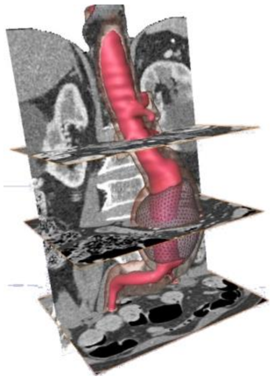
Applications



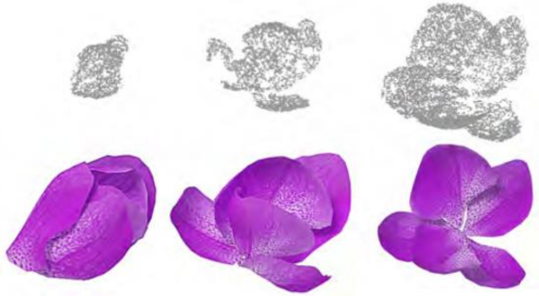
Computational engineering



Reverse engineering



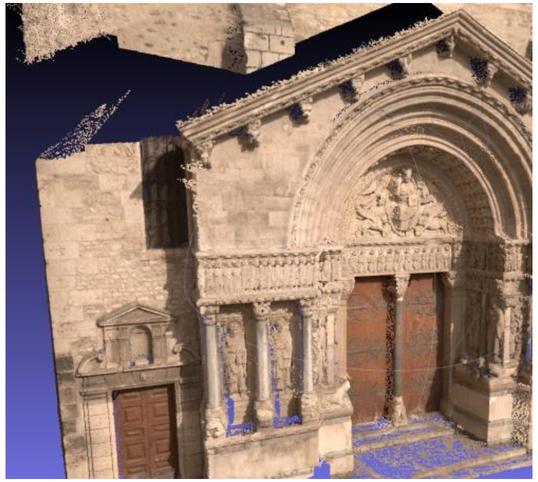
Computer-aided medicine



Biology
Zheng et al. *4D Reconstruction of Blooming Flowers.*

Scene interpretation
Choi et al. *Robust Reconstruction of Indoor Scenes.*

Underwater exploration
Geology / Archeology



Cultural Heritage
Data from Culture 3D Cloud [De Luca].

PROBLEM STATEMENT

Problem Statement

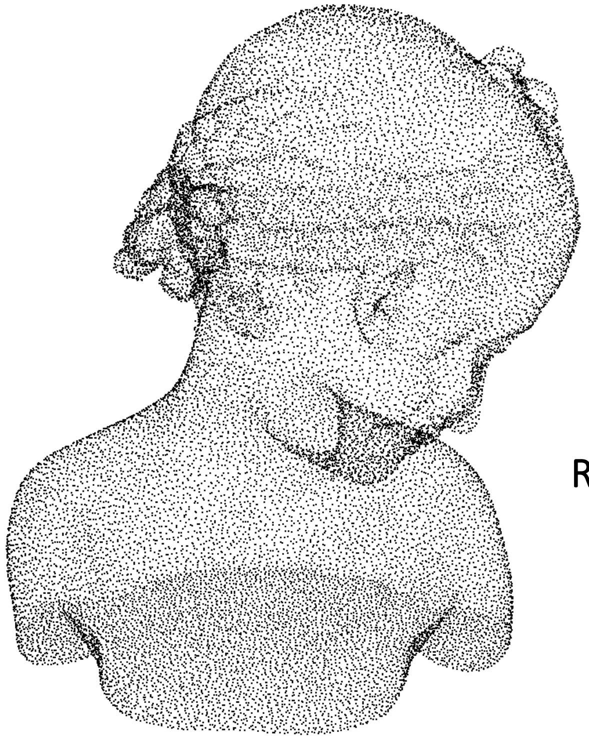
<u>Input</u>:

Dense point set *P* sampled over surface *S*

<u>Output</u>:

Surface: Approximation of *S* in terms of topology and geometry

Laser scanning



Point set

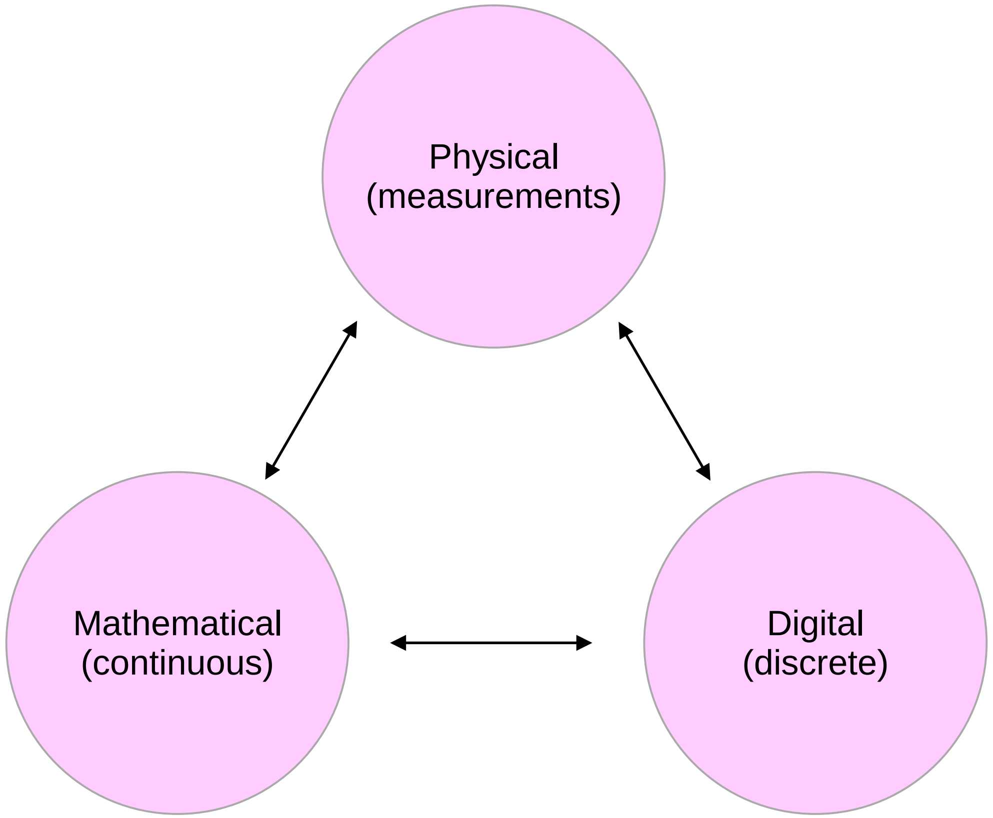
Reconstruction

Reconstructed surface

Scientific Challenge

Transitions

- Physical
- Mathematical
- Digital

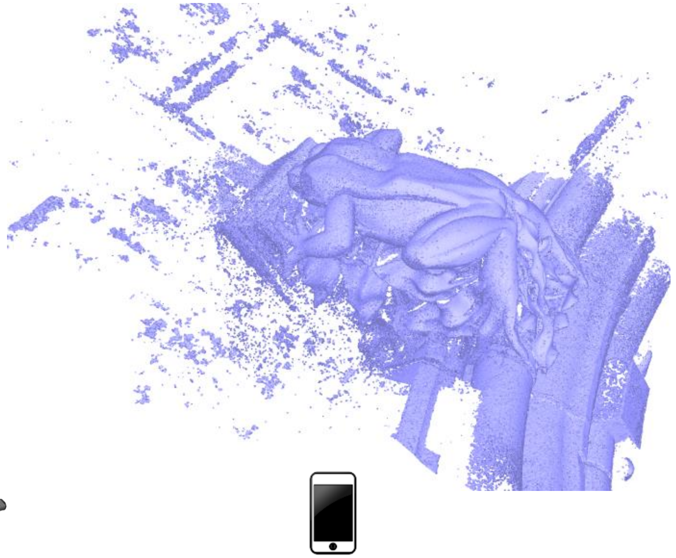


Real-World Problems

Input:

~~Dense~~ point set *P* sampled over surface *S*:

- Imperfect sampling
  - Non-uniform
  - Anisotropic
  - Missing data (holes)
- Uncertainty
  - Noise



DEMO

Real-World Problems

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~~Dense~~ point set *P* sampled over surface *S*:

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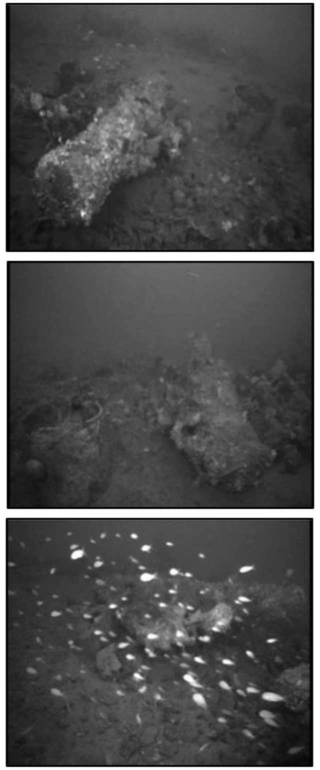

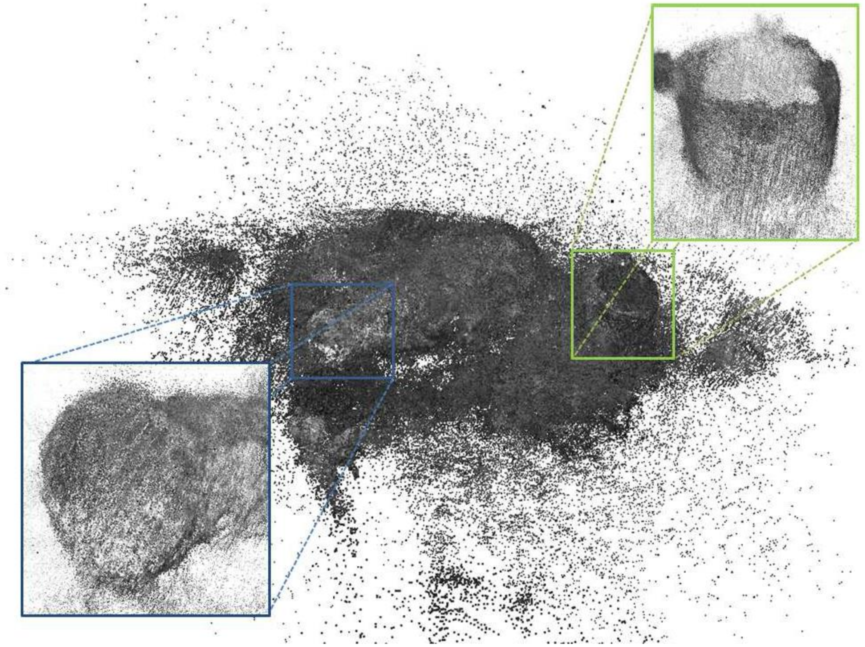

“La lune”: Data from Dassault Systèmes.
Sun King's flagship, sank off the Toulon coastline in 1664.

Real-World Problems

Input:

Point set *P* sampled over surface *S*:

- Imperfect sampling
  - Non-uniform
  - Anisotropic
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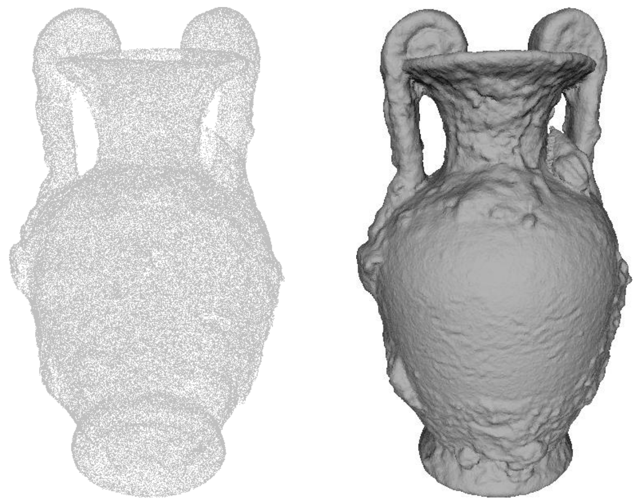
- Uncertainty
  - Noise
  - Outliers

Output:

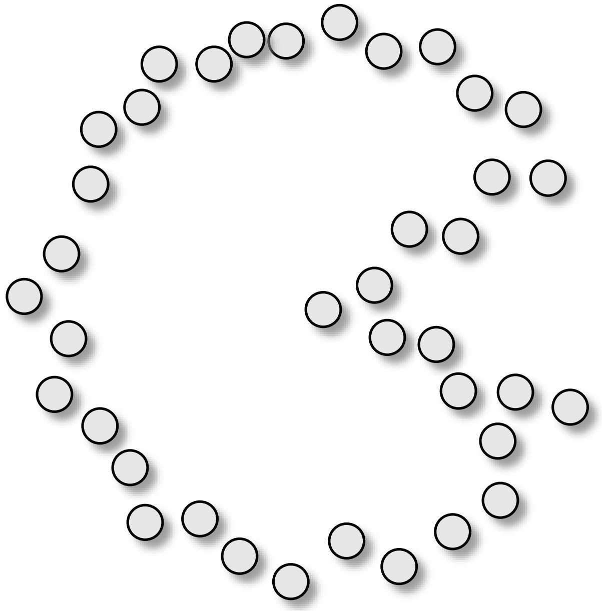
Surface: Approximation of *S* in terms of topology and geometry

Desired properties:

- Watertight
- Intersection free
- Data fitting vs smoothness

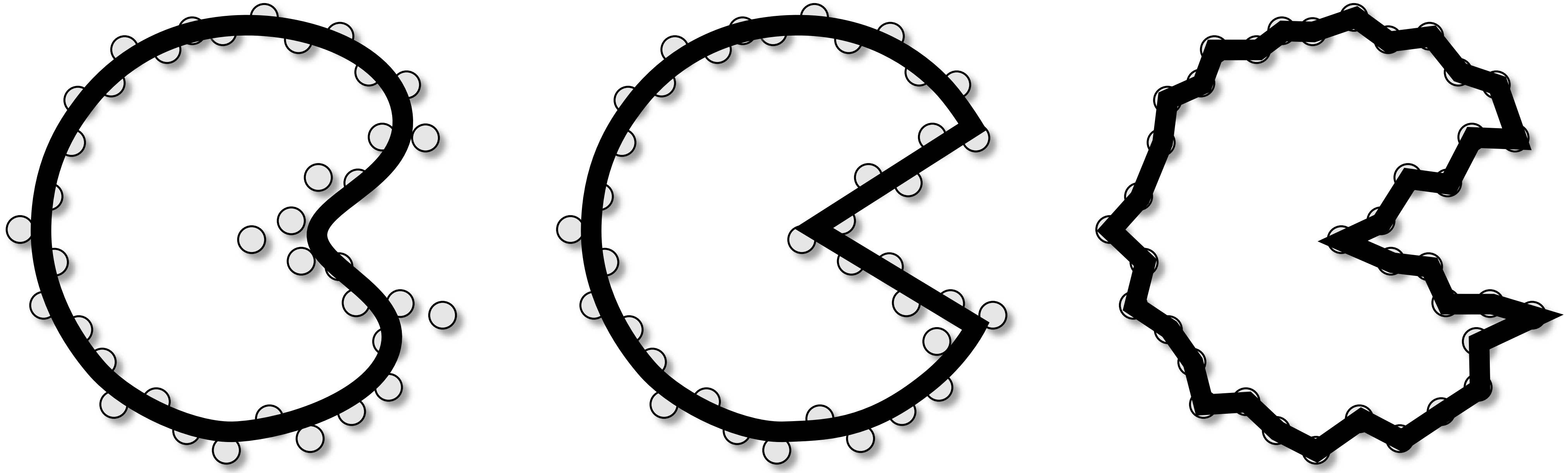


Ill-posed Problem



Many candidate shapes for the reconstruction problem.

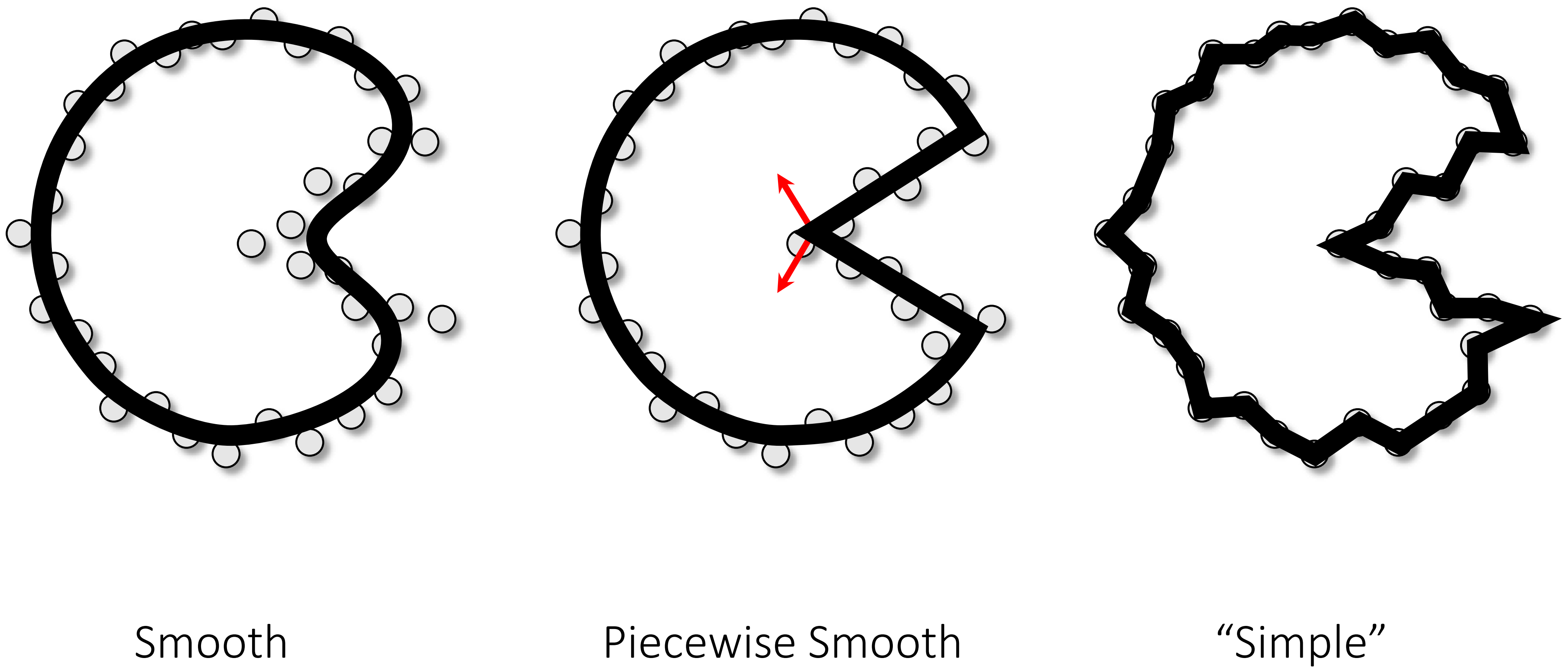
Ill-posed Problem



Many candidate shapes for the
reconstruction problem.

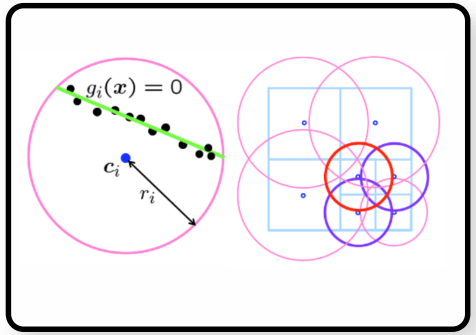
MAIN APPROACHES

Priors



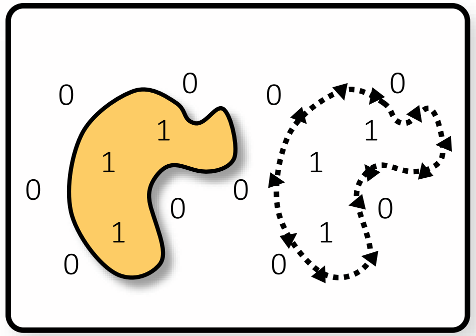
Surface Smoothness Priors

Local Smoothness



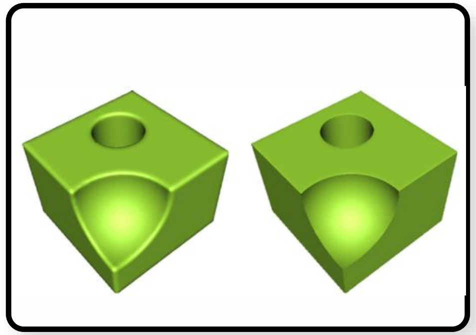
Local fitting
No control away from data
Solution by interpolation

Global Smoothness



Global: linear, eigen, graph cut, …
Robustness to missing data

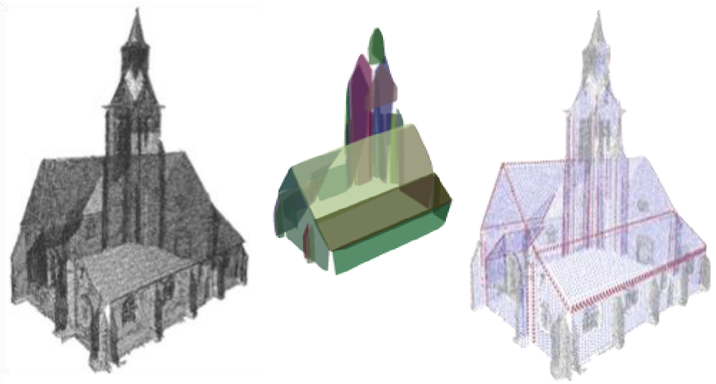
Piecewise Smoothness



Sharp near features
Smooth away from features

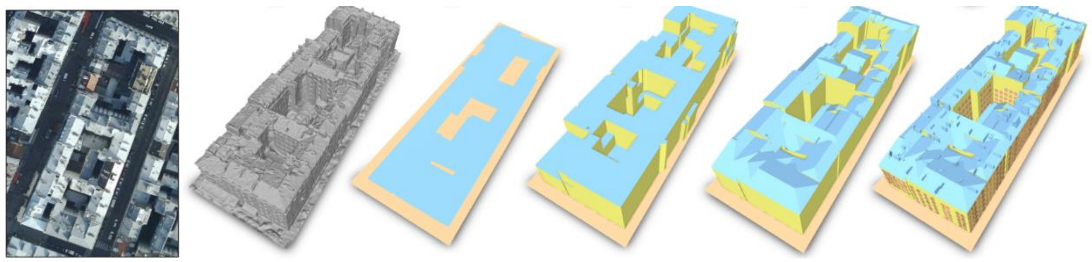
Domain-Specific Priors

Surface Reconstruction
by Point Set Structuring



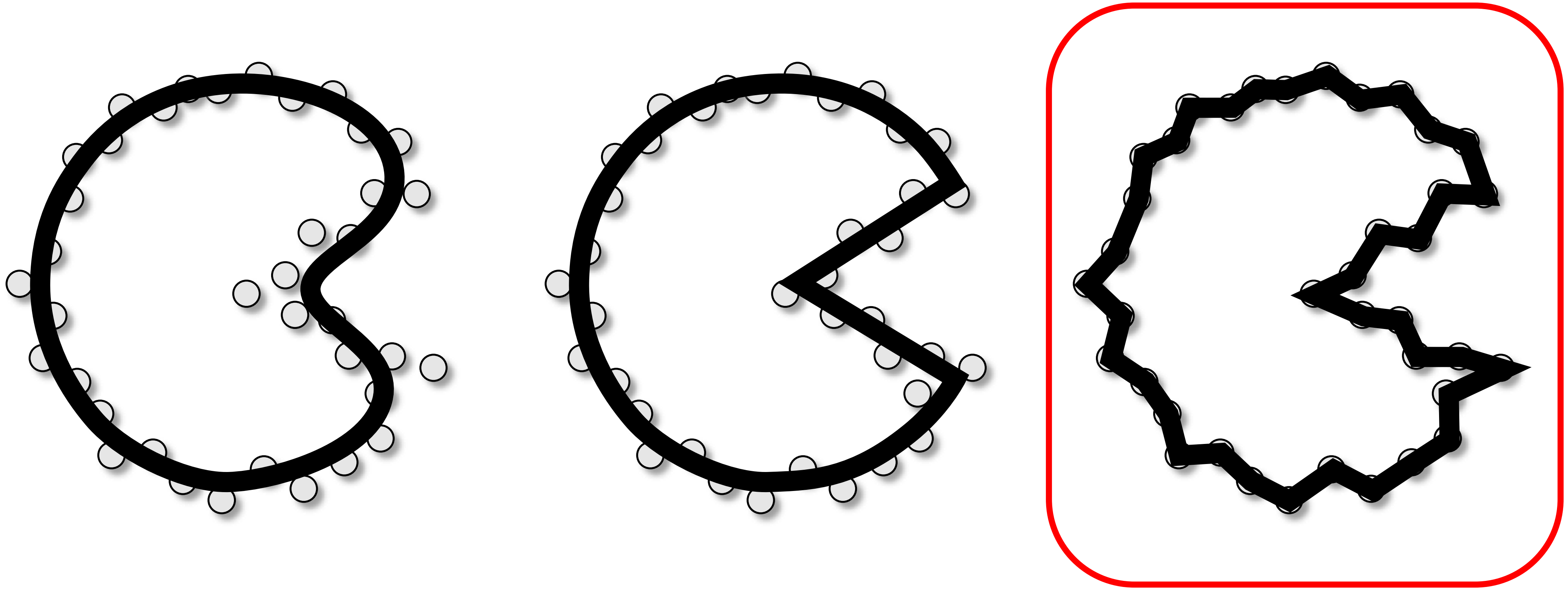
[**Lafarge - A**. EUROGRAPHICS 2013]

Reconstruction of Urban Scenes



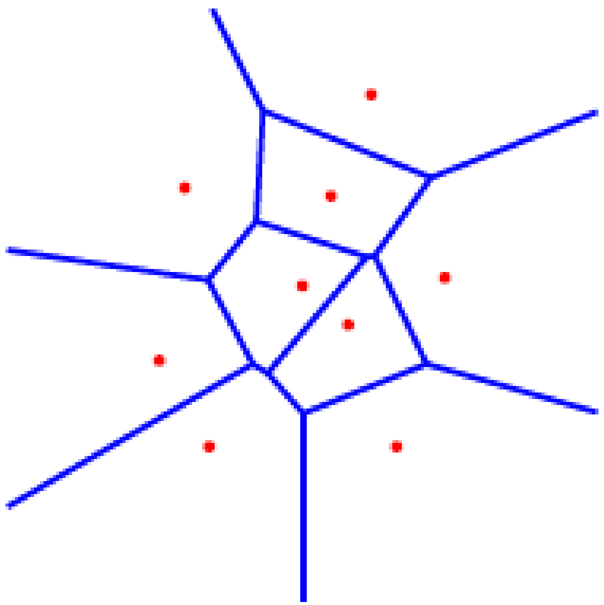
[**Verdie, Lafarge - A**. ACM Transactions on Graphics 2015]

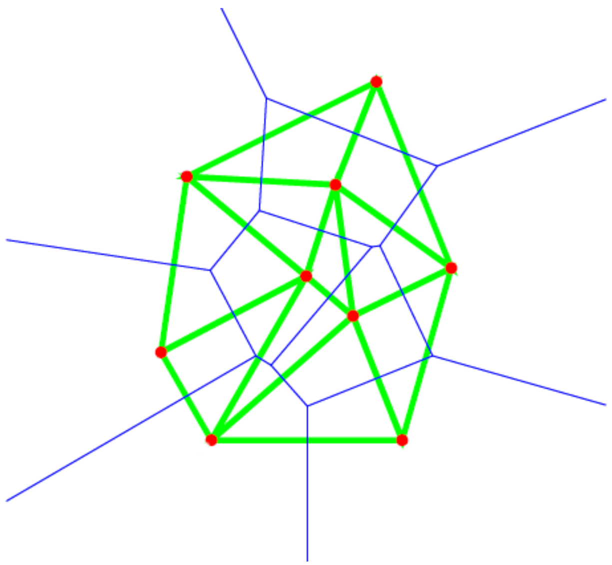
Priors



Smooth | Piecewise Smooth | “Simple”

Voronoi Diagram & Delaunay Triangulation





Let $\mathcal{E} = \{\mathbf{p_1}, \ldots, \mathbf{p_n}\}$ be a set of points (so-called sites) in $\mathbb{R}^d$. We associate to each site $\mathbf{p_i}$ its Voronoi region $V(\mathbf{p_i})$ such that:

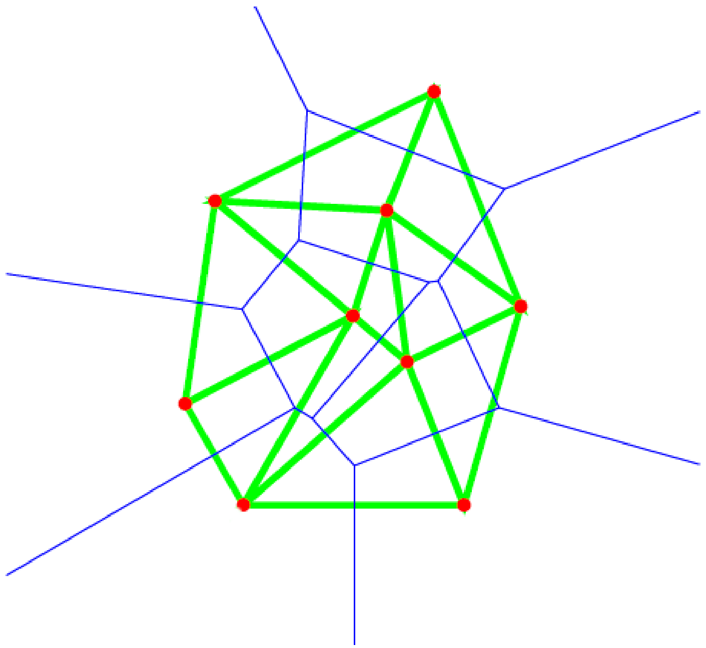
$$V(\mathbf{p_i}) = \{\mathbf{x} \in \mathbb{R}^d : \|\mathbf{x} - \mathbf{p_i}\| \leq \|\mathbf{x} - \mathbf{p_j}\|, \forall j \leq n\}.$$

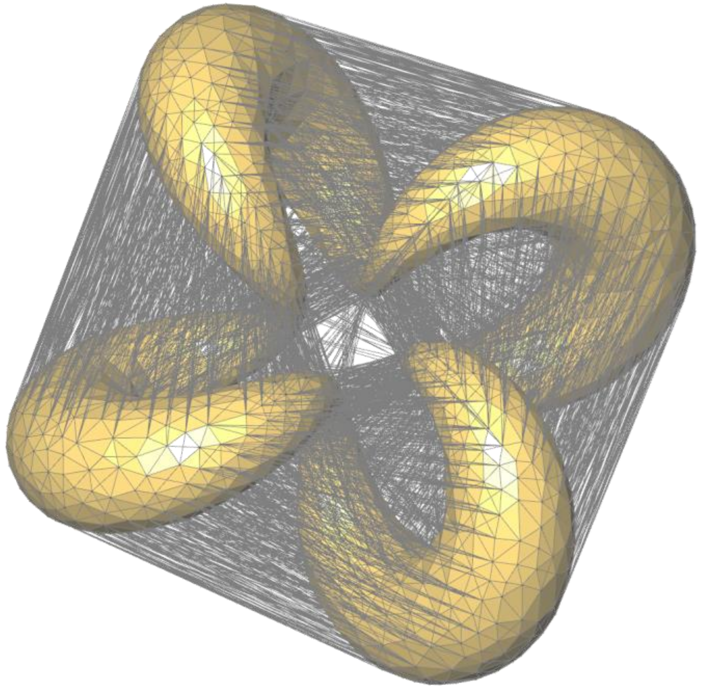
Delaunay triangulation: simplicial complex such that k+1 points form a Delaunay simplex if their Voronoi cells have nonempty intersection.

DEMO

Delaunay-based Reconstruction

Key idea: assuming dense enough sampling, reconstructed triangles are Delaunay triangles.





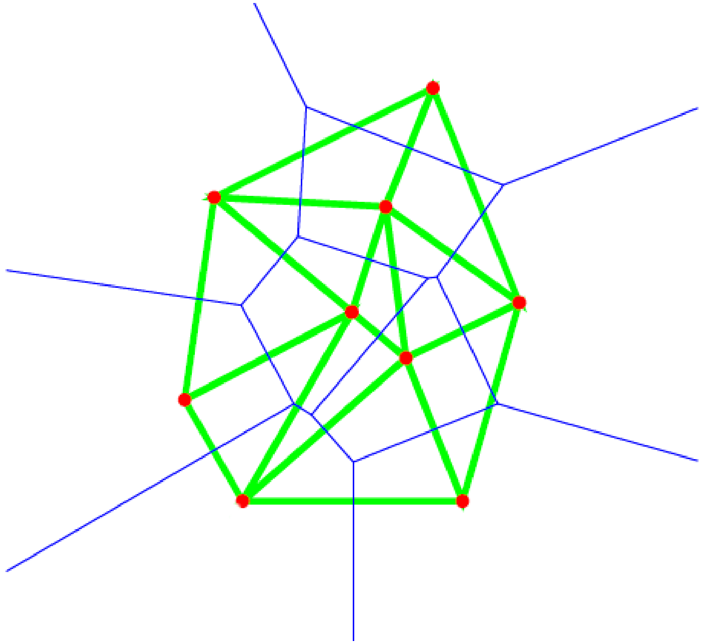
Delaunay-based Reconstruction

Key idea: assuming <u>dense enough</u> sampling, reconstructed triangles are Delaunay triangles.

?

First define

- Medial axis
- Local feature size
- Epsilon-sampling



Medial Axis (2D)

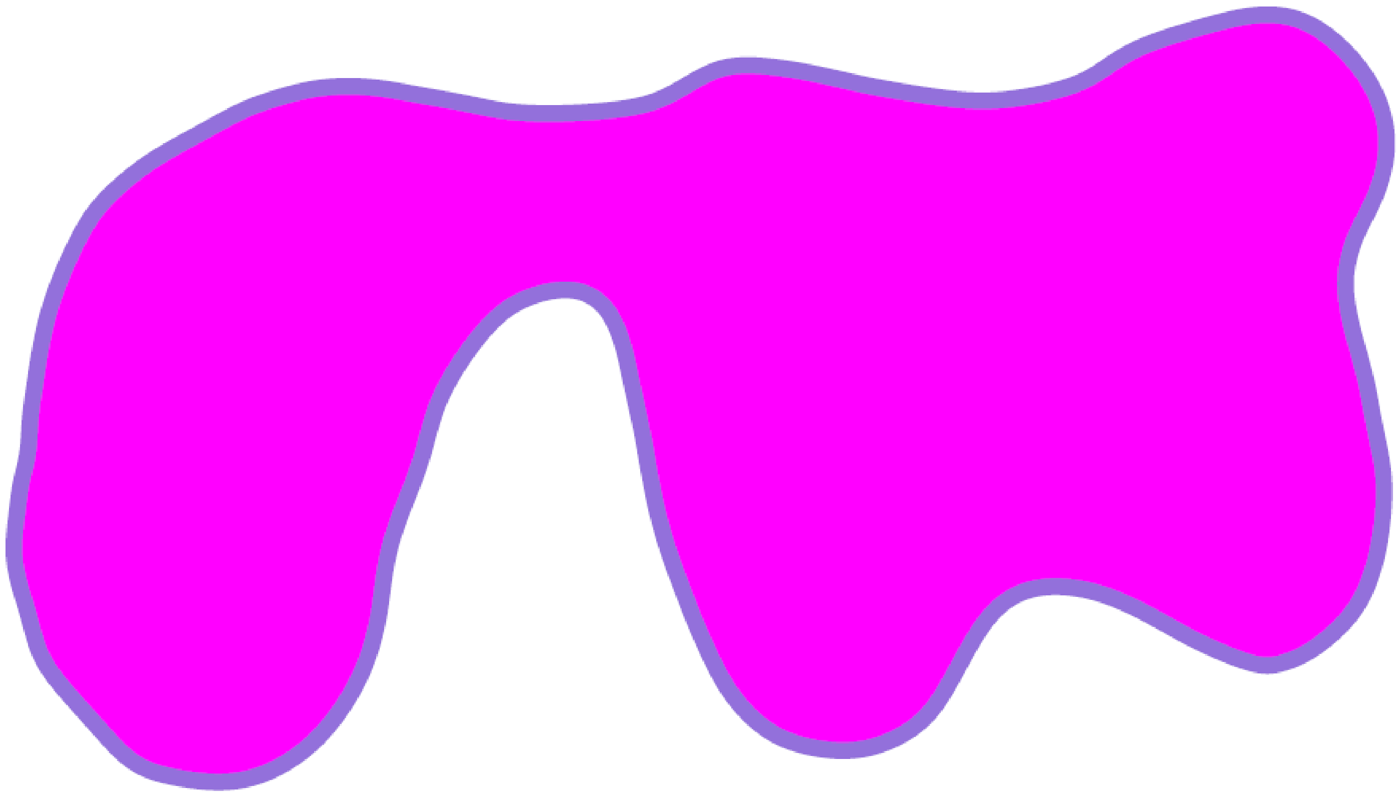


Figure from O. Devillers

Medial Axis

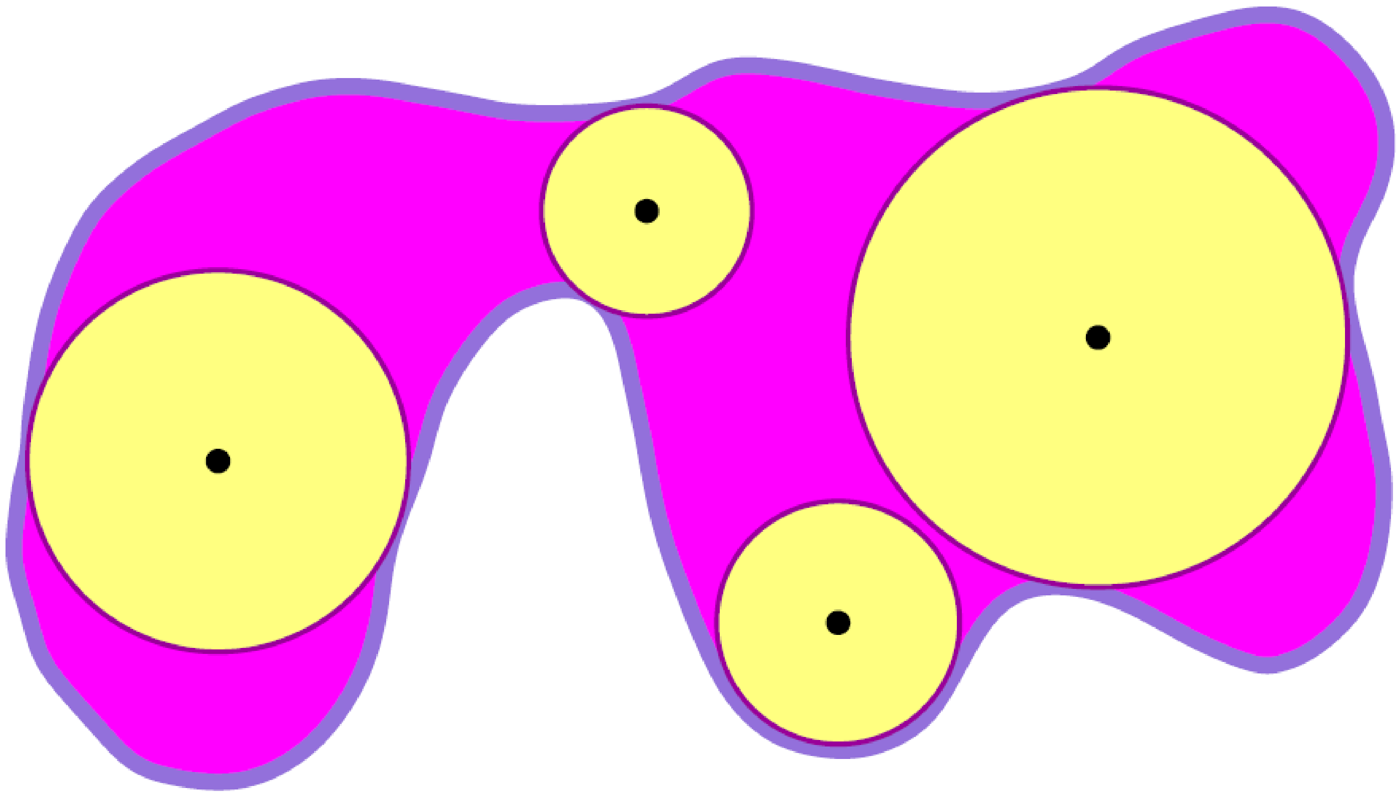


Figure from O. Devillers

Medial Axis

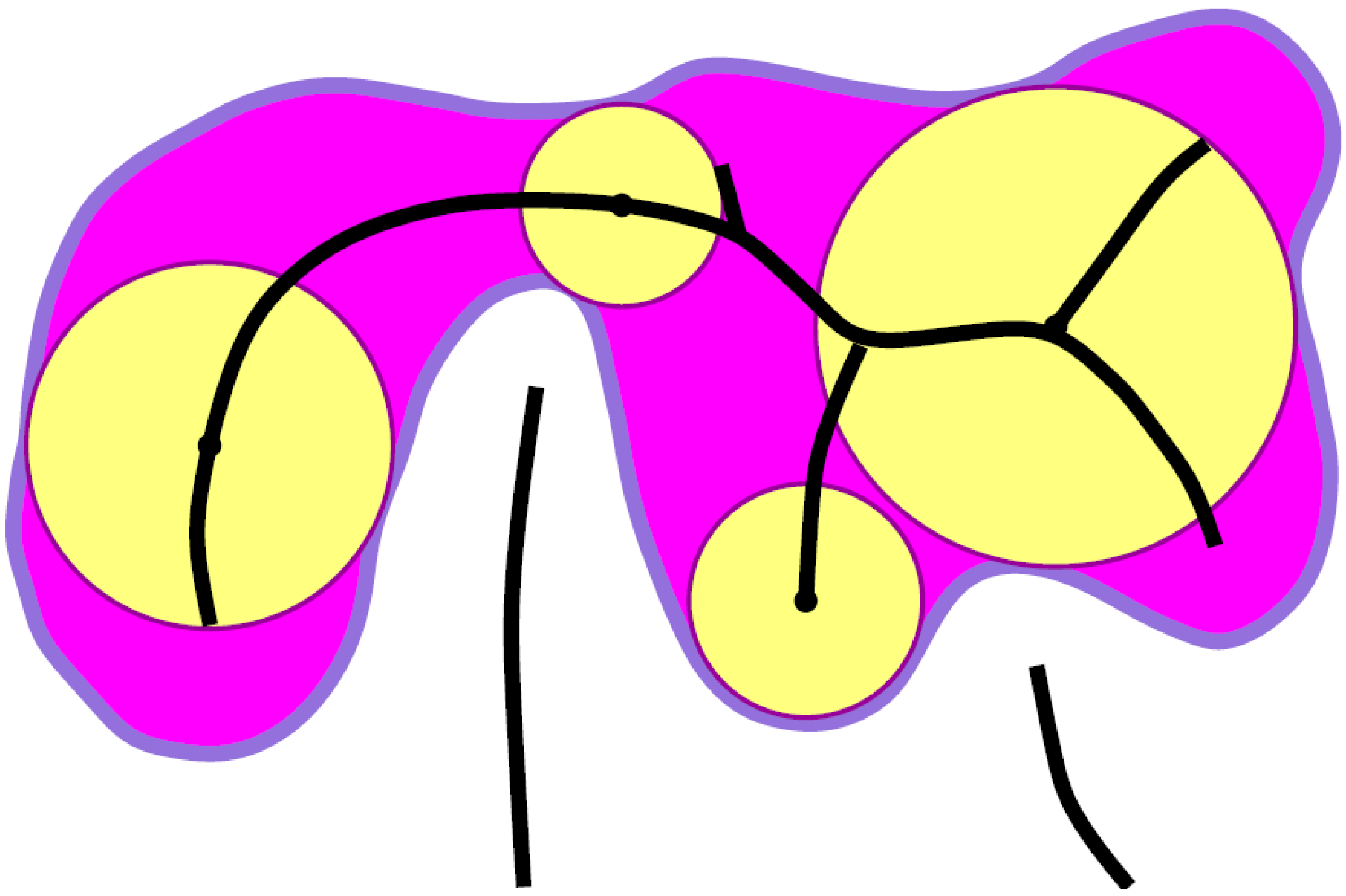
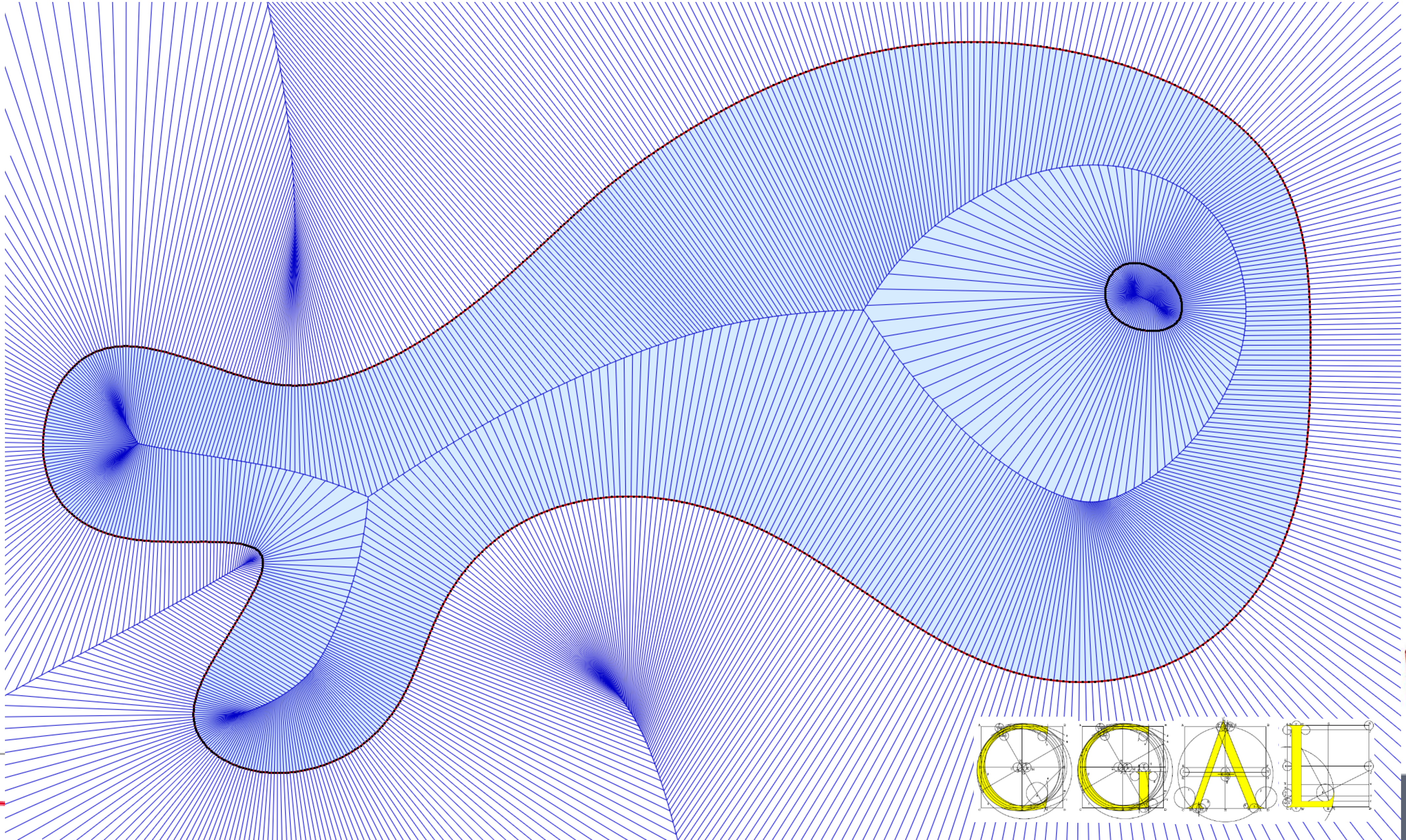


Figure from O. Devillers

Voronoi Diagram & Medial Axis



Local Feature Size

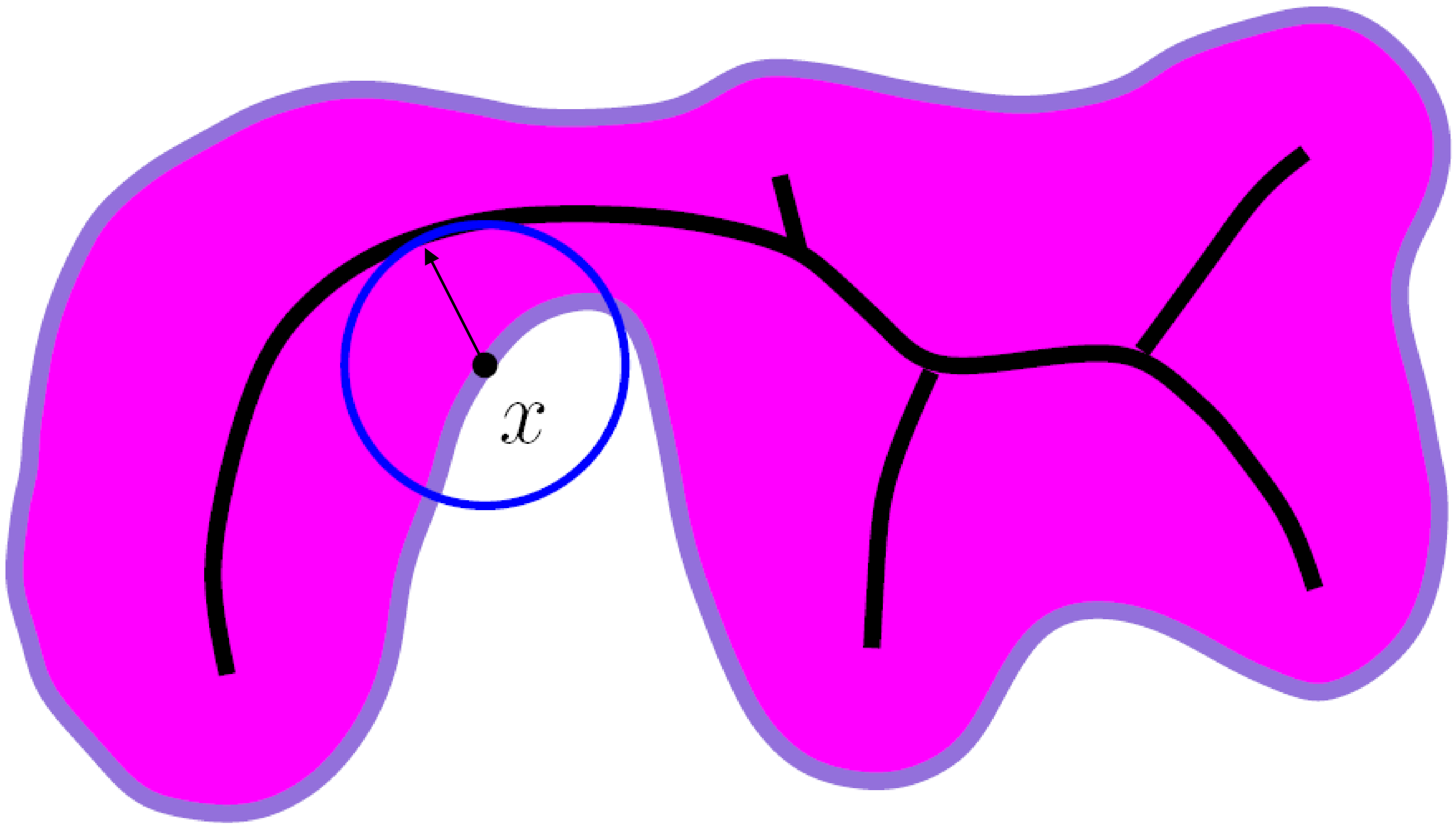


Figure from O. Devillers

Epsilon-Sampling

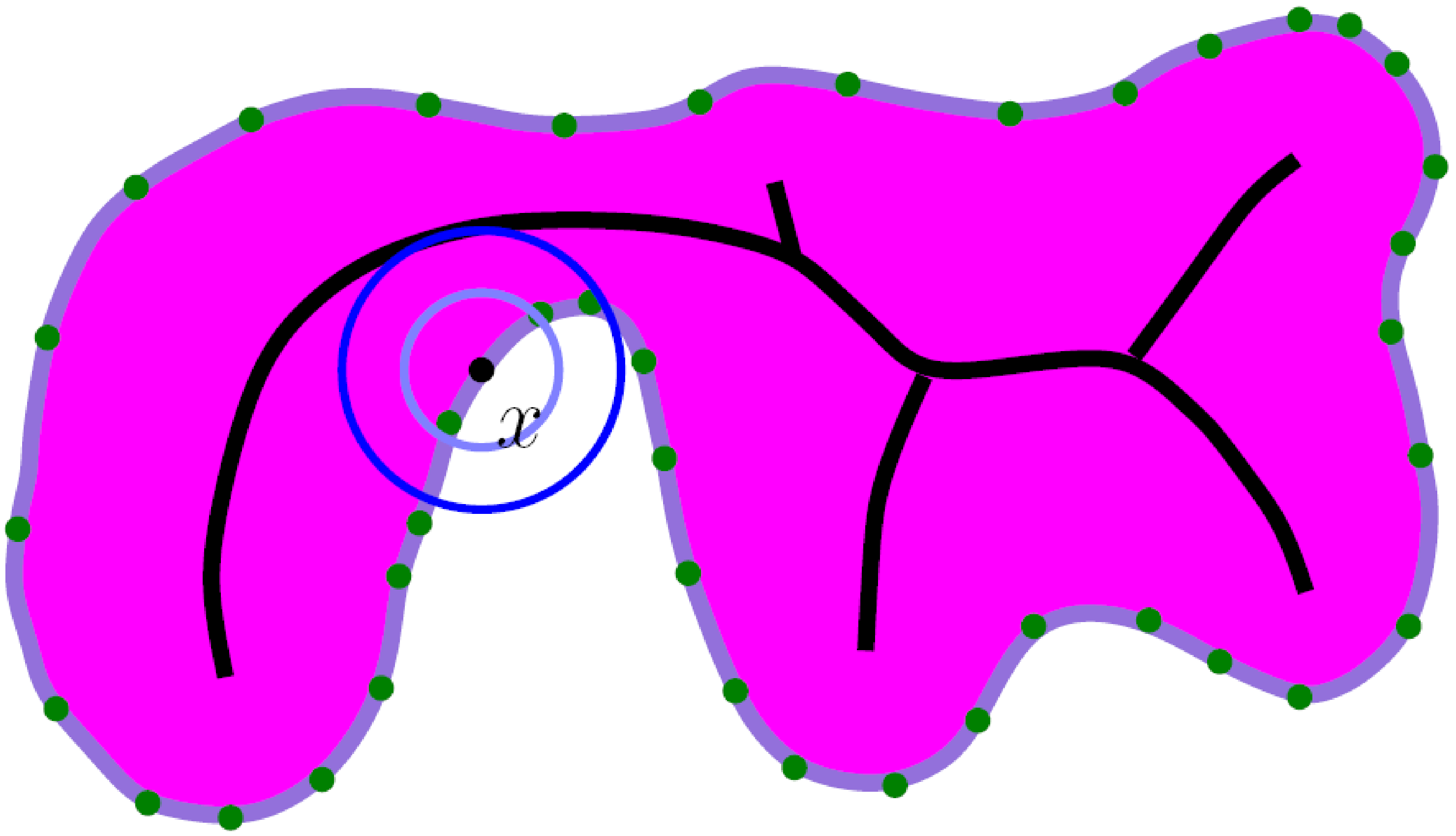


Figure from O. Devillers

Crust Algorithm [Amenta et al.]

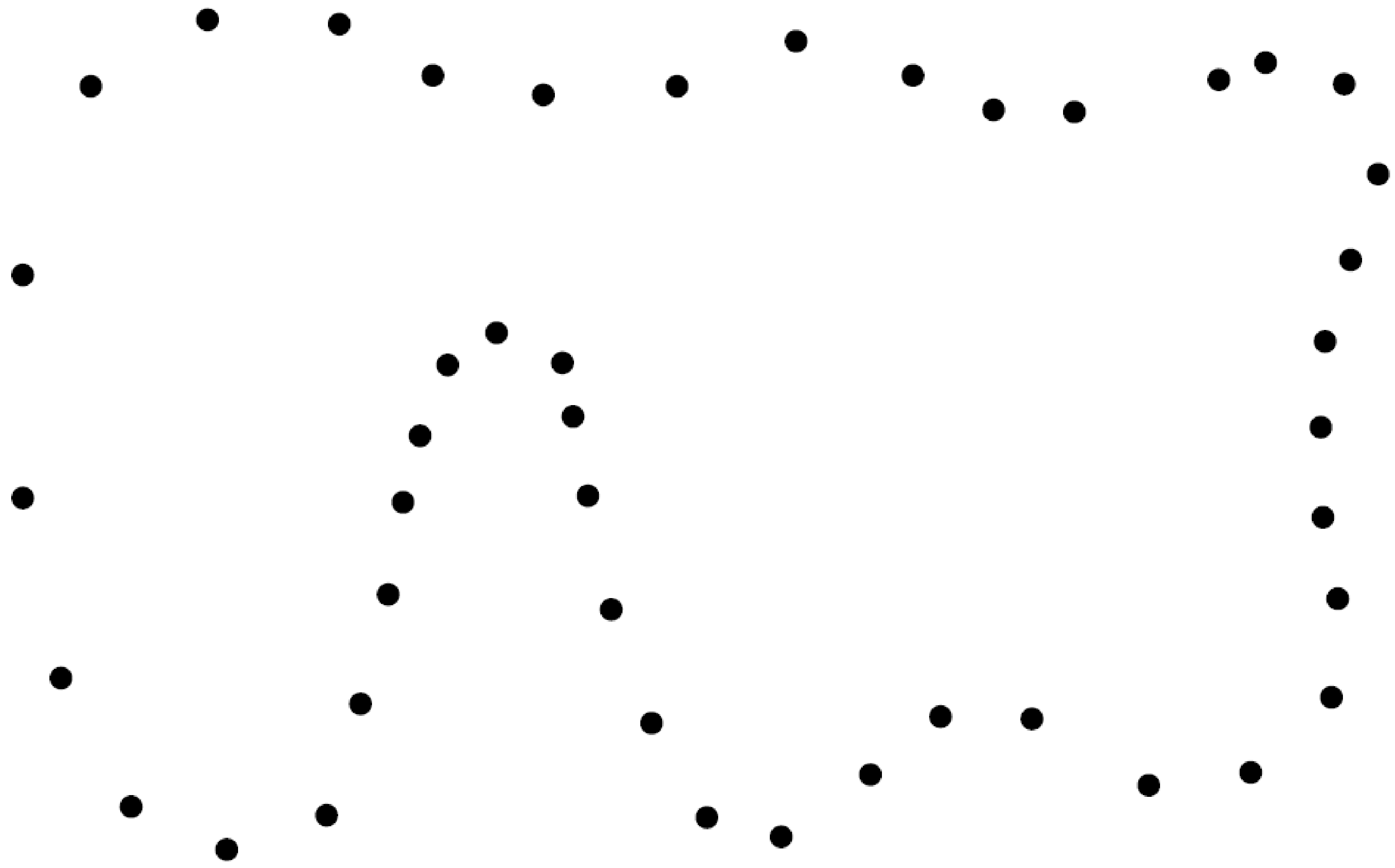


Figure from O. Devillers

Delaunay Triangulation

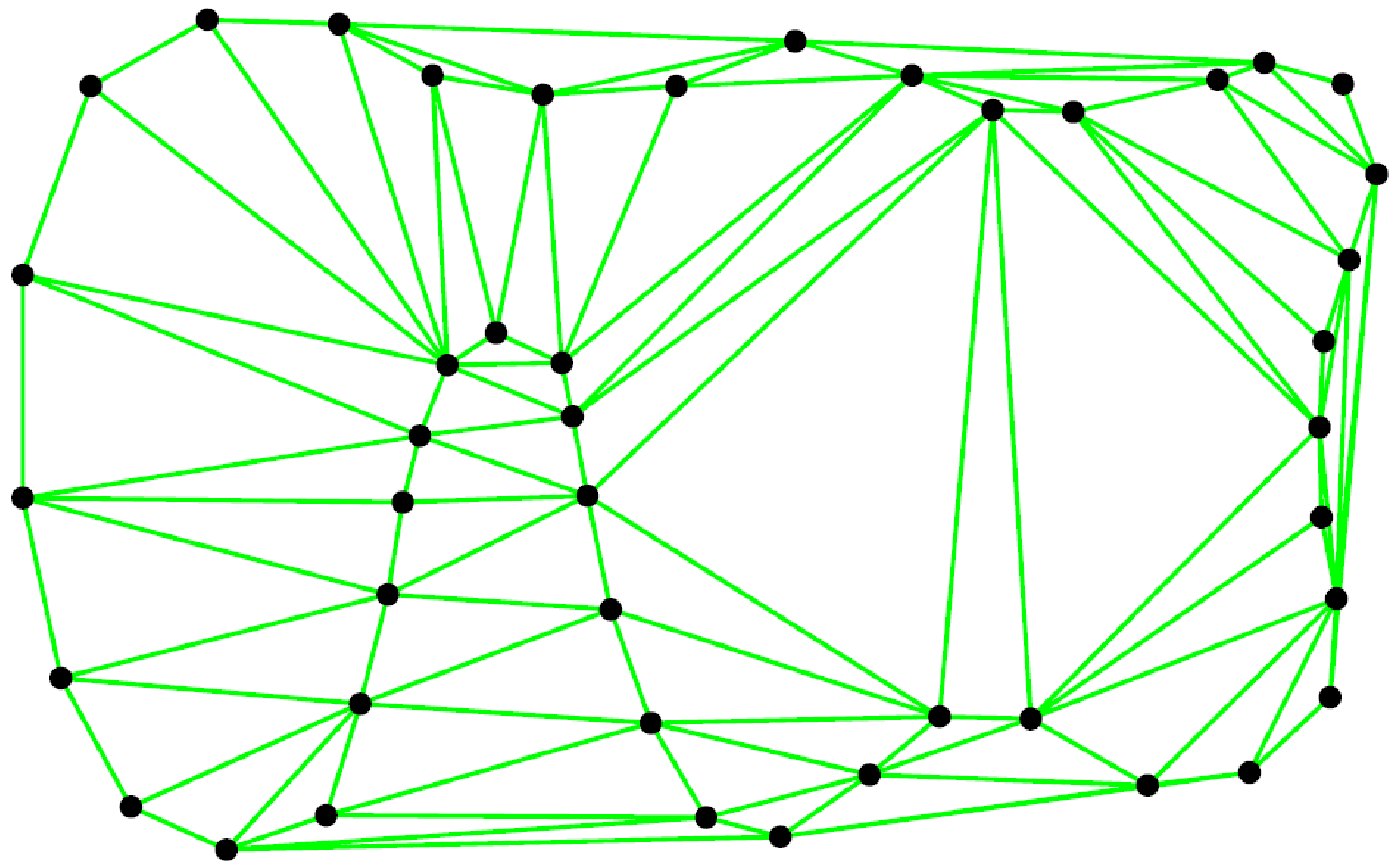


Figure from O. Devillers

Delaunay Triangulation & Voronoi Diagram

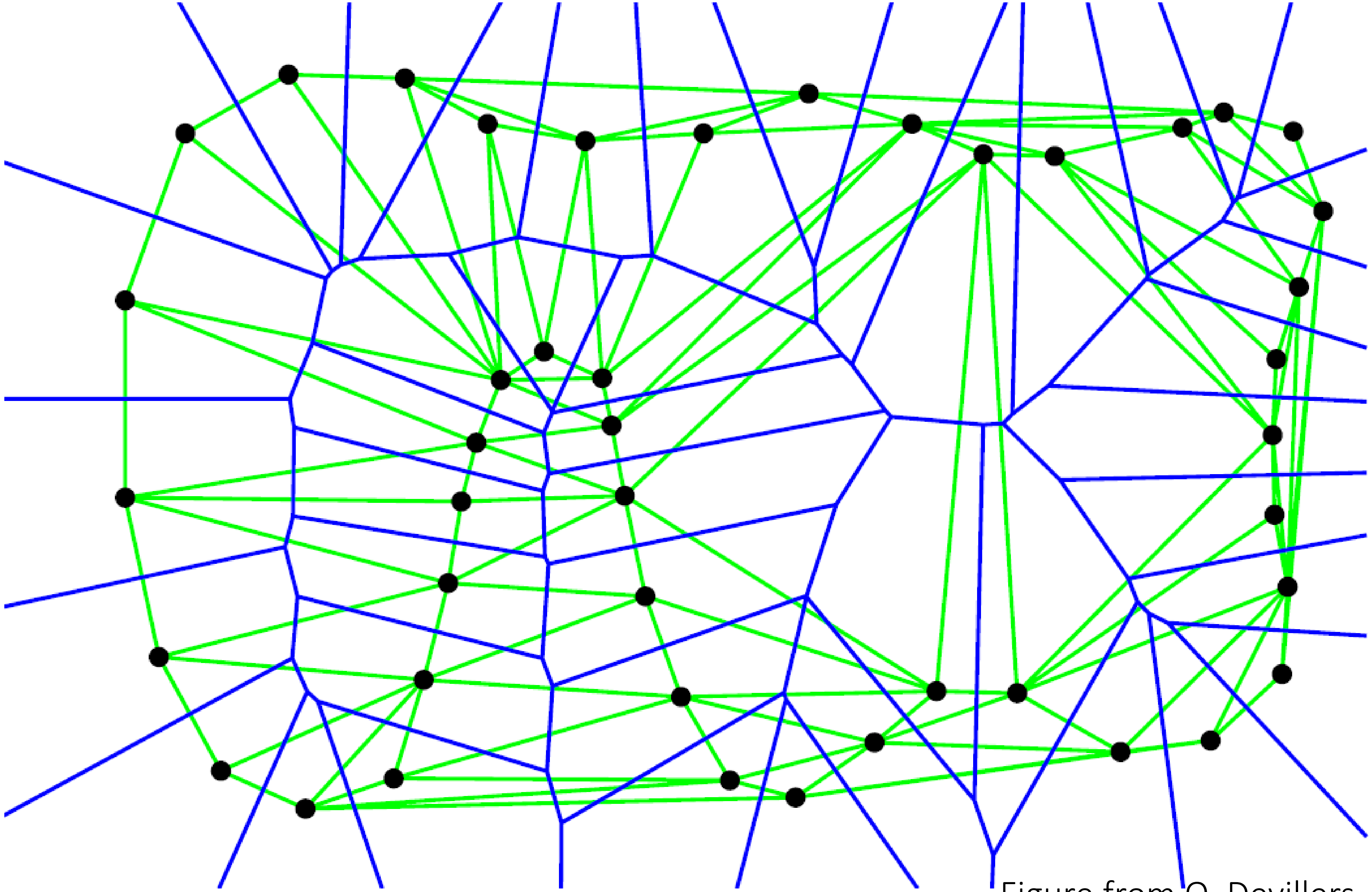


Figure from O. Devillers

Voronoi Vertices

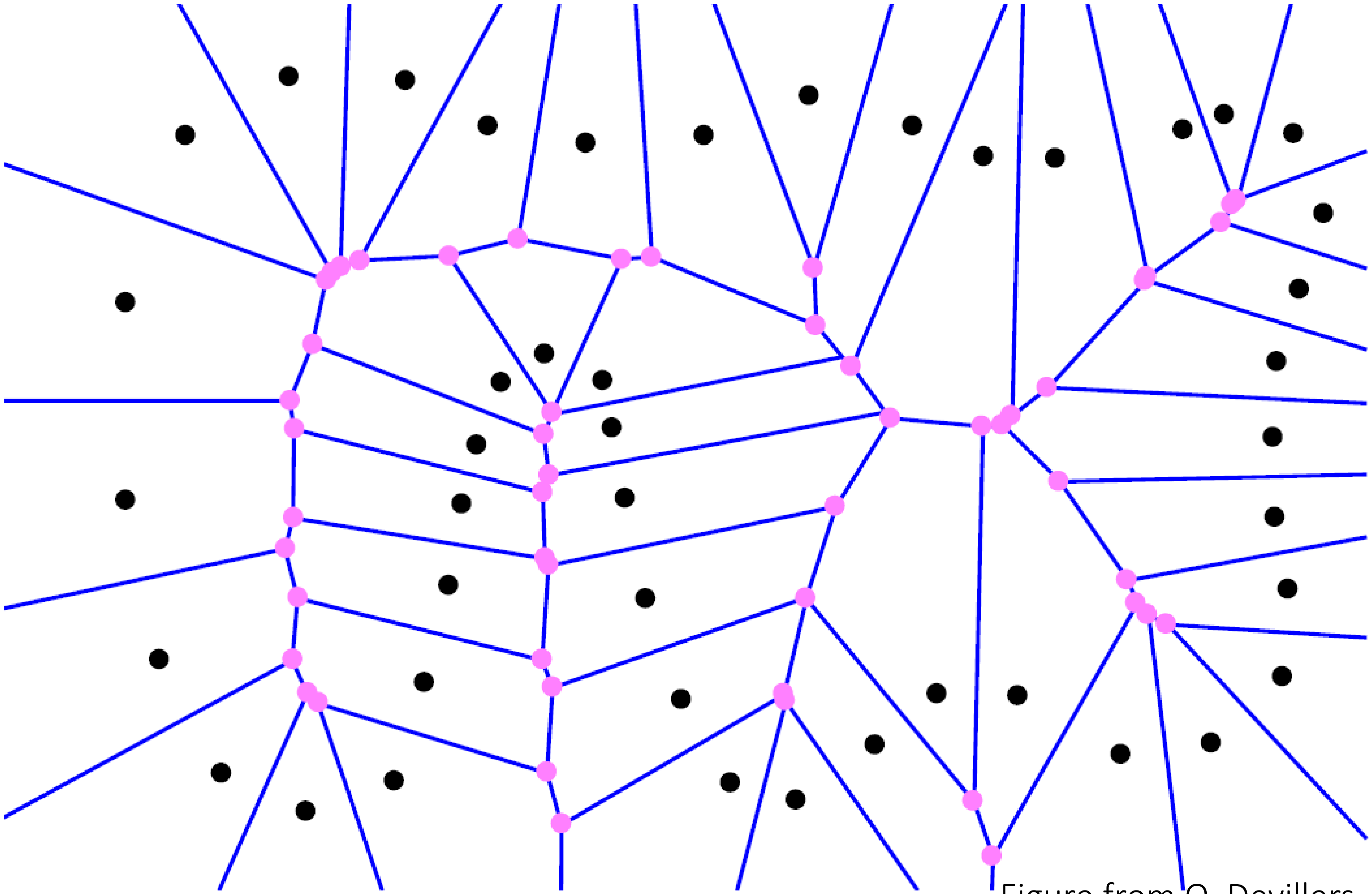


Figure from O. Devillers

Augmented Delaunay Triangulation

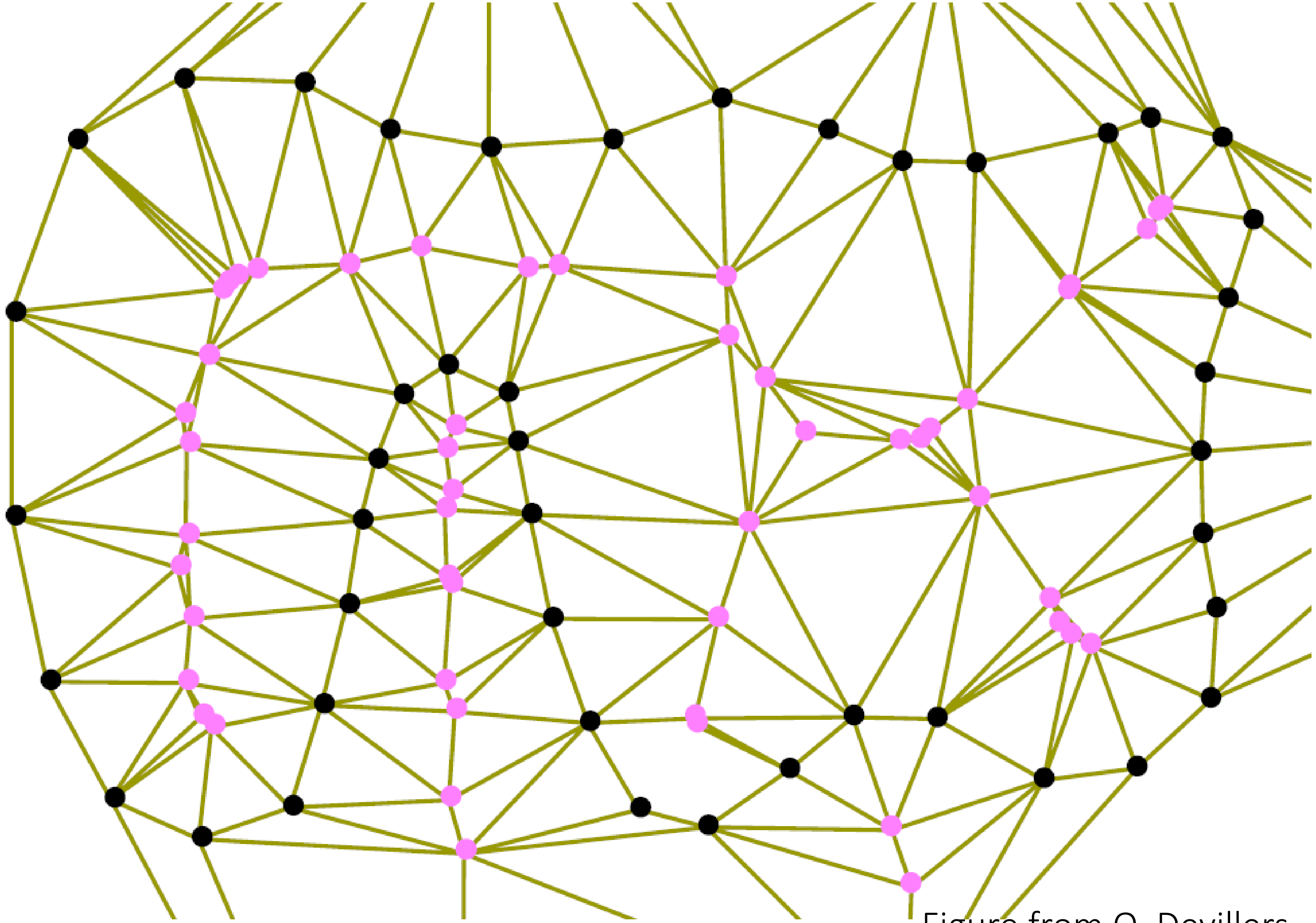


Figure from O. Devillers

Crust

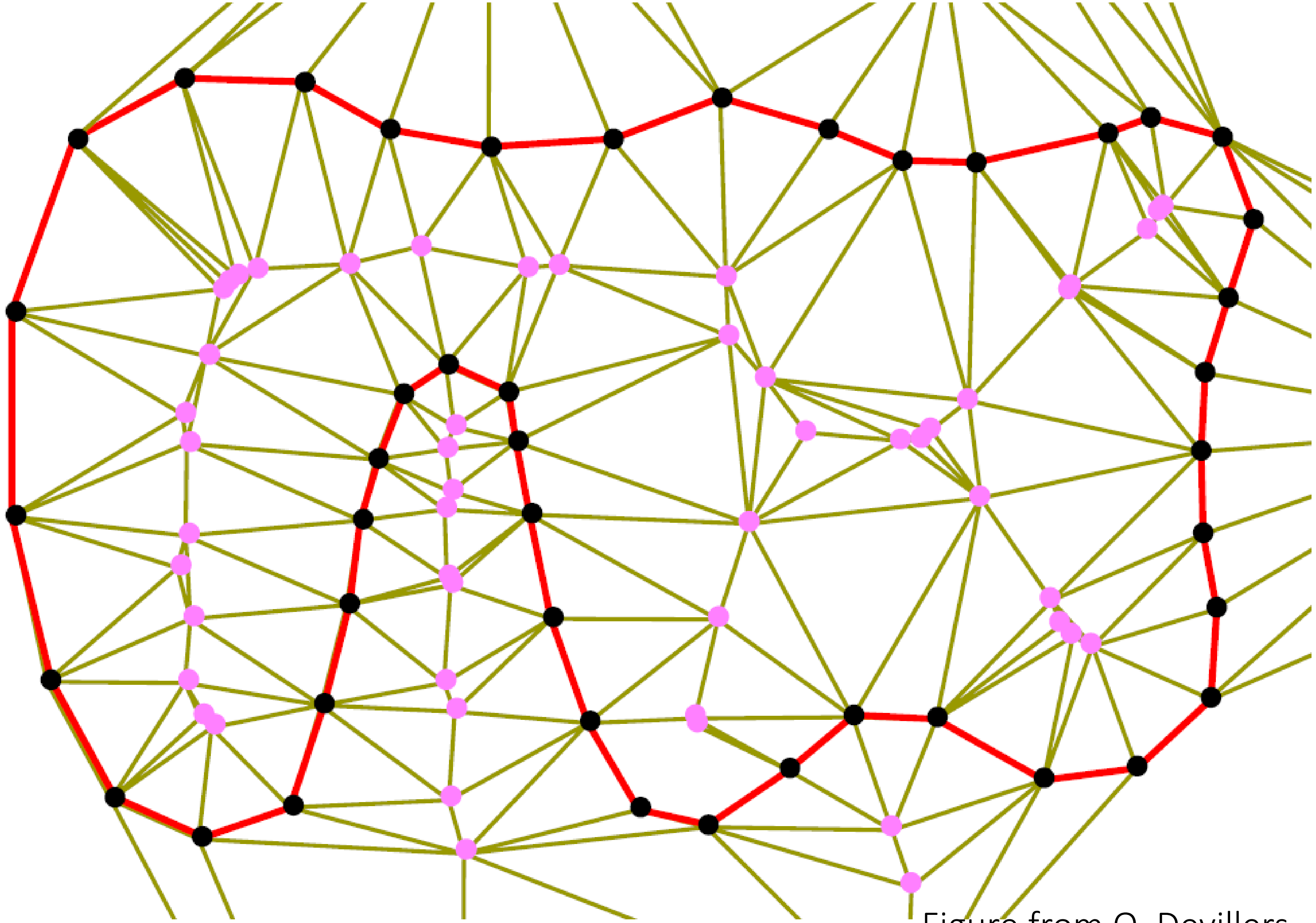


Figure from O. Devillers

Crust

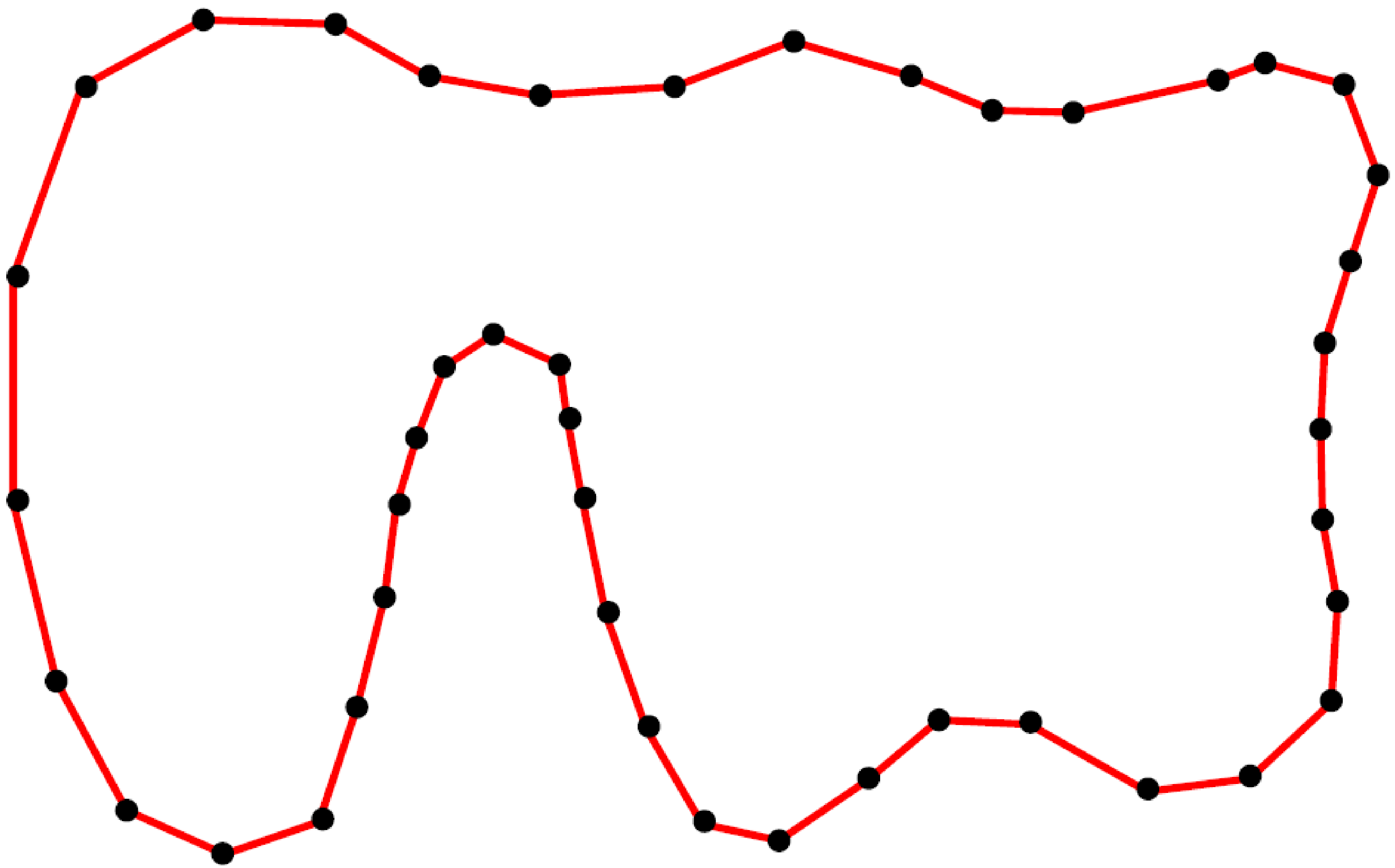


Figure from O. Devillers

Delaunay-based Reconstruction

Several Delaunay algorithms are provably correct

- Boissonnat
- Amenta, Bern, Eppstein
- Attali
- Dey, Goswami
- Cazals & Giesen
- …

Dey. *Curve and surface reconstruction: algorithms with mathematical analysis.*

Delaunay-based Reconstruction

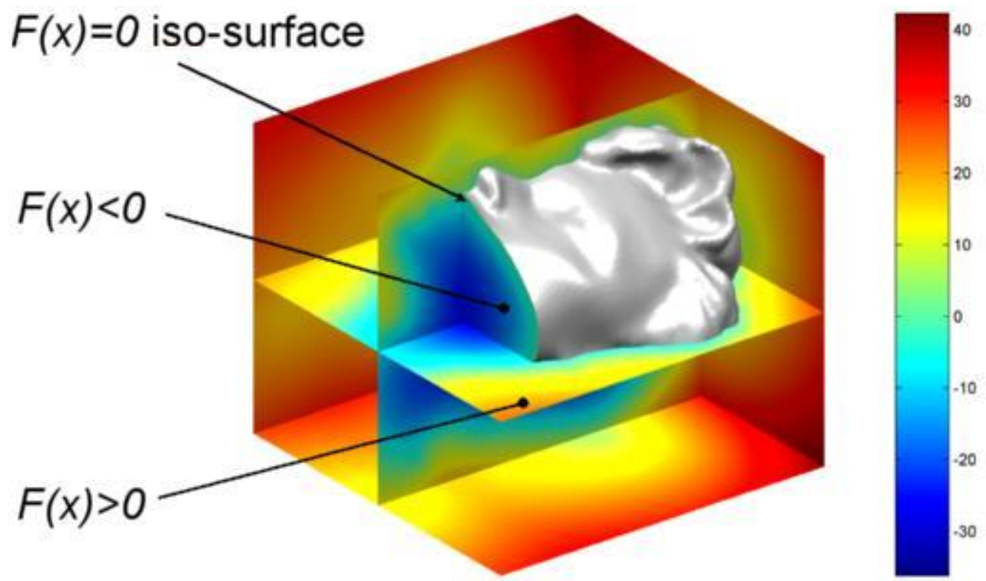
Several Delaunay algorithms are **provably correct**… in the absence of noise and undersampling.

perfect data ?

Delaunay-based Reconstruction

Several Delaunay algorithms are **provably correct**… in the absence of noise and undersampling.

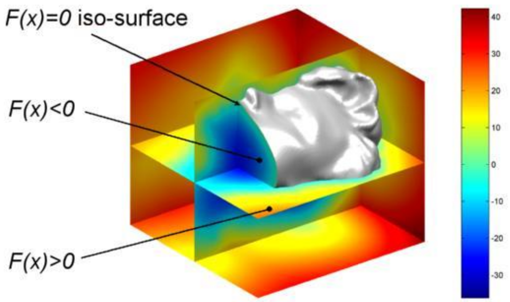
Motivates reconstruction by fitting approximating implicit surfaces



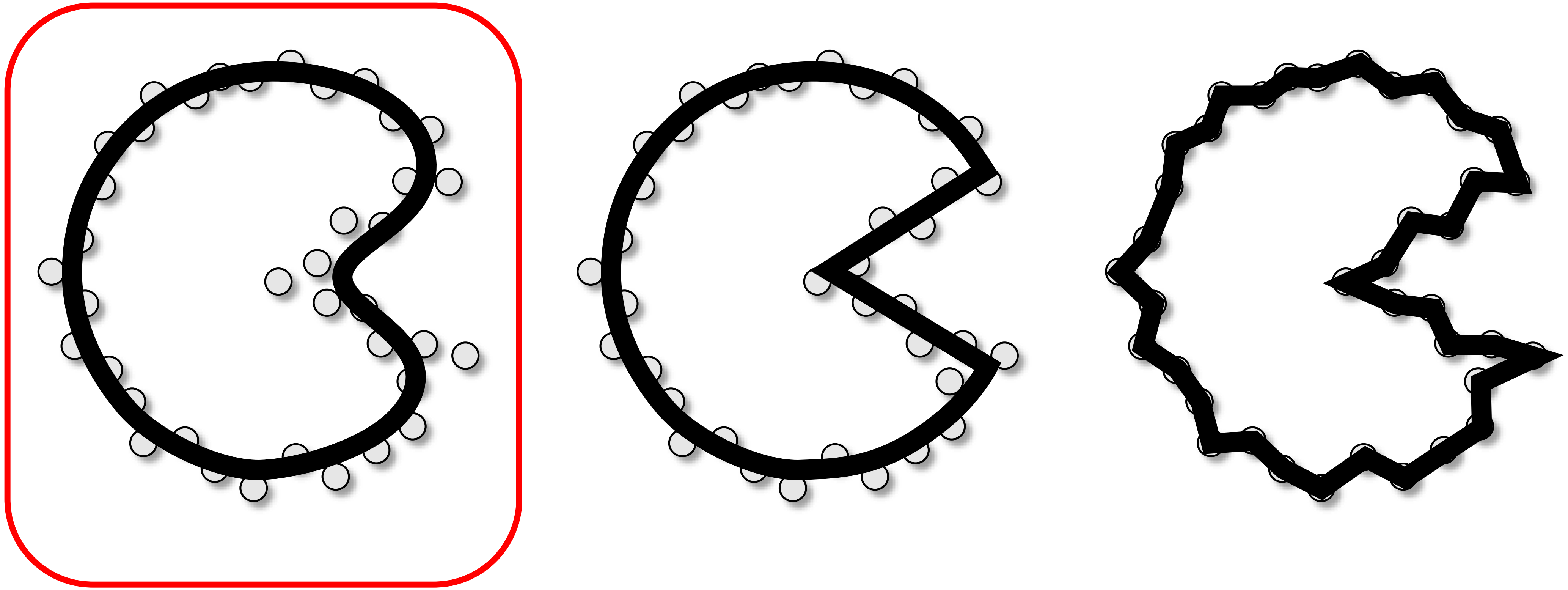
Implicit Surface Approaches

Solve for scalar function (IR^3 -> IR) defined as approximate

- Signed distance to inferred surface *S*
  [Hoppe 92, Carr et al. 01, Belyaev et al. 02]
- Unsigned distance to *S*
  [Hornung-Kobbelt 06]
- Indicator (characteristic) function of inferred solid
  [Kahzdan et al. 06]



Priors



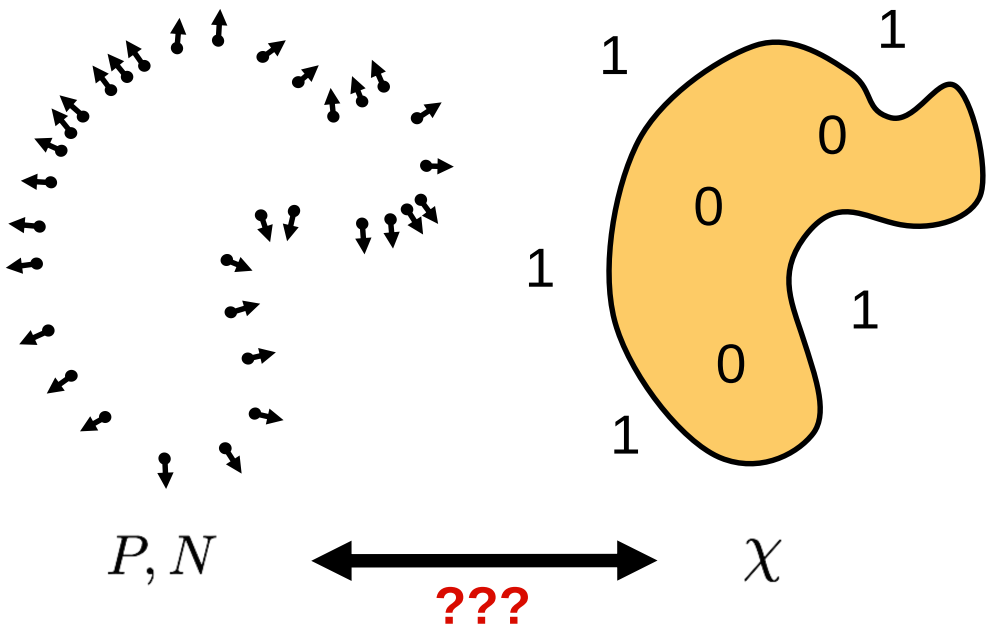
Smooth

Piecewise Smooth

"Simple"

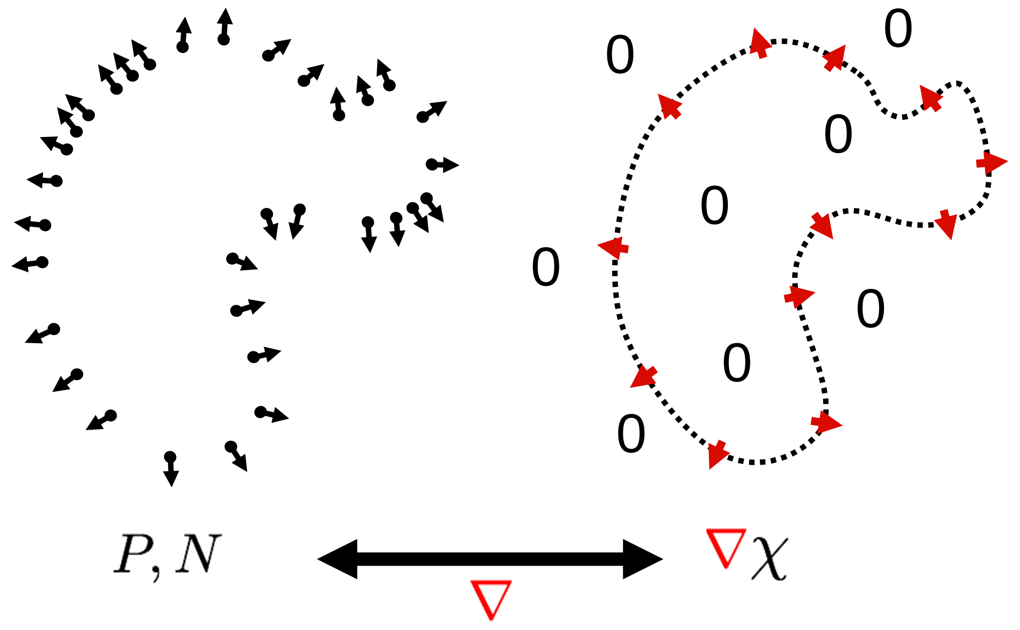
Indicator Function

Compute indicator function from oriented points (points + normals)



Poisson Surface Reconstruction

Compute indicator function from oriented points



splatted normals

$$\min_{\chi} \int \|\nabla\chi(\mathbf{x}) - \mathcal{N}(\mathbf{x})\|_2^2 \, dx$$

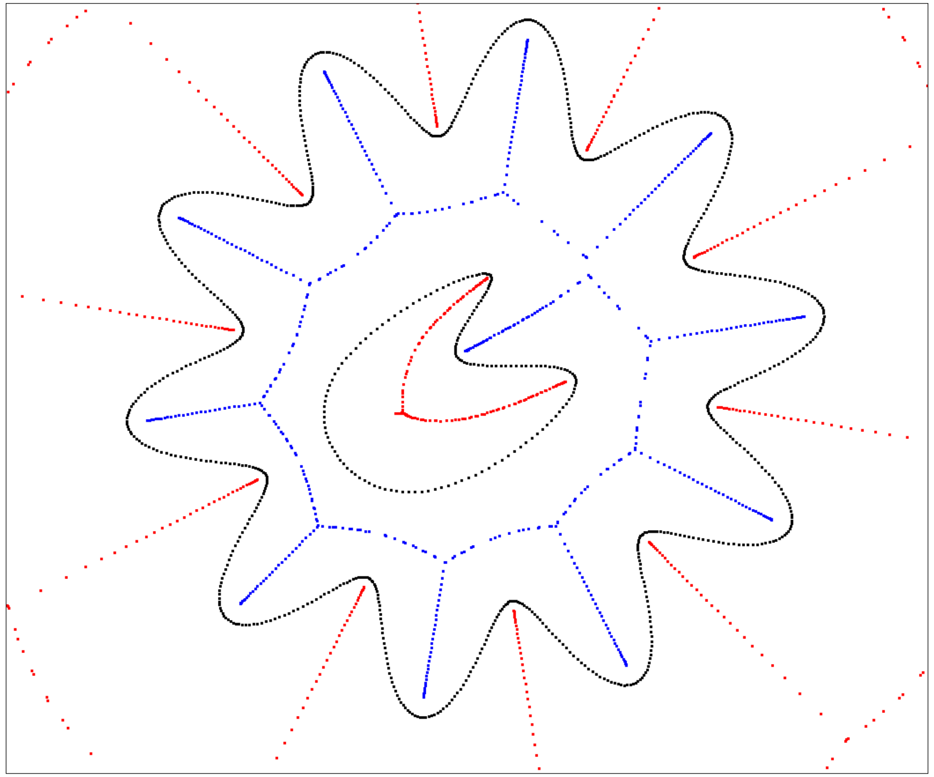
variational calculus

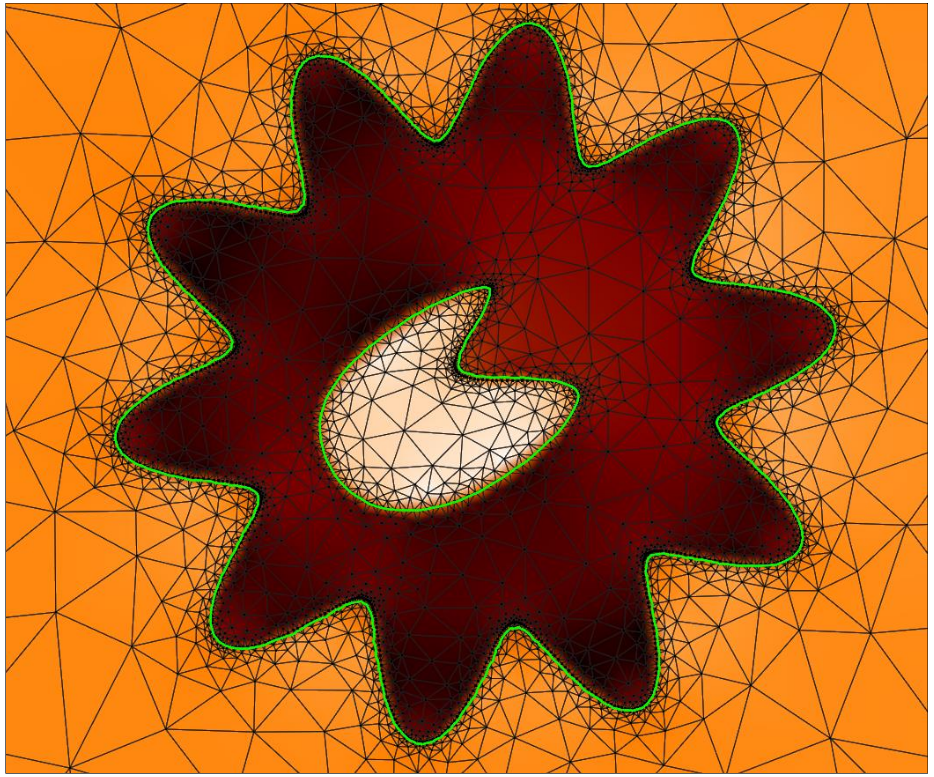
$$\Delta\chi = \nabla \cdot \mathcal{N}$$

sparse linear system

Poisson Surface Reconstruction.
Kazhdan, Bolitho, Hoppe.
EUROGRAPHICS Symposium on Geometry Processing 2006.

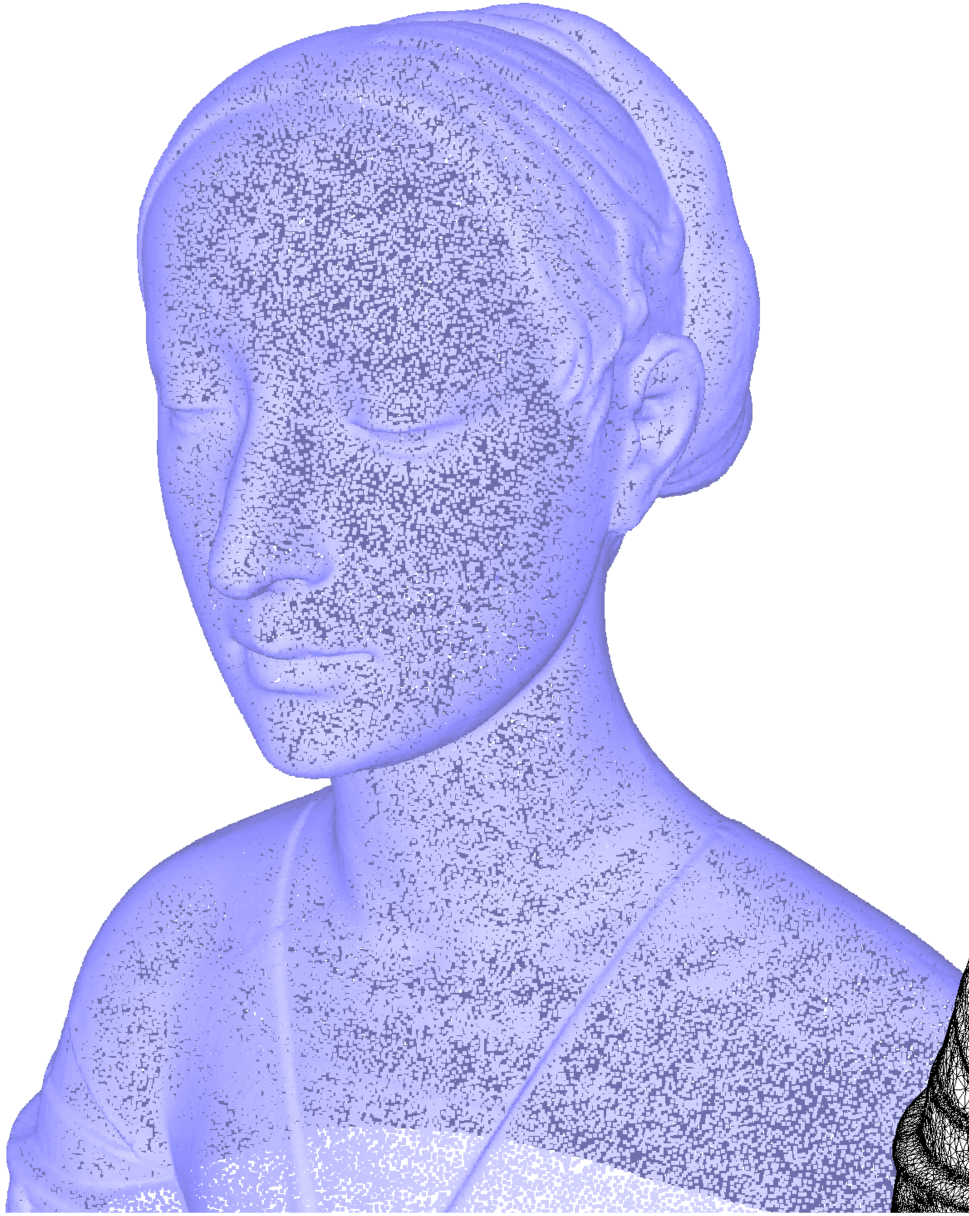
2D Poisson Reconstruction

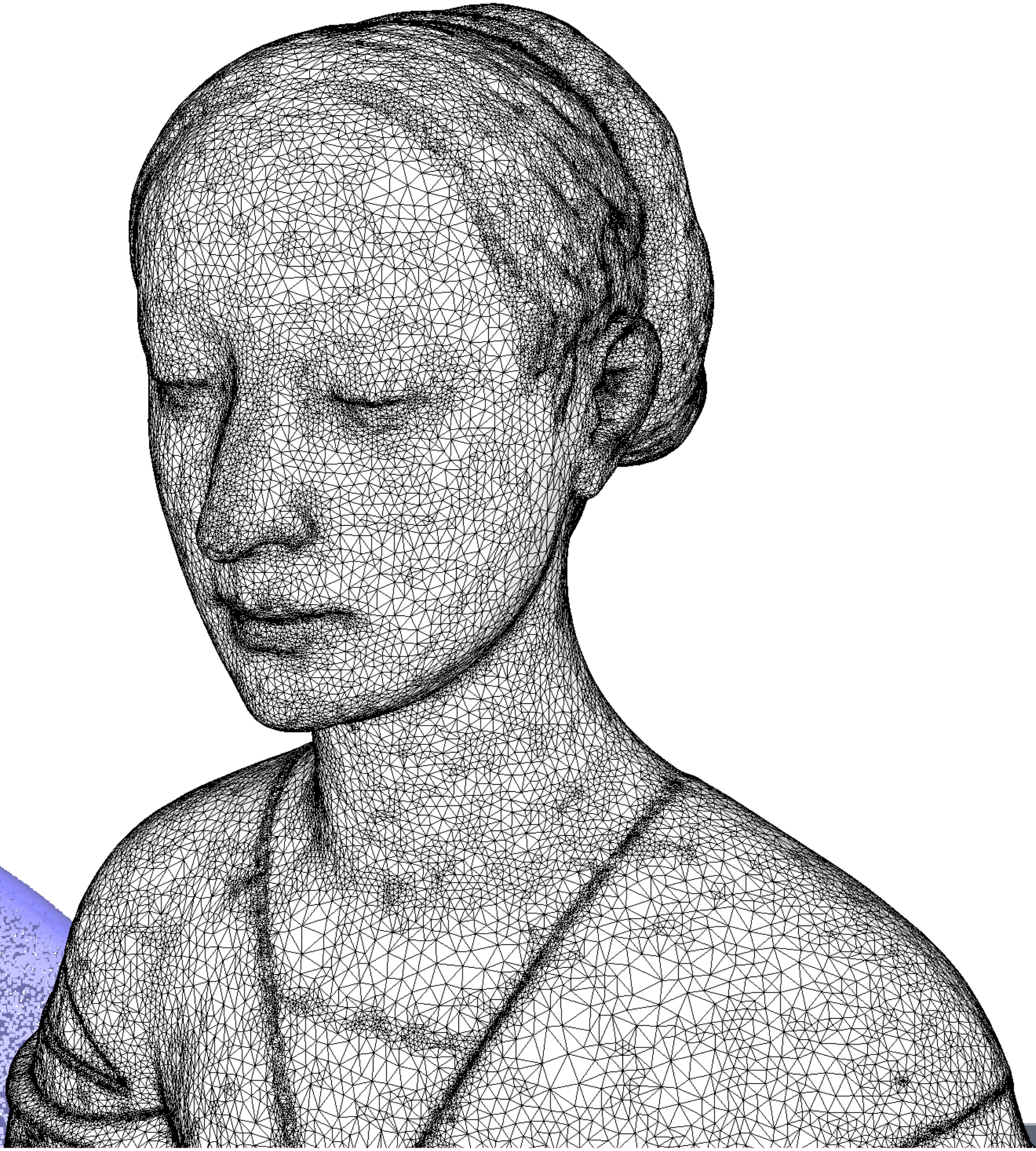




DEMO

3D Poisson Reconstruction



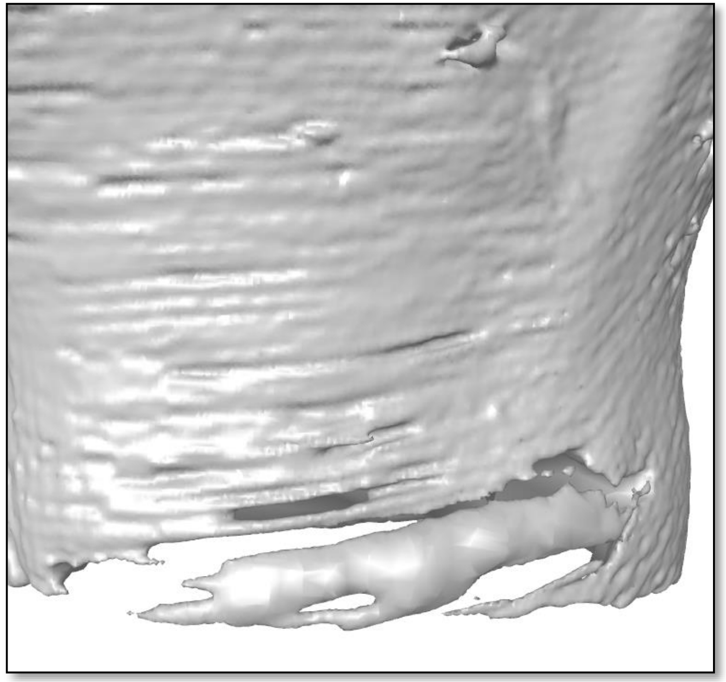


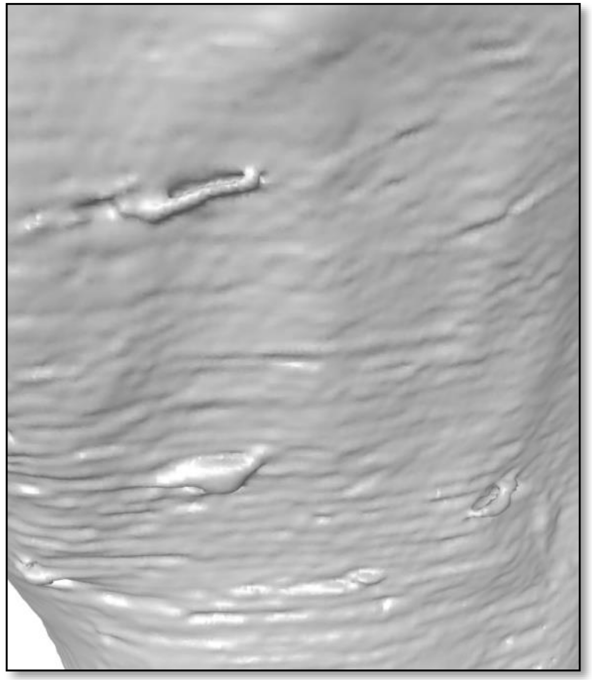
DEMO

Oriented point set (data from CNR Pisa)

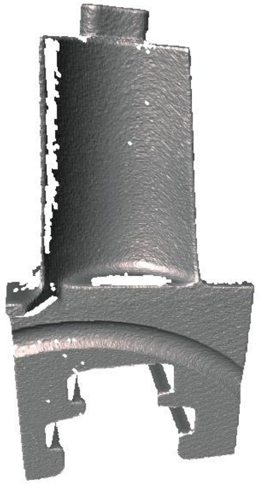
Reconstructed surface (via CGAL library)

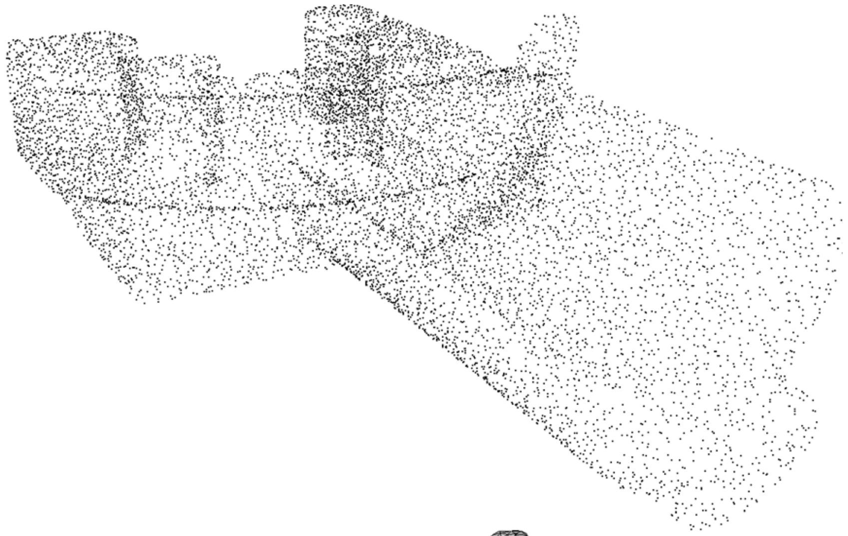
Failure Case 1

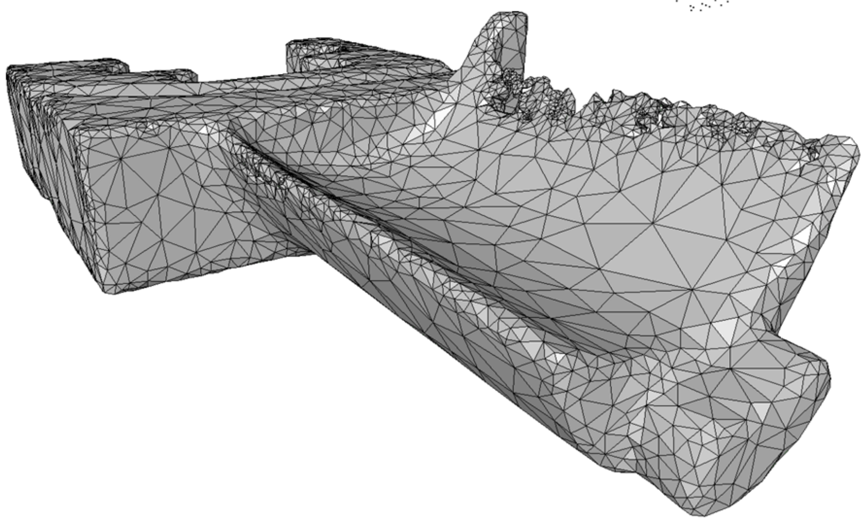


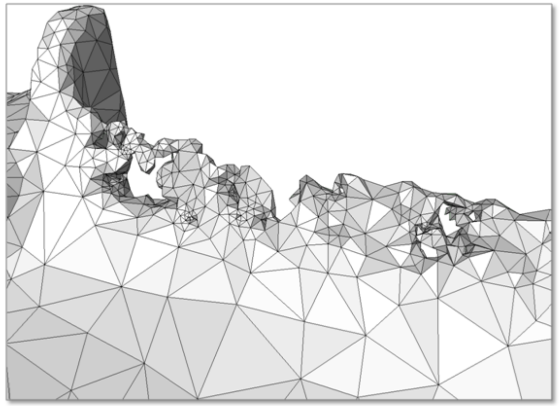


Failure Case 2



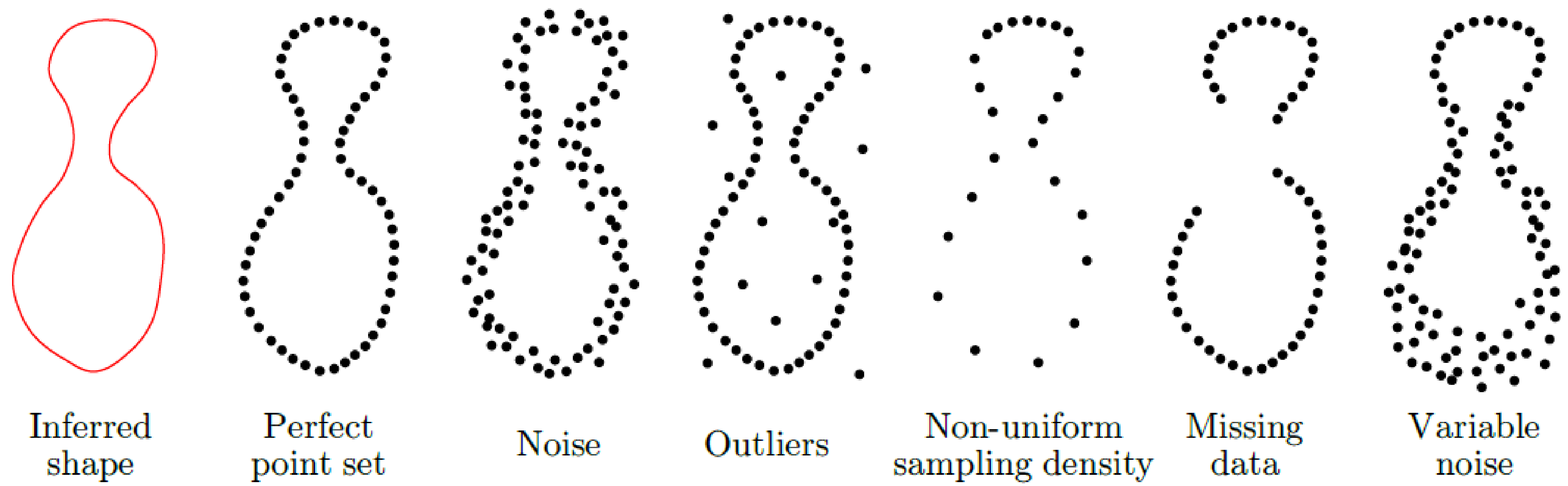






QUEST FOR ROBUSTNESS

Quest for Robustness



Poisson Reconstruction

Requires <u>oriented normals</u>, as many other implicit approaches.

Poisson Reconstruction

Requires <u>oriented normals</u>, as many other implicit approaches.

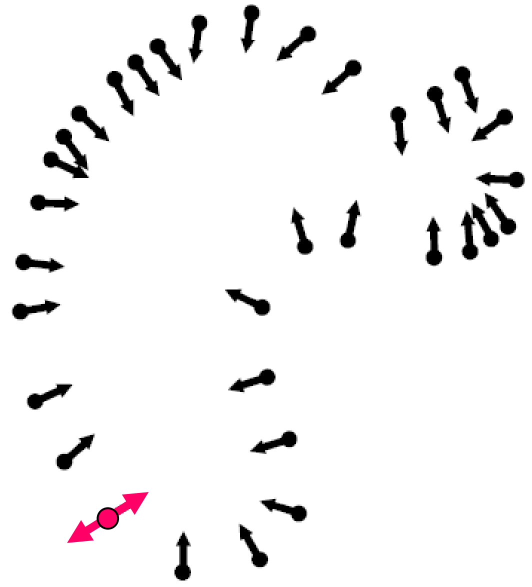
Normal estimation

Normal orientation

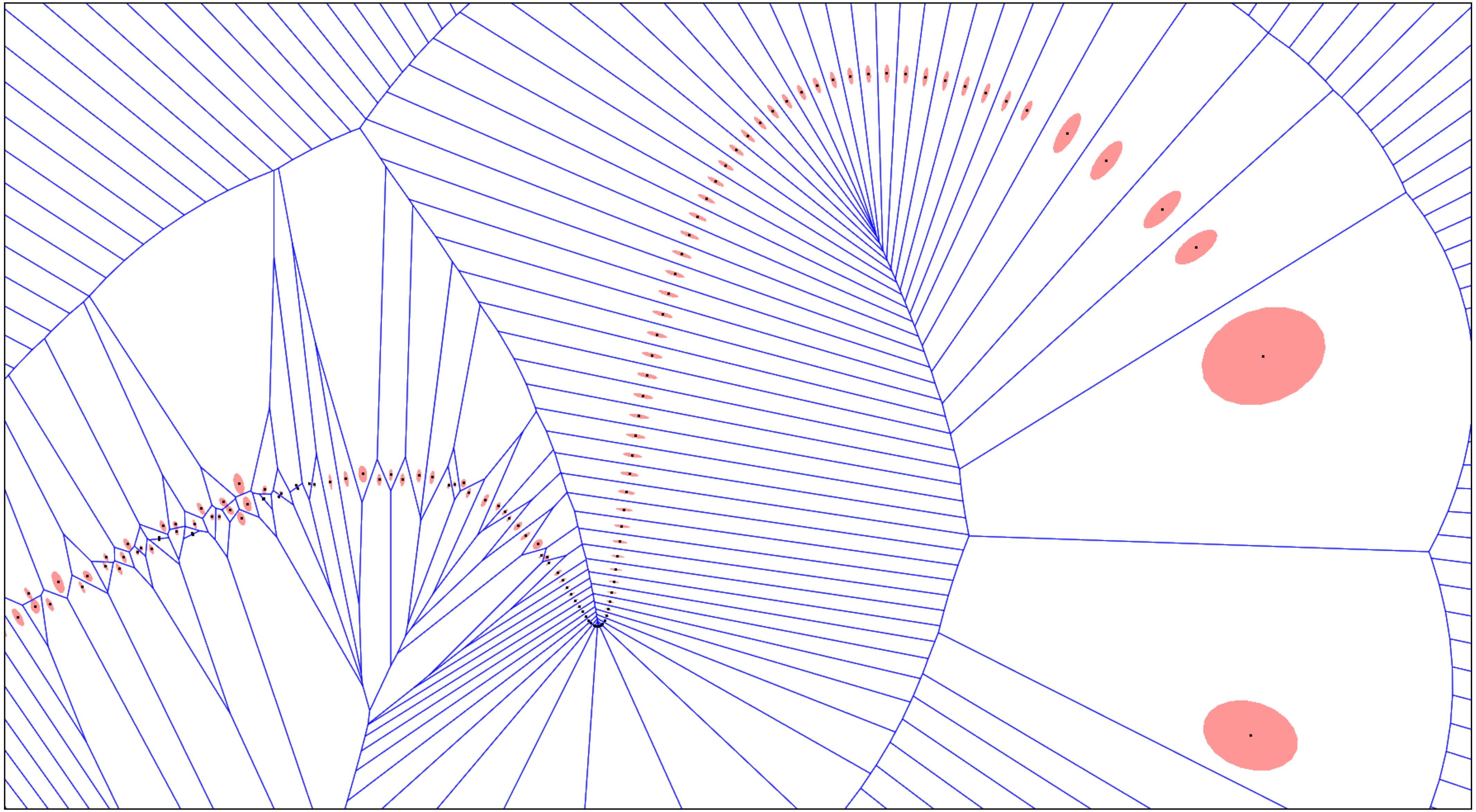
} ill-posed problems

Poisson Reconstruction

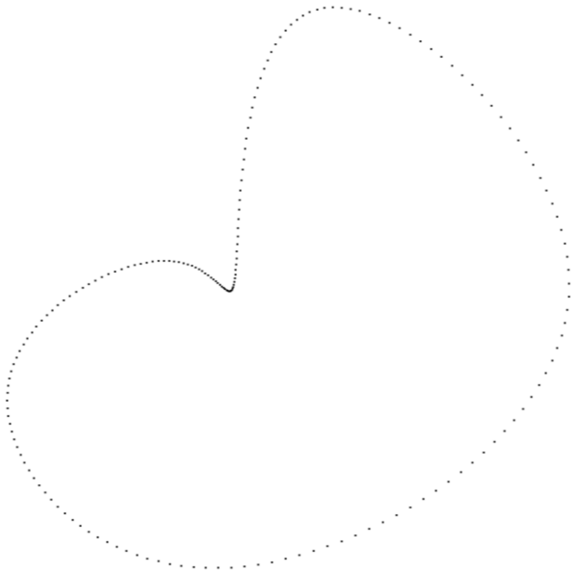
Can we deal with unoriented normals?



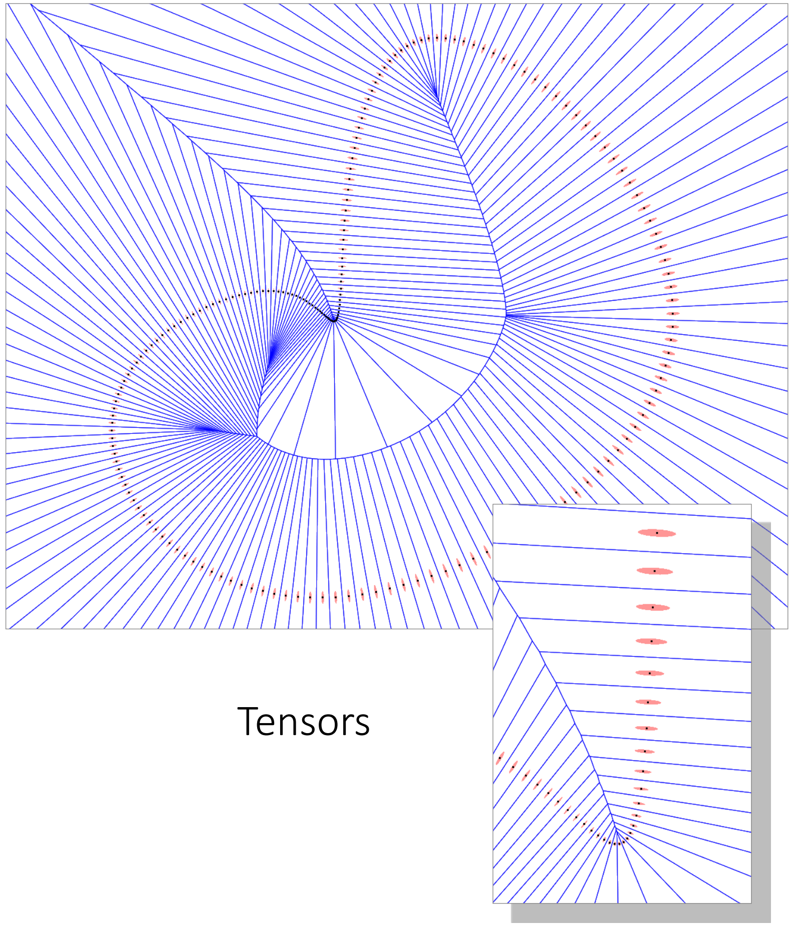
Unoriented Normals?



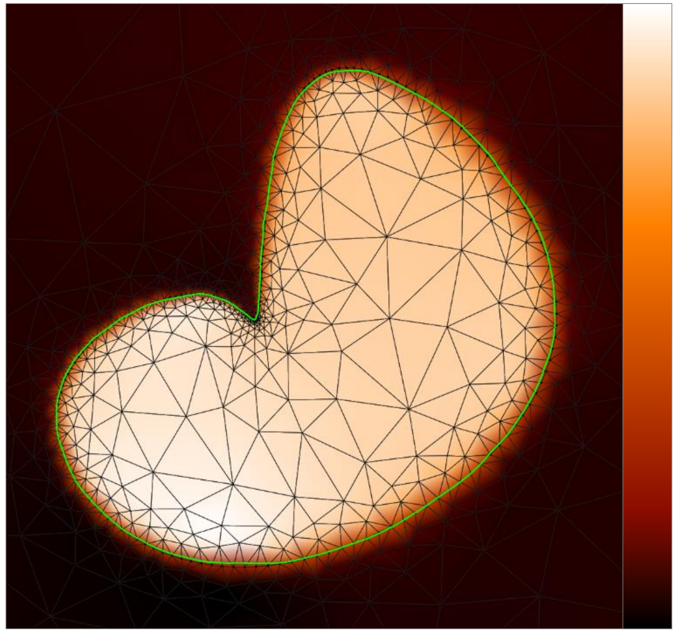
Spectral Reconstruction



Point set



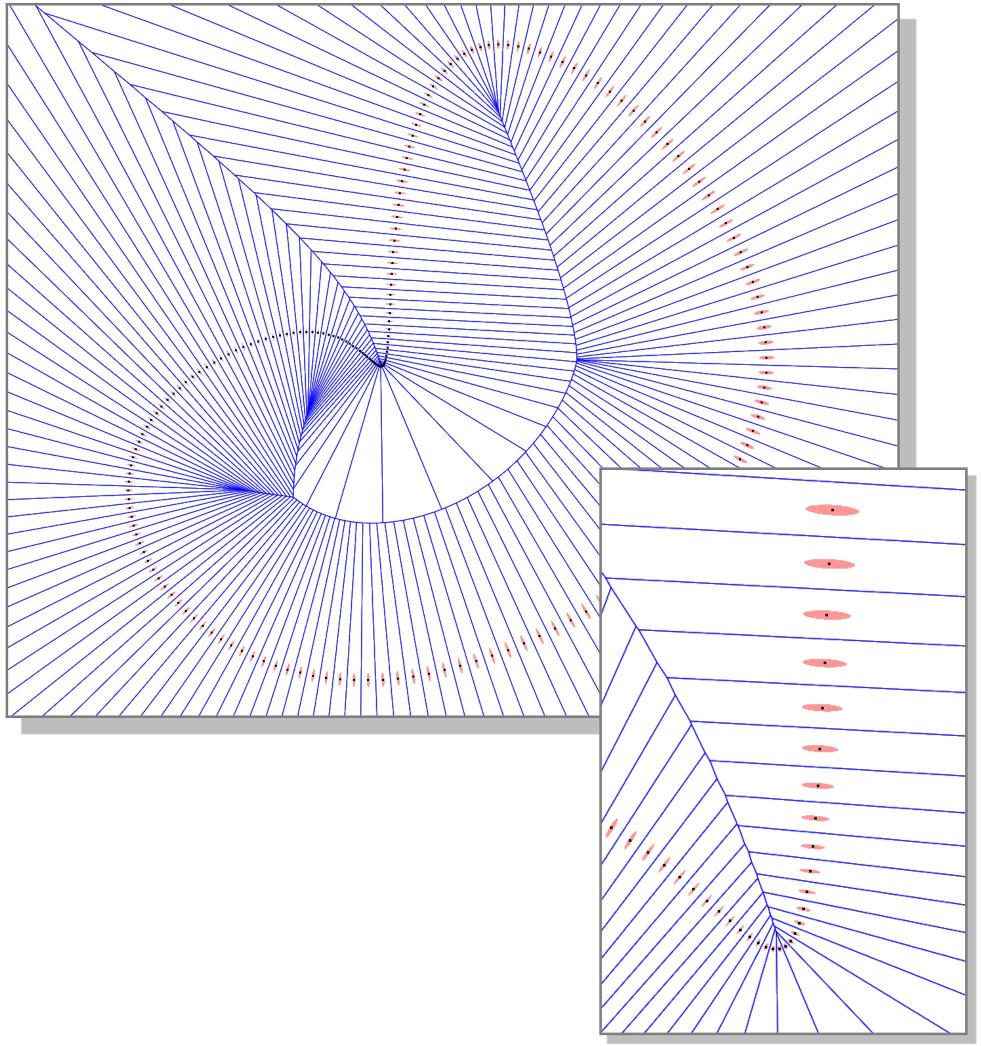
Tensors

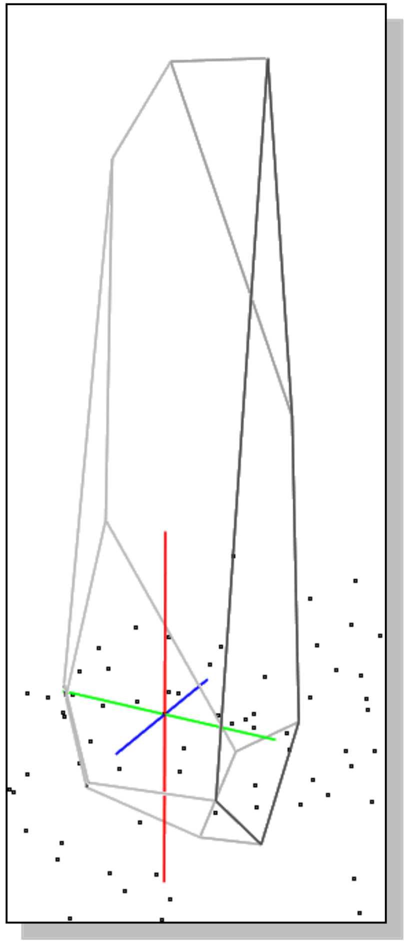


Implicit function
+ contouring

Voronoi-based Variational Reconstruction of Unoriented Point Sets.
A., Cohen-Steiner, Tong, Desbrun.
EUROGRAPHICS Symposium on Geometry Processing 2007.

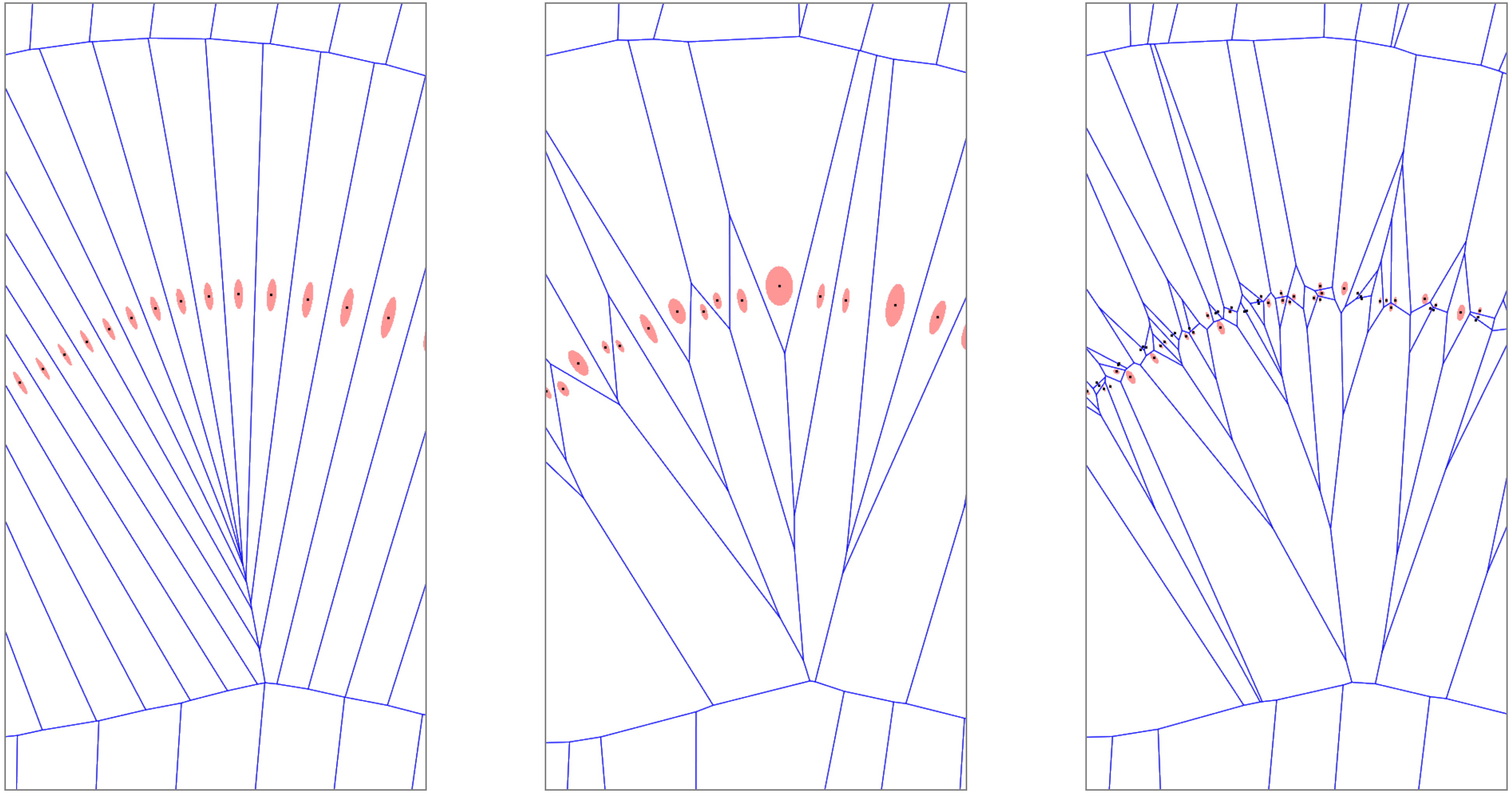
Tensor Estimation



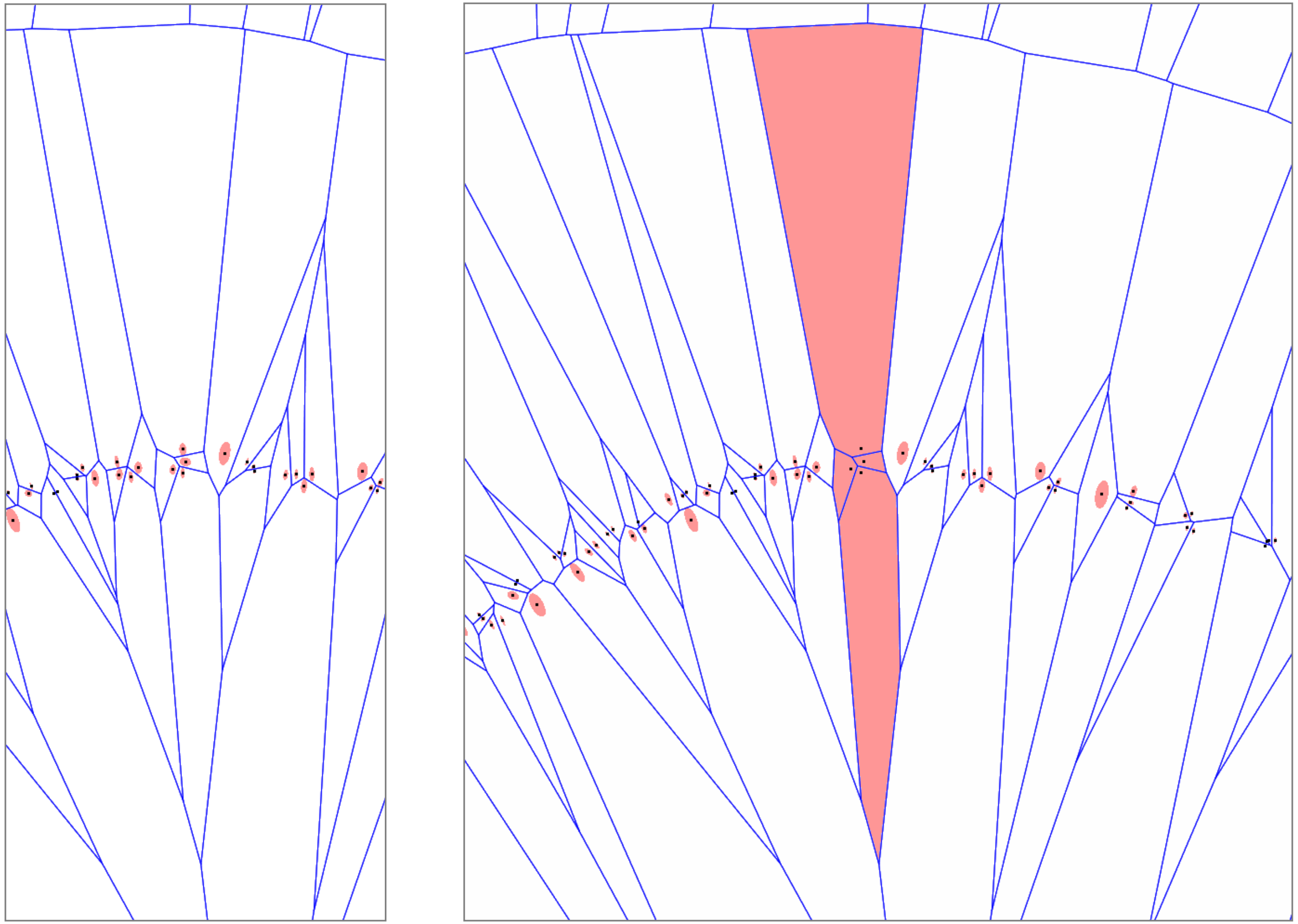


$$\int_{\Omega}(X-p)(X-p)^T dV$$

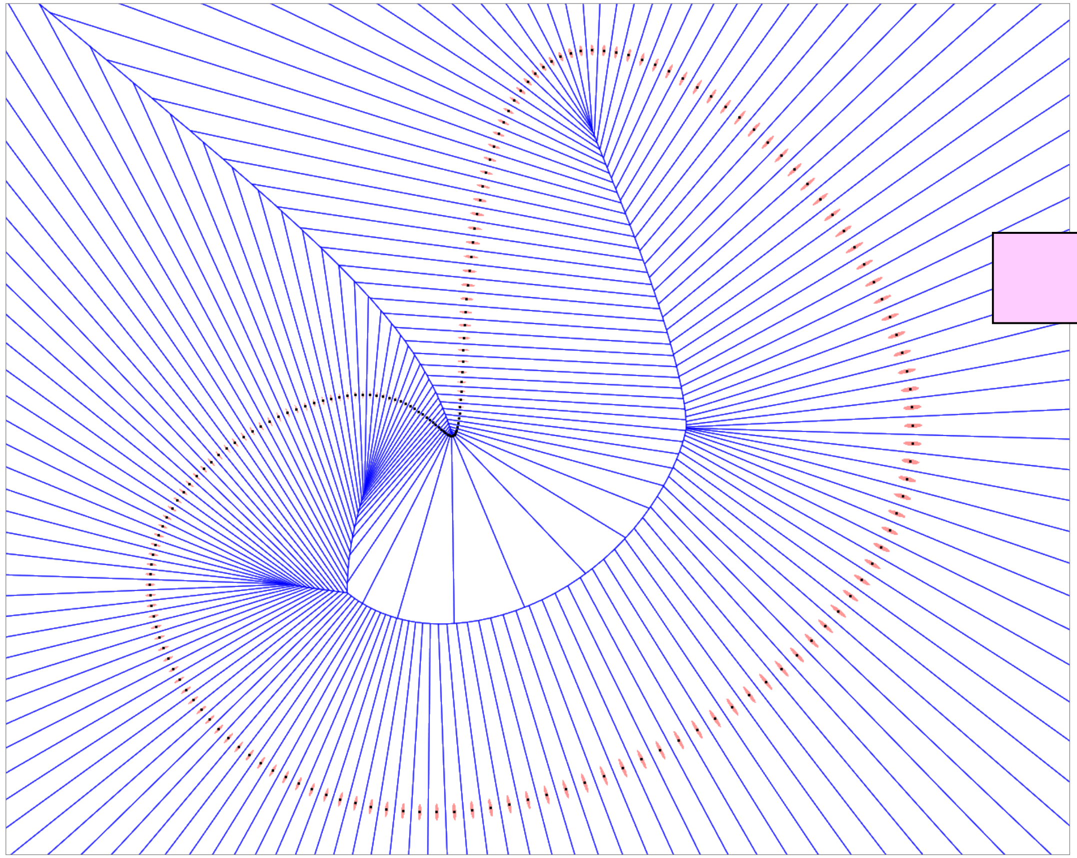
Noise-free vs Noisy



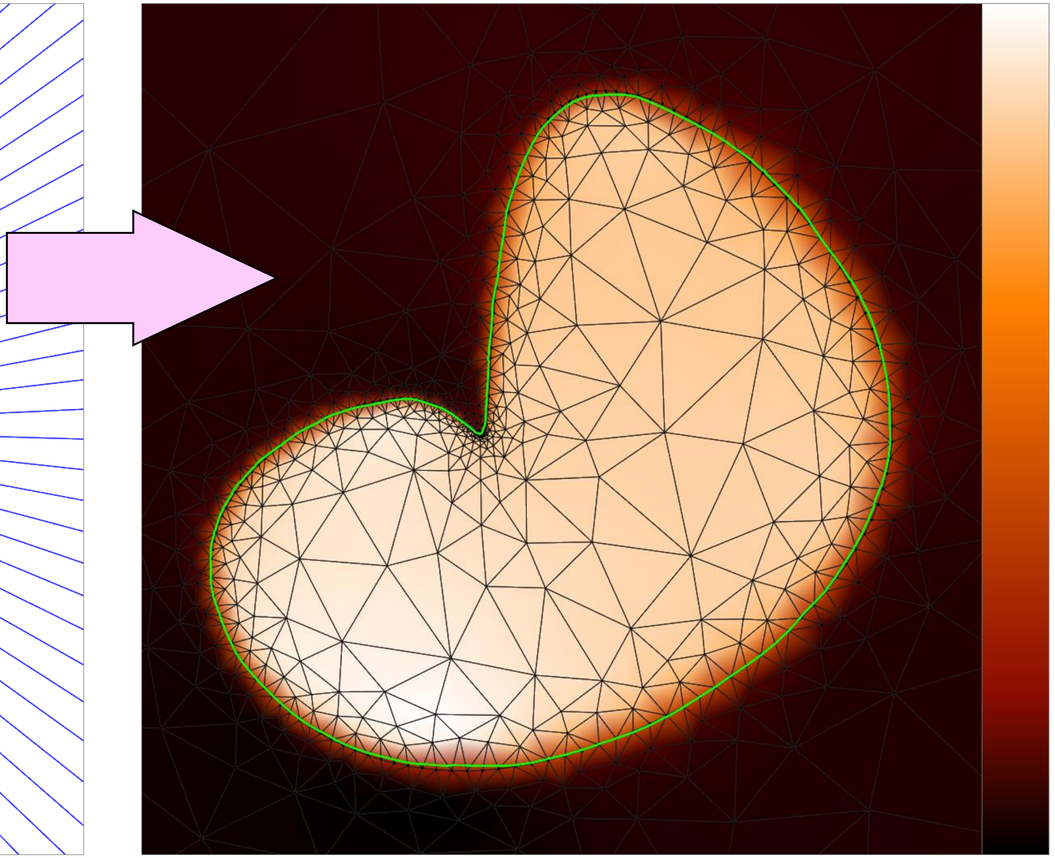
Dealing with Noise



Implicit Function



Tensors



Implicit function

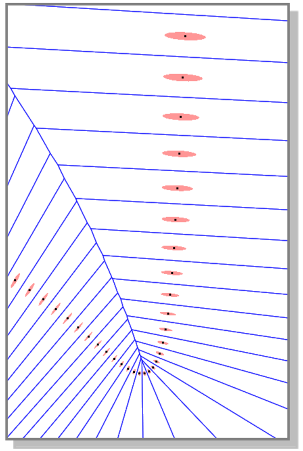
Formulation

Find implicit function f such that its gradient ∇f best aligns to the principal component of the tensors.

Given a tensor field C, find the *maximizer* f of:

$$E_C^D(f) = \int_\Omega \nabla f^t \, C \, \nabla f \text{ subject to: } \int_\Omega \left[|\Delta f|^2 + \varepsilon |f|^2 \right] = 1$$

Anisotropic Dirichlet energy
Measures alignment with tensors



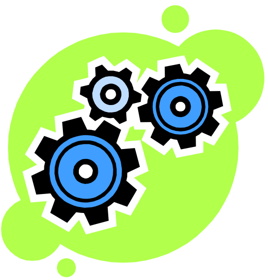
Biharmonic energy
Measures smoothness of ∇f

Formulation

Find implicit function *f* such that its gradient ∇f best aligns to the principal component of the tensors.

Given a tensor field C, find the *maximizer* f of:

$$E_C^D(f) = \int_\Omega \nabla f^t \, C \, \nabla f \text{ subject to: } \int_\Omega \left[|\Delta f|^2 + \varepsilon |f|^2 \right] = 1$$

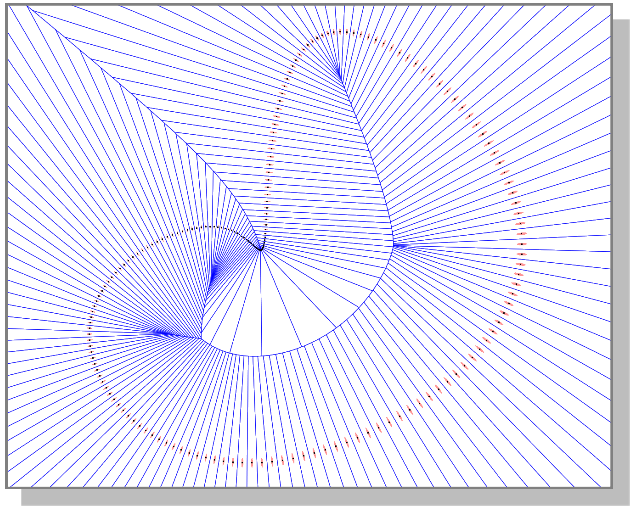


Biharmonic energy
Favors smoothness of ∇f

fitting vs
smoothness

Anisotropic Dirichlet energy
Rewards alignment with tensors

Rationale

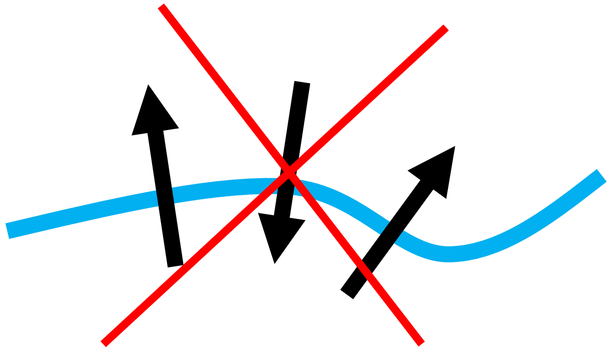


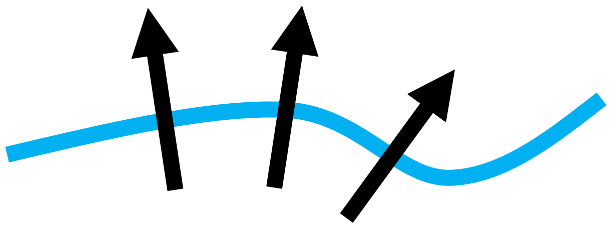
On areas with:

- anisotropic tensors: favors alignment
- isotropic tensors: favors smoothness

Large aligned gradients + smoothness

leads to consistent orientation of ∇f





Generalized Eigenvalue Problem

Given a tensor field C, find the *maximizer* f of:

$$E_C^D(f) = \int_\Omega \nabla f^t \, C \, \nabla f \text{ subject to: } \int_\Omega [|\Delta f|^2 + \varepsilon |f|^2] = 1$$

A: anisotropic Laplacian operator

$$E_C^D(F) \approx F^t \, A \, F$$

B: isotropic Bilaplacian operator

$$E^B(f) \approx F^t \, B \, F$$

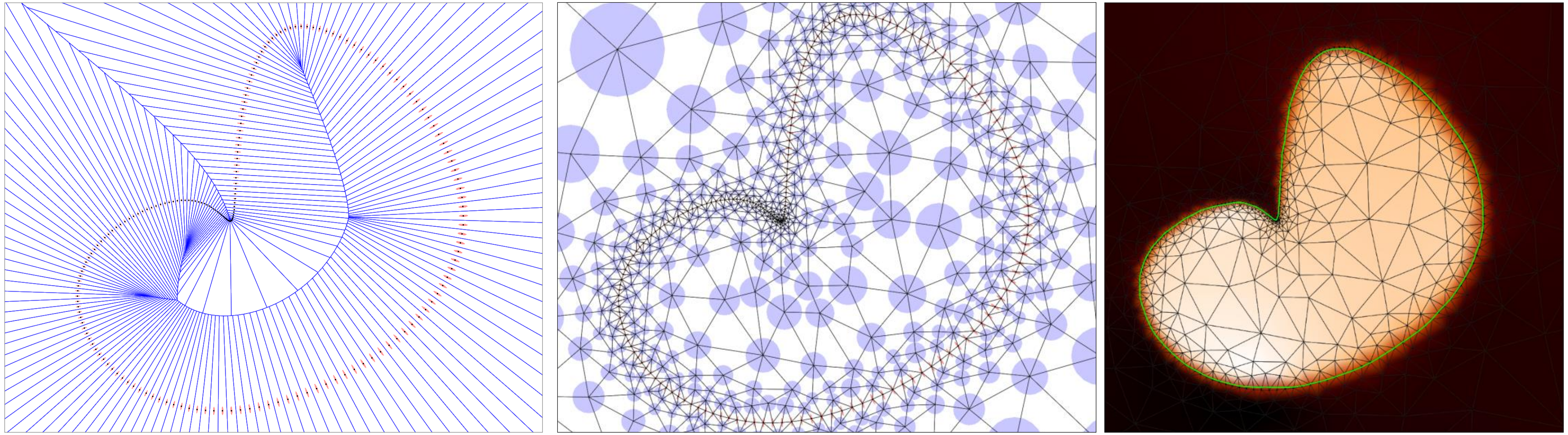
$$\boxed{AF = \lambda BF}$$

max

Eigenvector

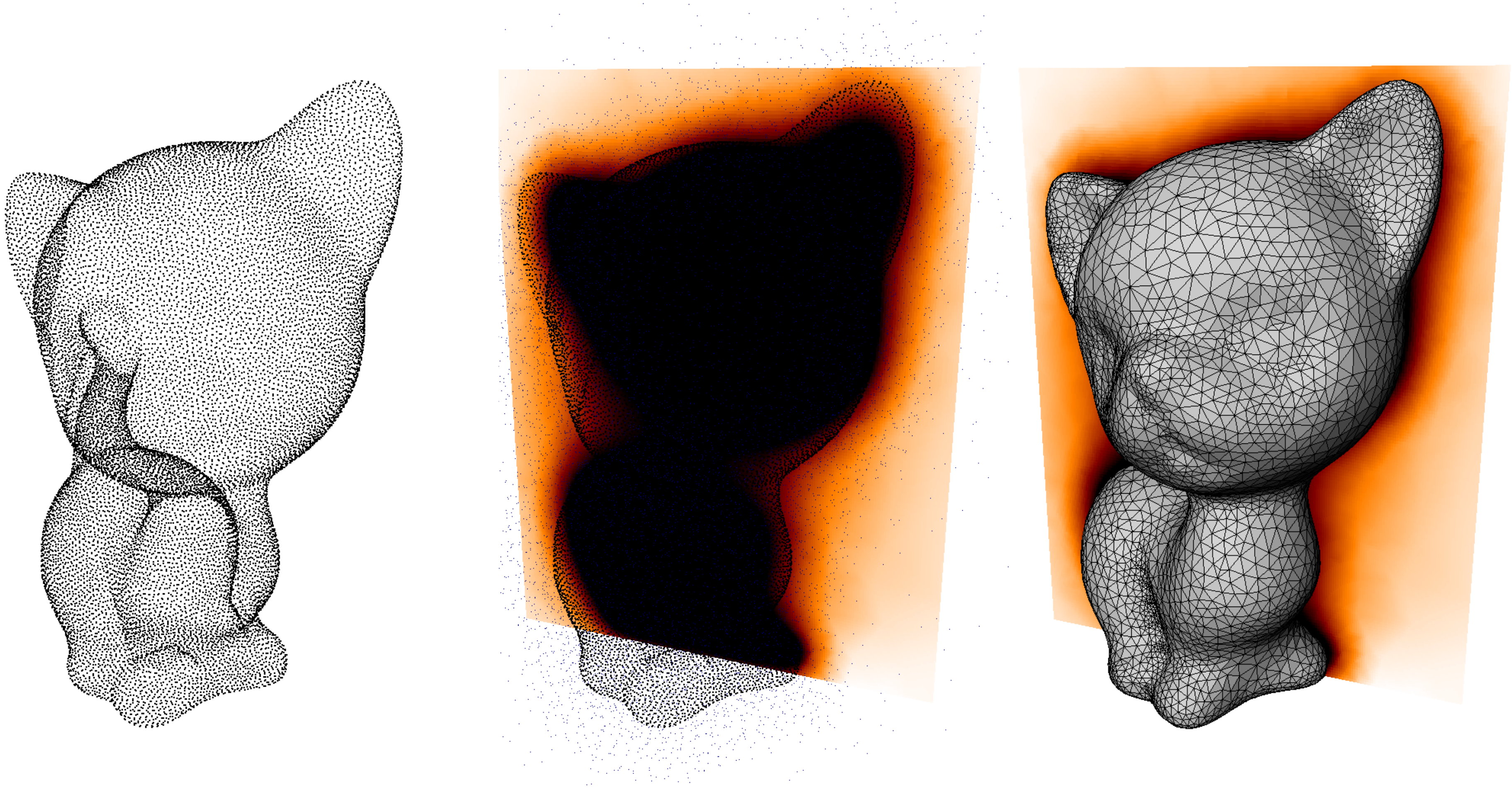
Generalized Eigenvalue Problem

$$AF = \lambda BF$$

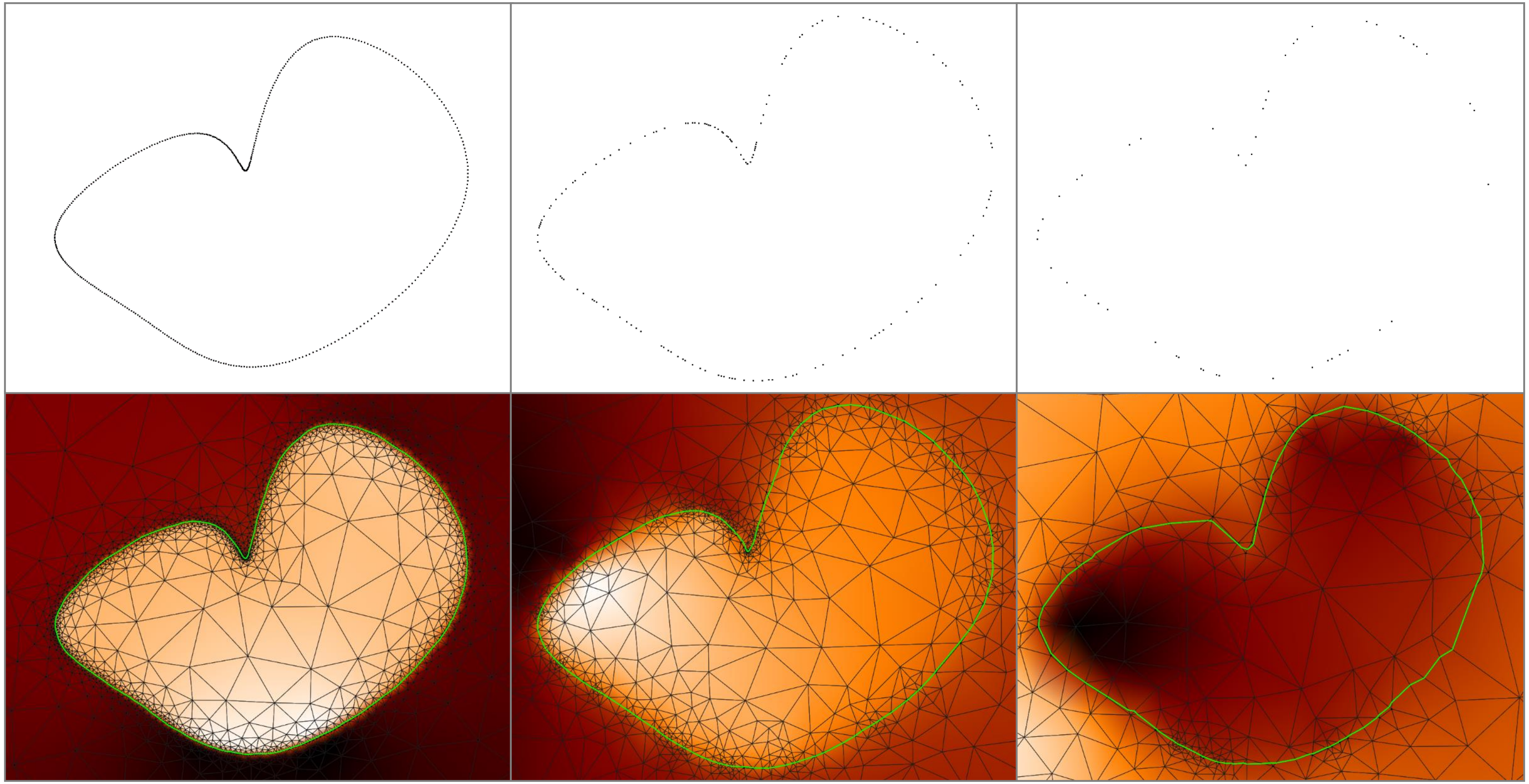


Eigenvector

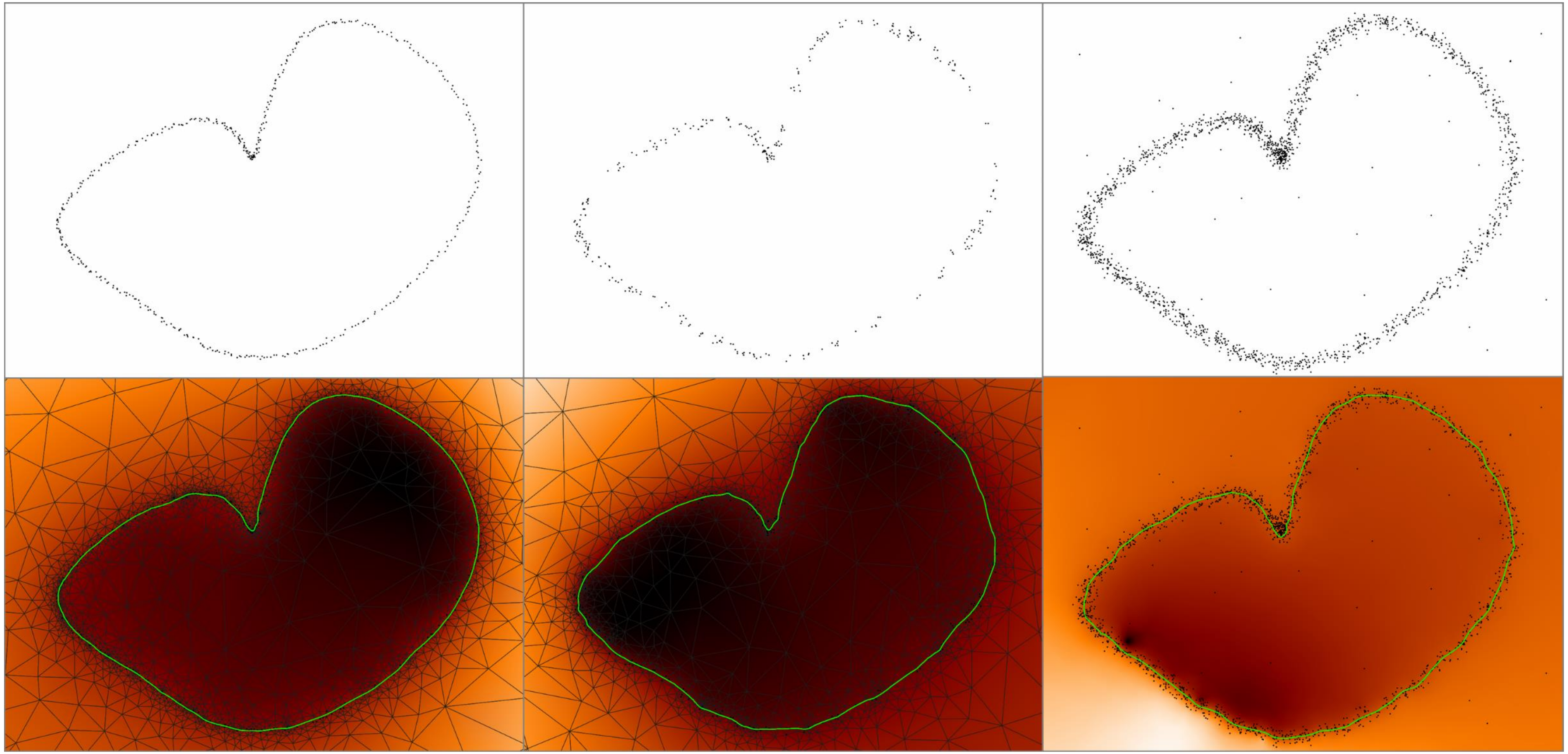
Implicit Reconstruction



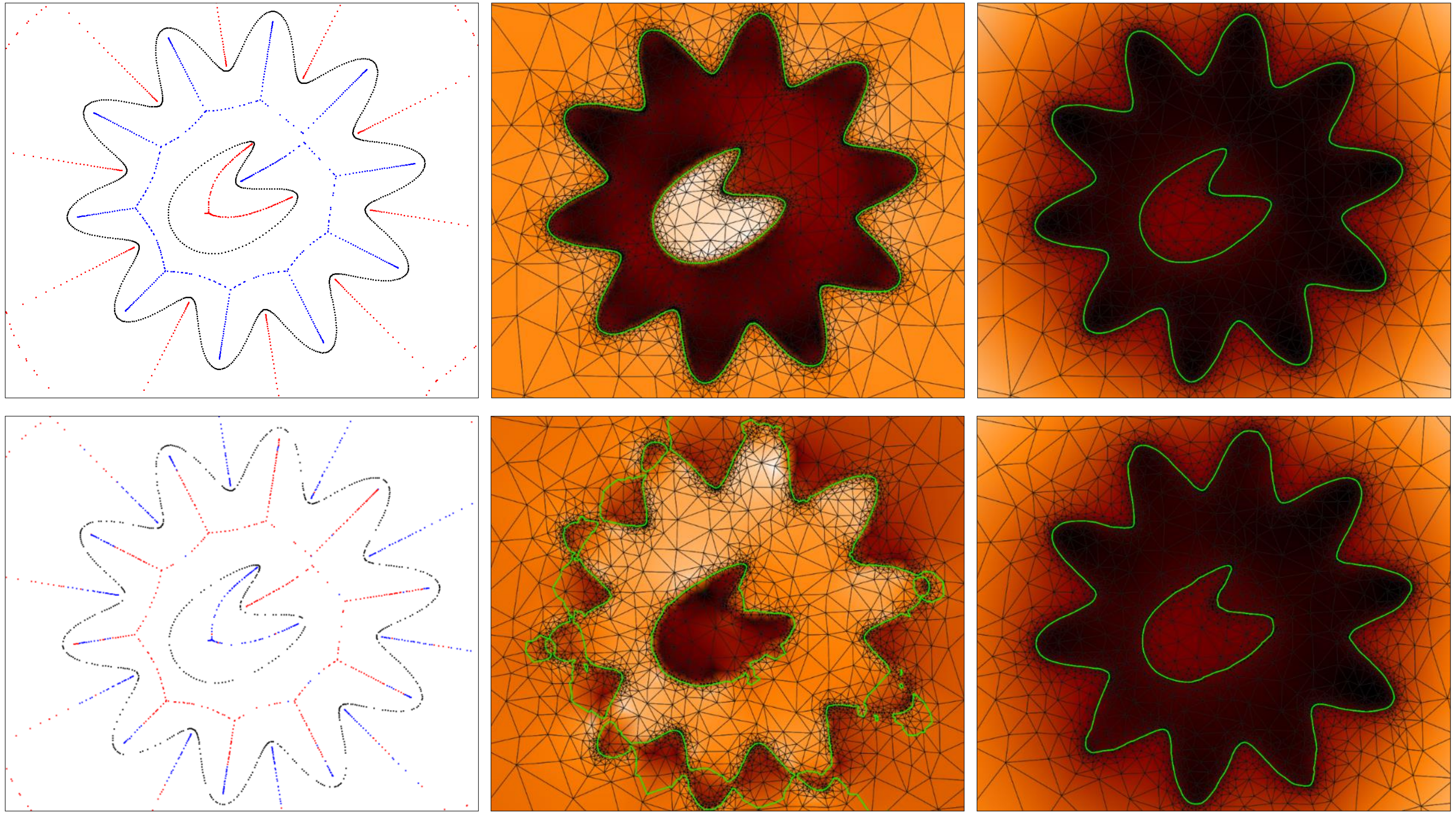
Robustness to Sparse Sampling



Robustness to Noise

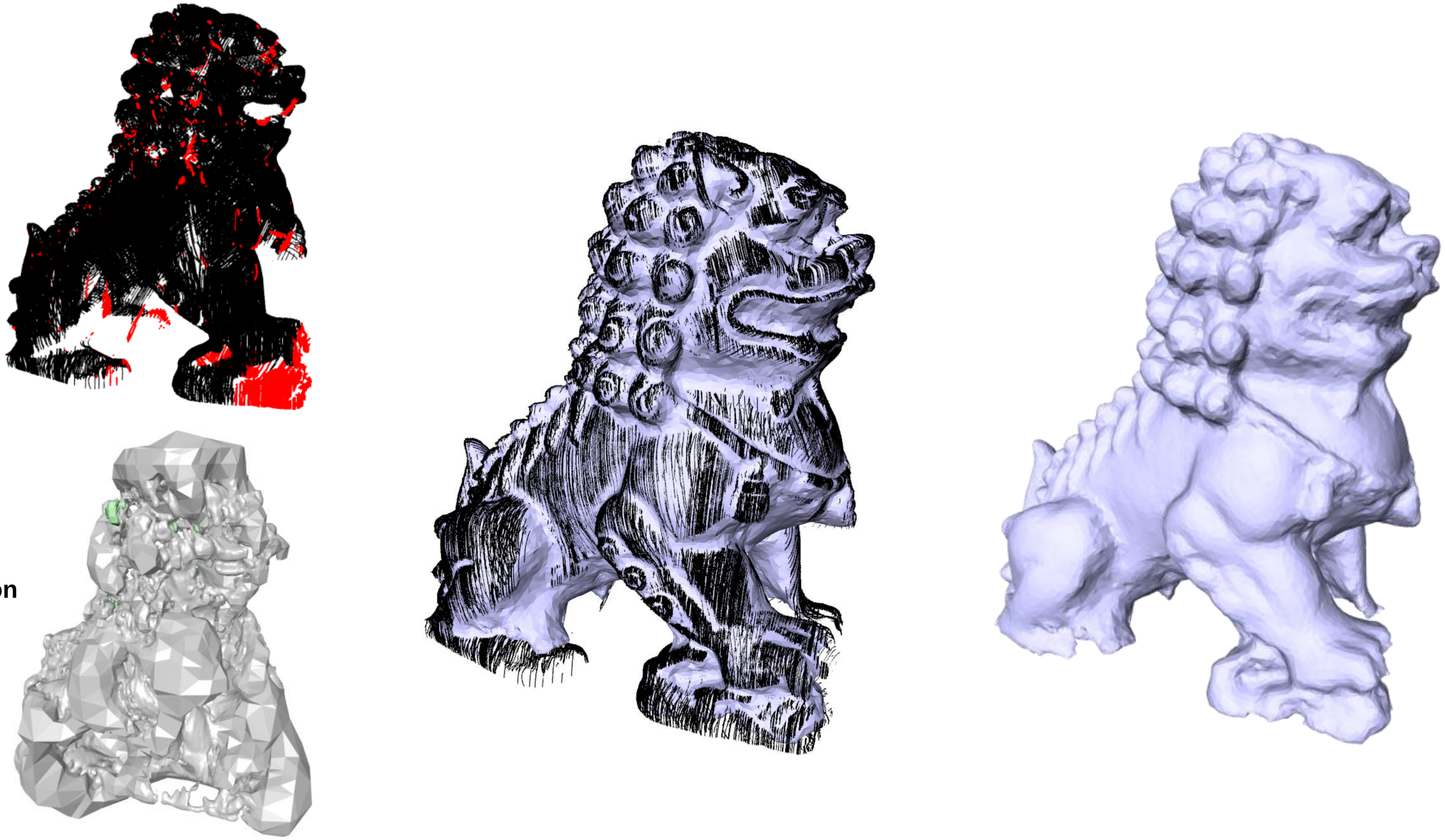


vs Poisson Reconstruction



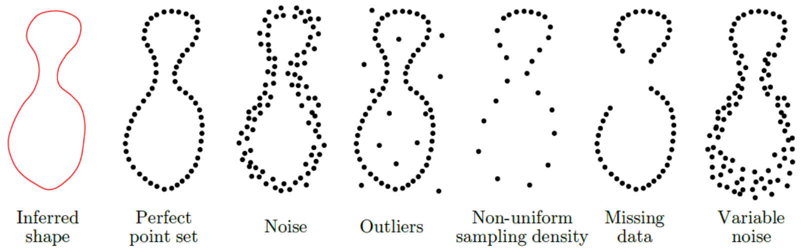
Oriented points | Poisson | Spectral

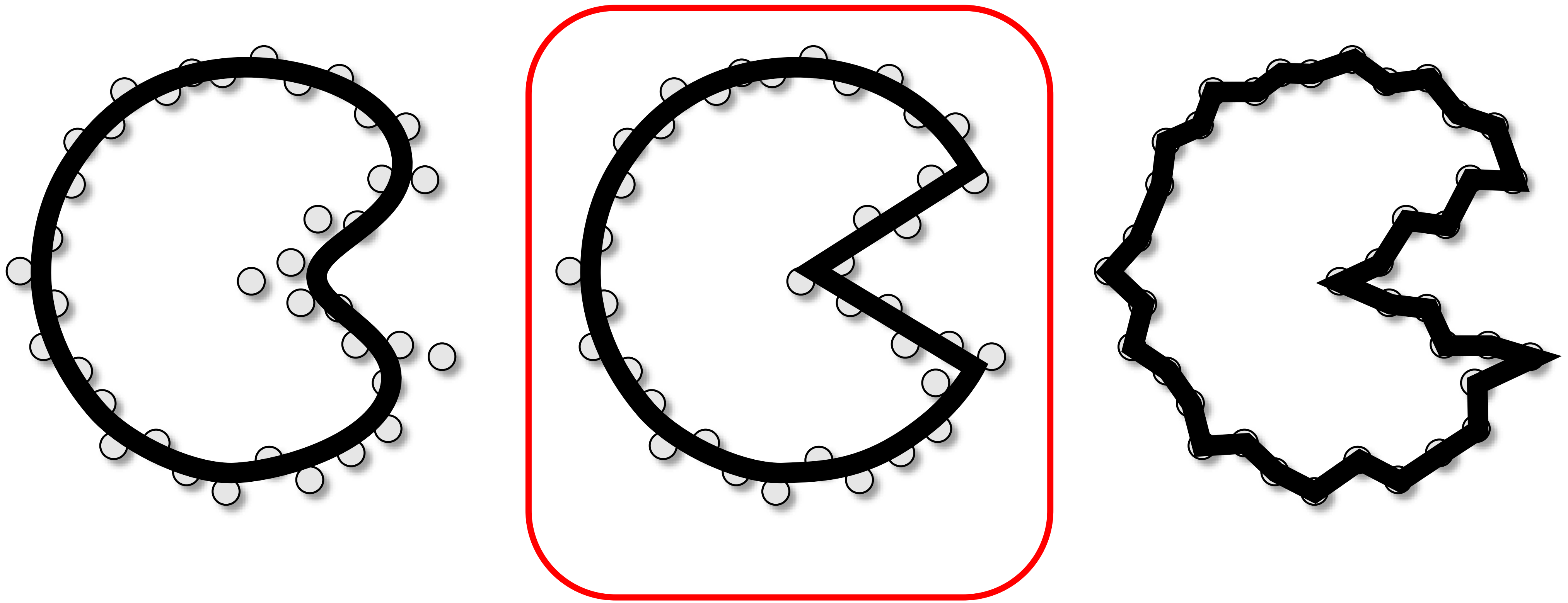
vs Poisson Reconstruction



Poisson

Priors



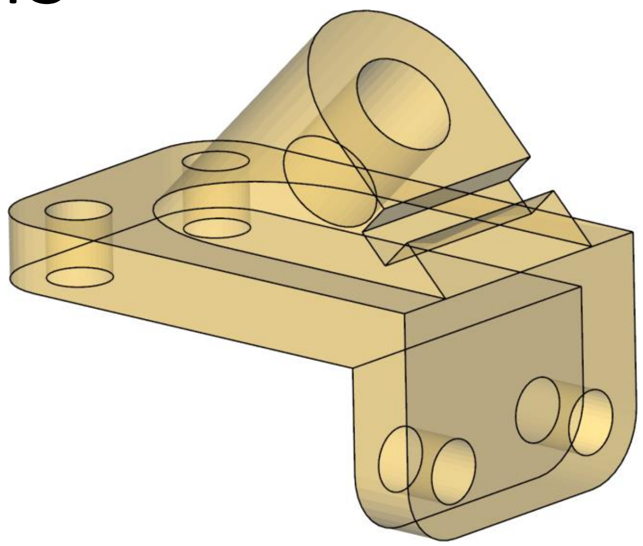


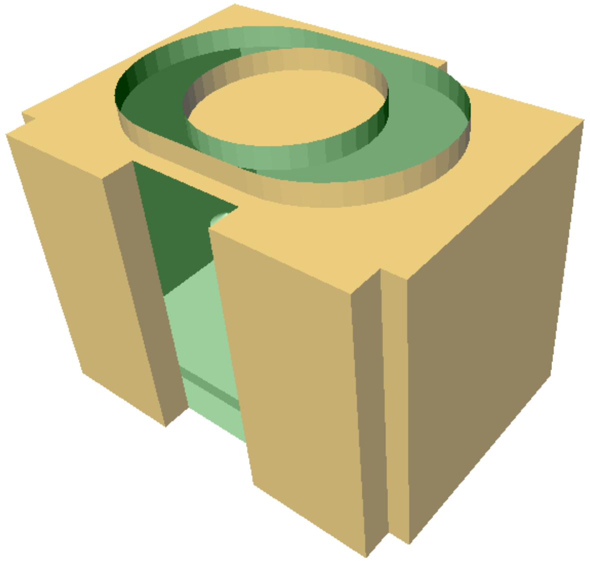
Smooth   Piecewise Smooth   “Simple”

Motivations

Complex shapes:

- Sharp features
- Boundaries
- Non-manifold features

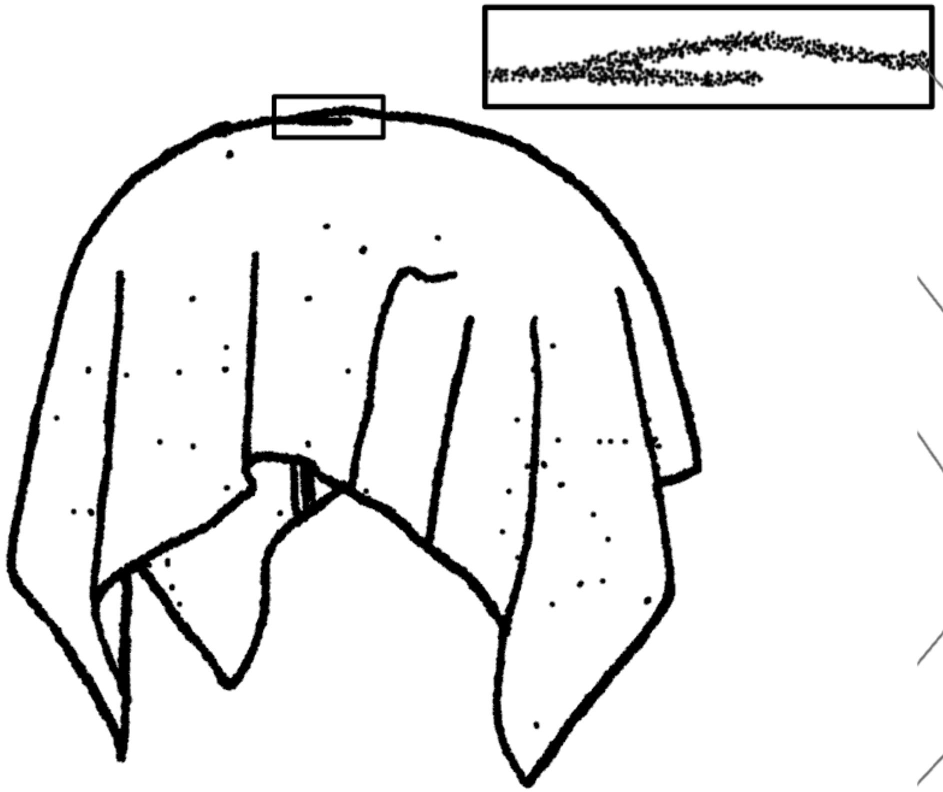


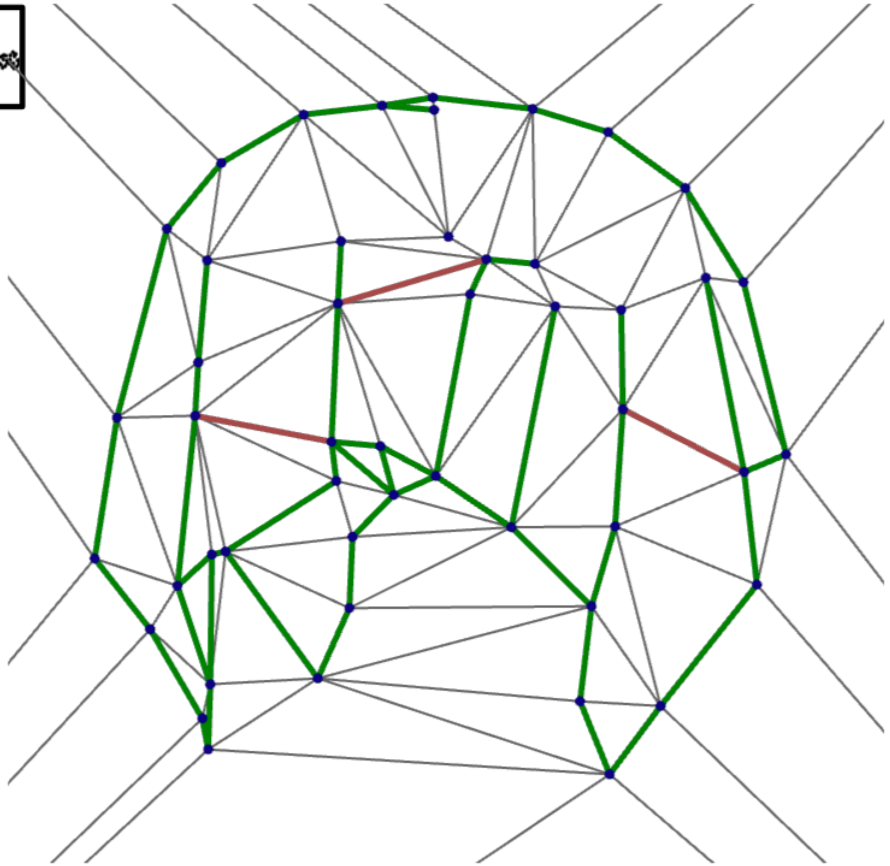


Calls for feature preservation

Approach in 2D

Given a point set S, find a coarse triangulation T such that S is well approximated by uniform measures on the 0- and 1-simplices of T.





Approach in 2D

Given a point set S, find a coarse triangulation T such that S is well approximated by uniform measures on the 0- and 1-simplices of T.

How to measure distance D(S,T)?

$\Rightarrow$ optimal transport between measures

How to construct T that minimizes D(S,T)?

optimal location problem $\Rightarrow$ greedy decimation

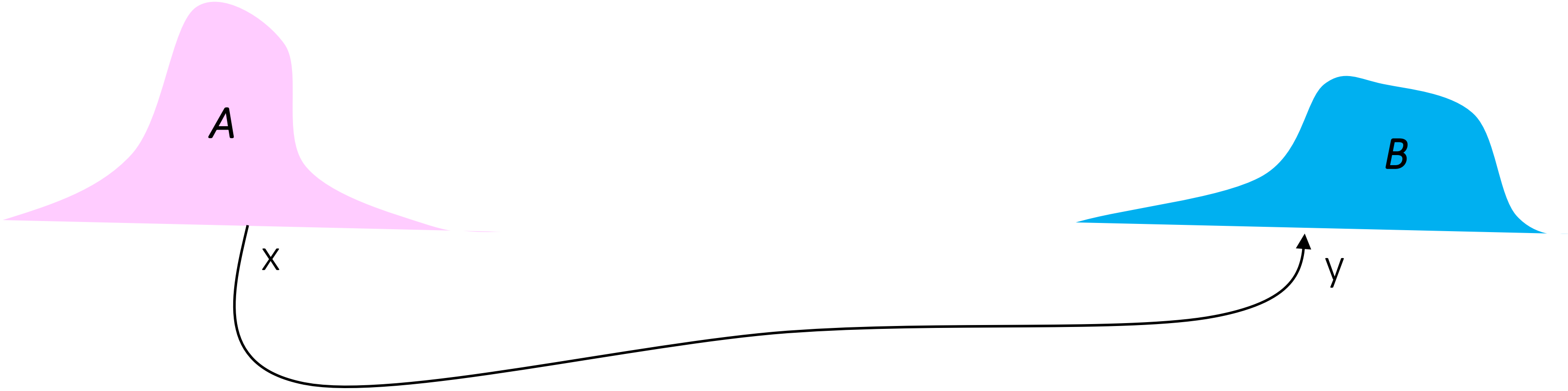
- Mérigot
- Peyré
- Schmitzer
- Cuturi
- Solomon
- …

Distance between Measures (1D)

Transport plan: π on $\mathbb{R} \times \mathbb{R}$ whose marginals are A and B

Transport cost: $$W_2(A, B, \pi) = \left(\int_{\mathbb{R}\times\mathbb{R}} \|x - y\|^2 d\pi(x, y) \right)^{1/2}$$

Optimal transport: $$W_2(A, B) = \inf_{\pi} W_2(A, B, \pi)$$



Distance between Measures (1D)

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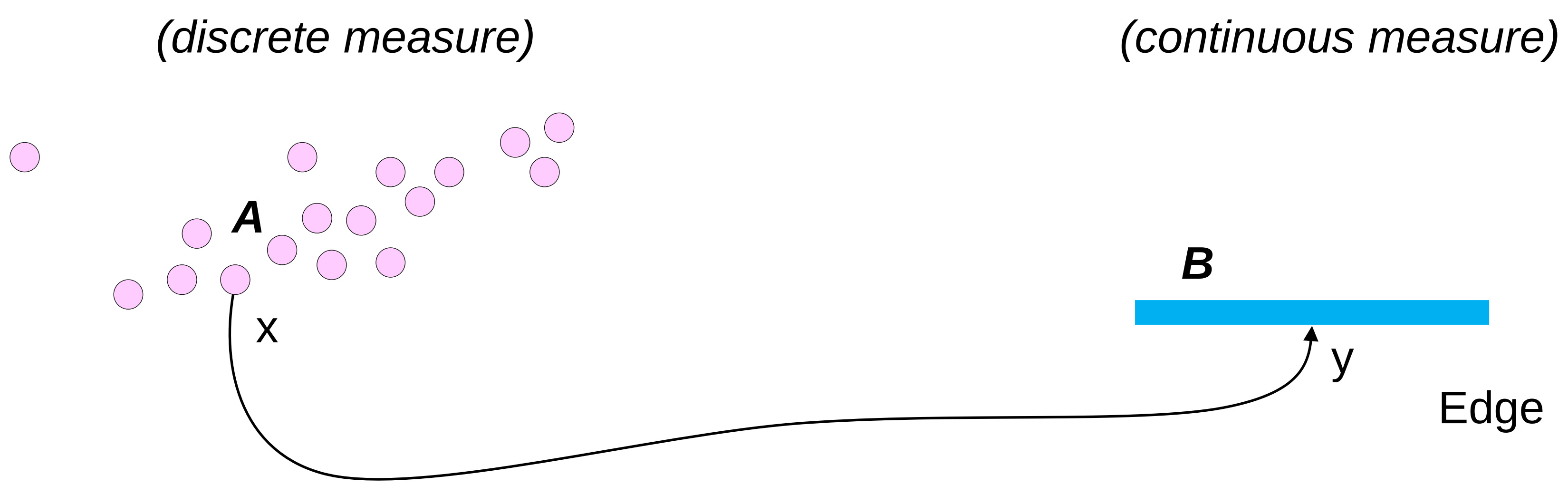
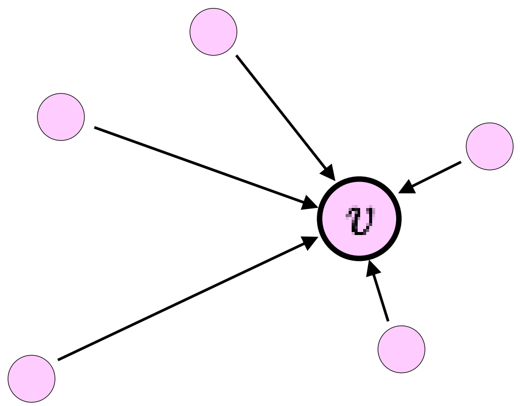
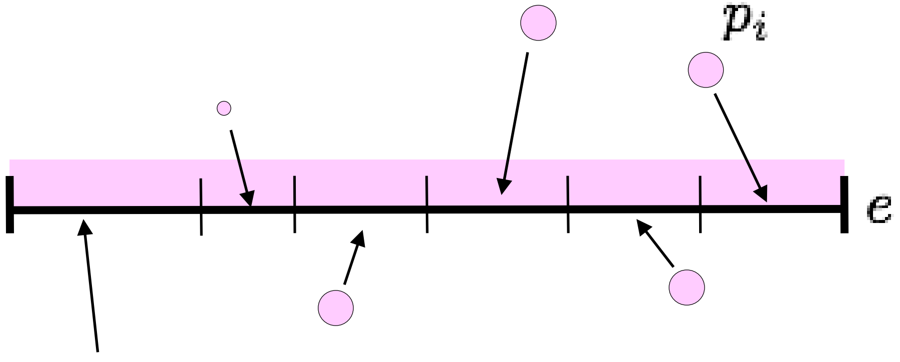


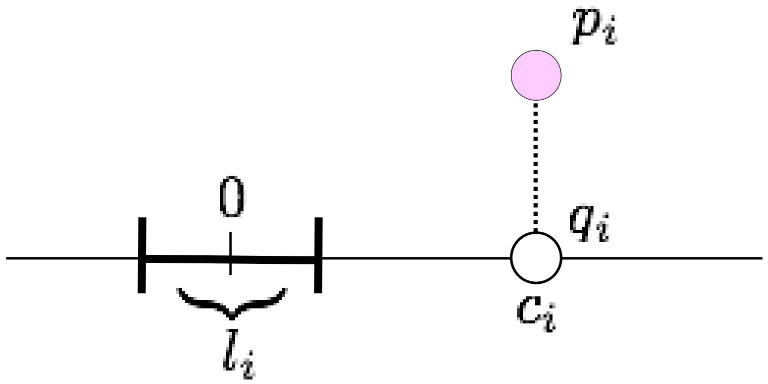
Piecewise Uniform Measures



Vertex



Edge

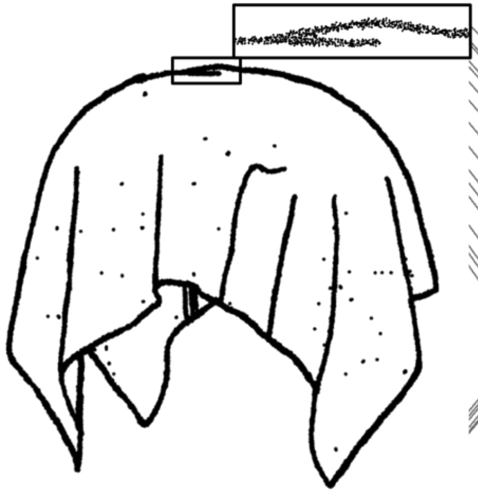


$$W_2(v, S_v) = \sqrt{\sum_{p_i \in S_v} m_i \|p_i - v\|^2}.$$

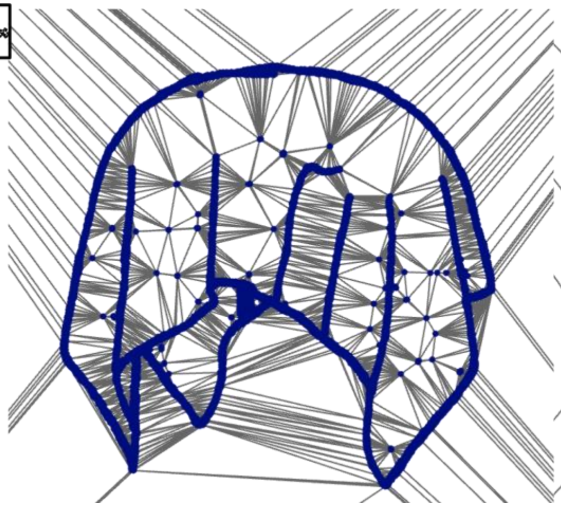
$$N(e, S_e) = \sqrt{\sum_{p_i \in S_e} m_i \|p_i - q_i\|^2}$$

$$T(e, S_e) = \sqrt{\sum_{p_i \in S_e} \frac{M_e}{|e|} \int_{-l_i/2}^{l_i/2} (x - c_i)^2 \, dx} = \sqrt{\sum_{p_i \in S_e} m_i \left(\frac{l_i^2}{12} + c_i^2 \right)}$$

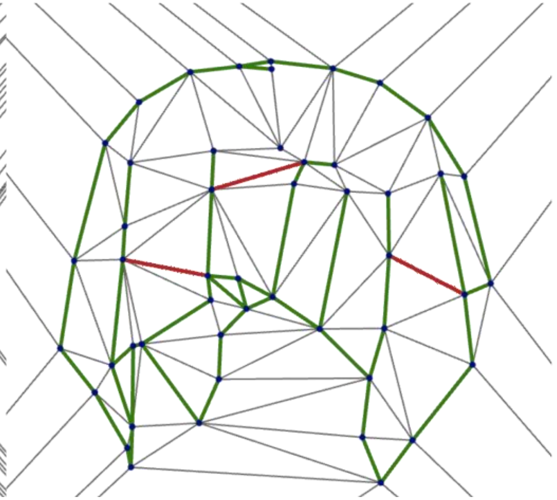
Algorithm Overview



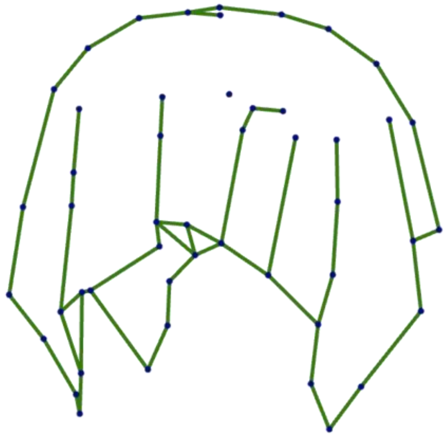
Input point set



Delaunay triangulation

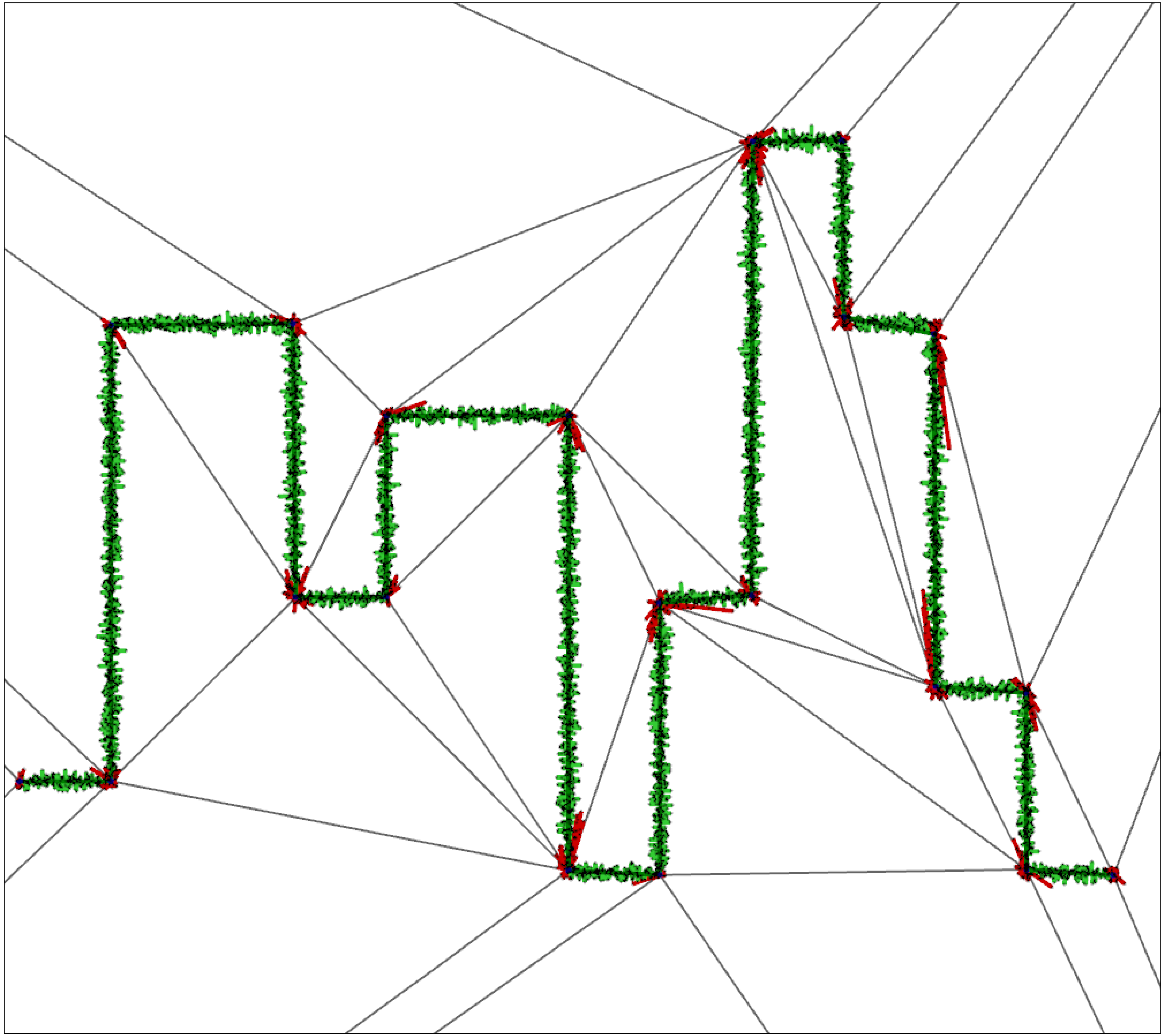


After decimation

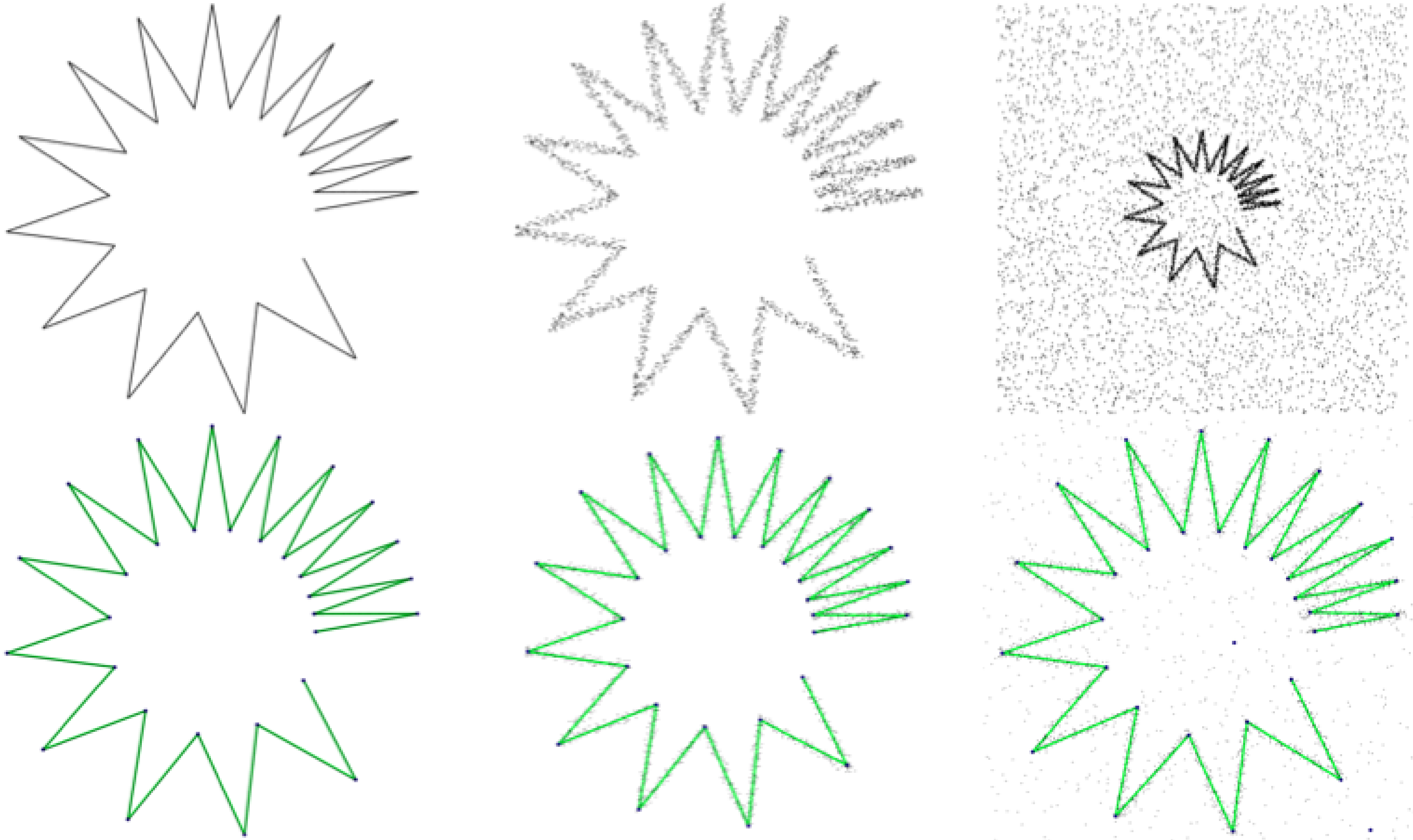


Output after edge filtering

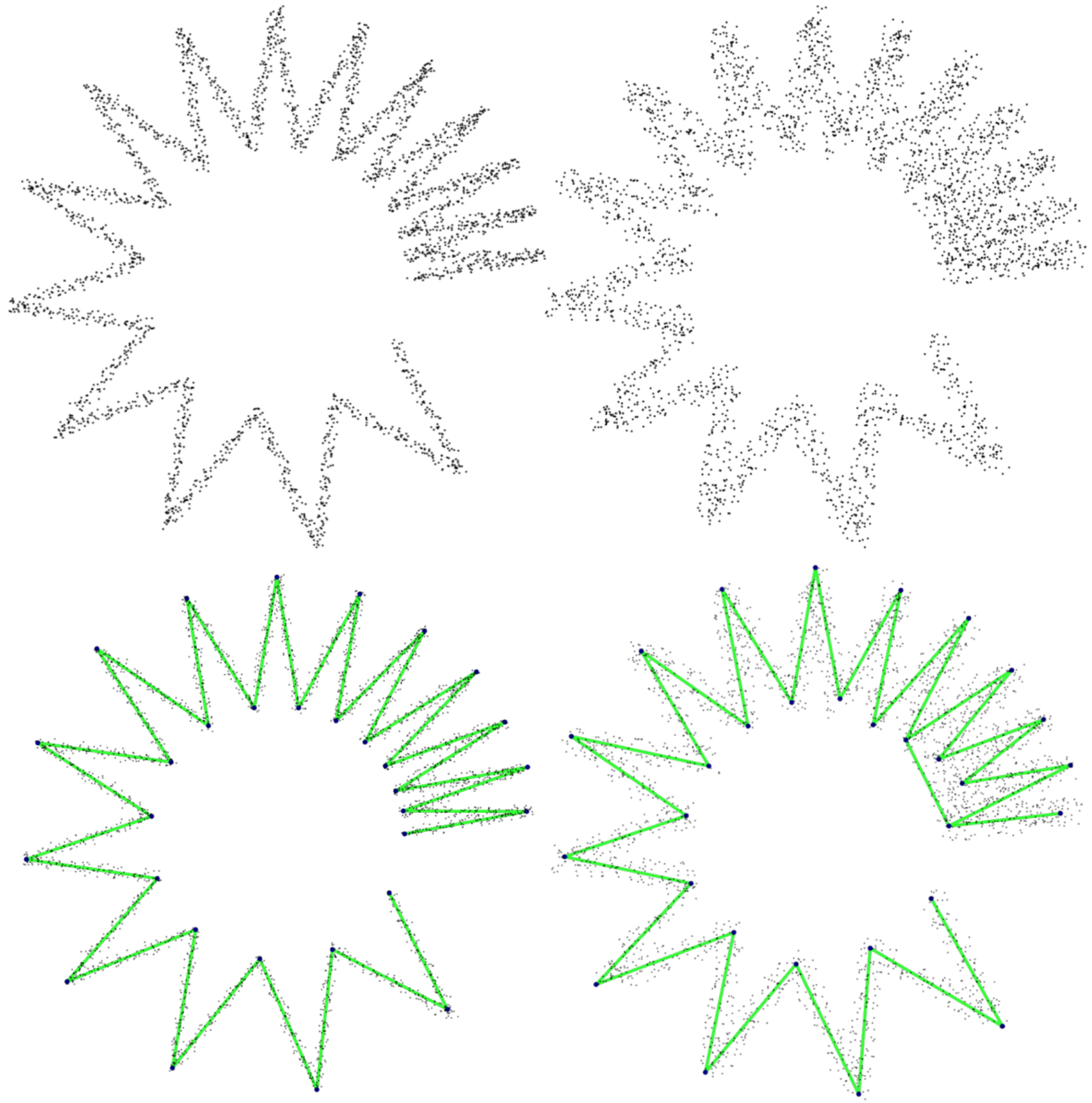
An Optimal Transport Approach to Robust Reconstruction and Simplification of 2D Shapes.
De Goes, Cohen-Steiner, A., Desbrun.
EUROGRAPHICS Symposium on Geometry Processing 2011.



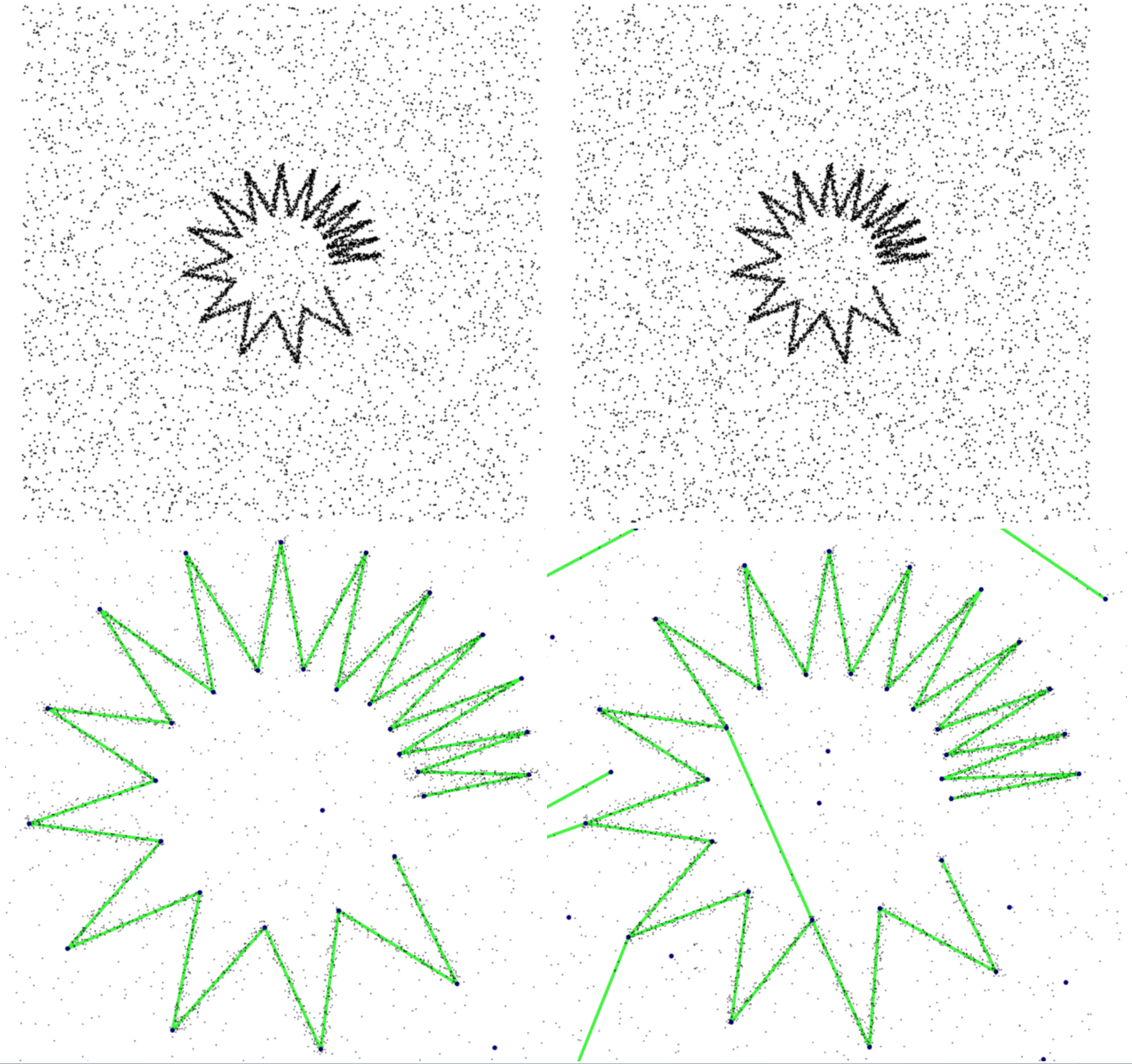
Robustness



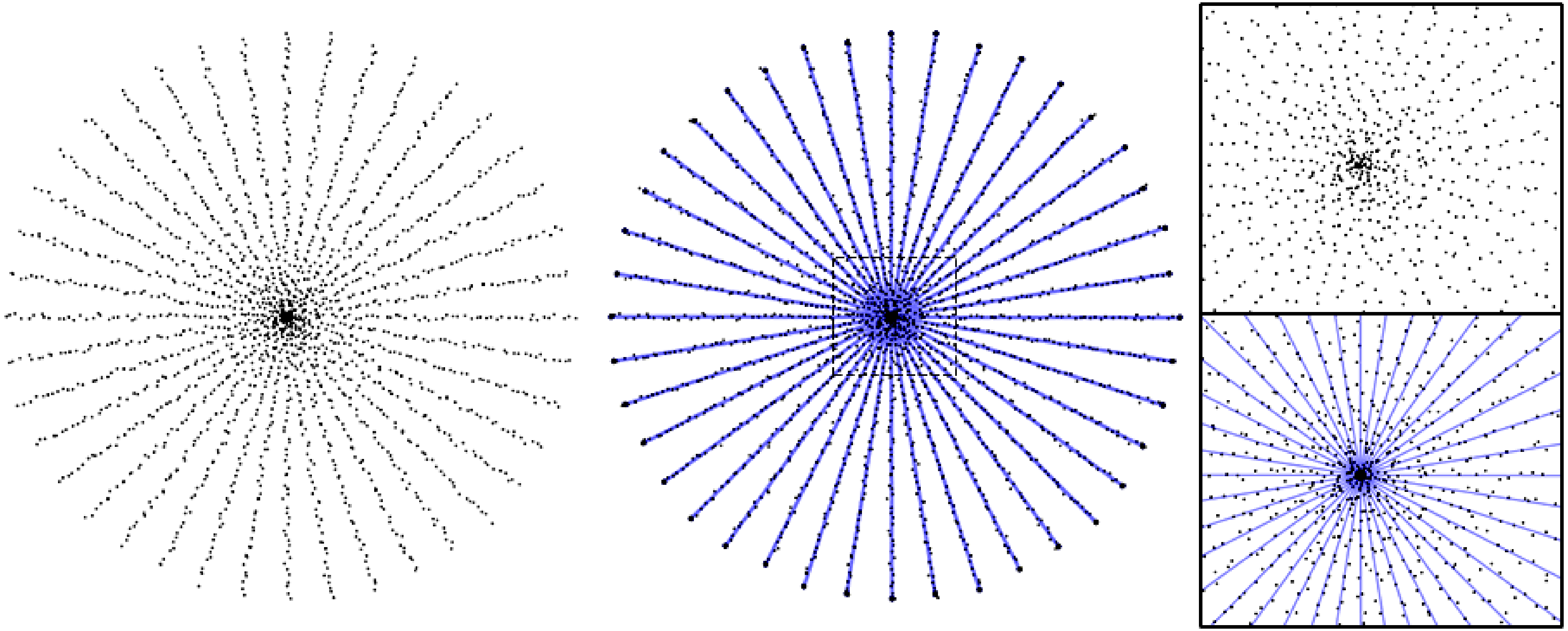
More Noise



More Outliers

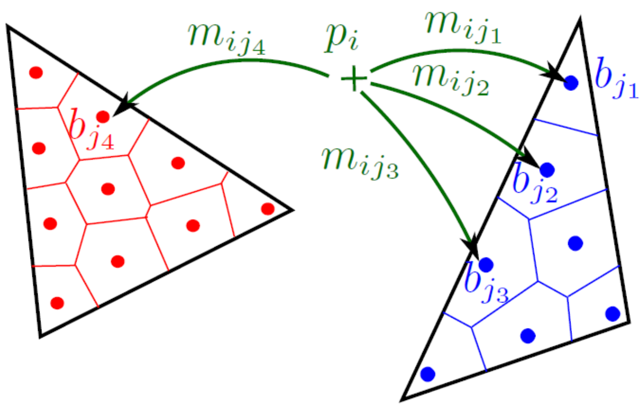


Features and Robustness

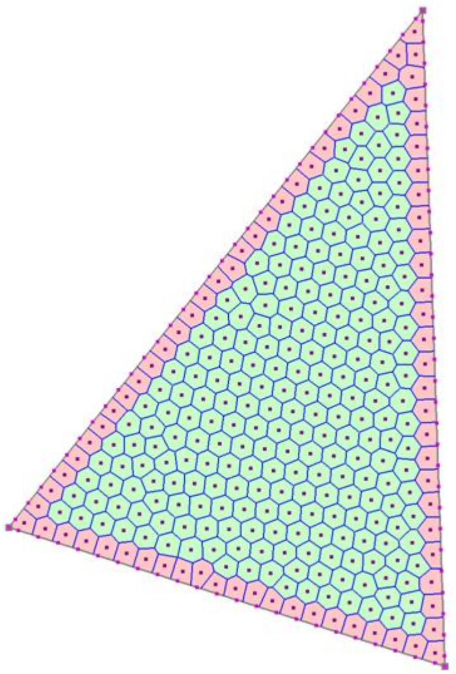


Surface Reconstruction?

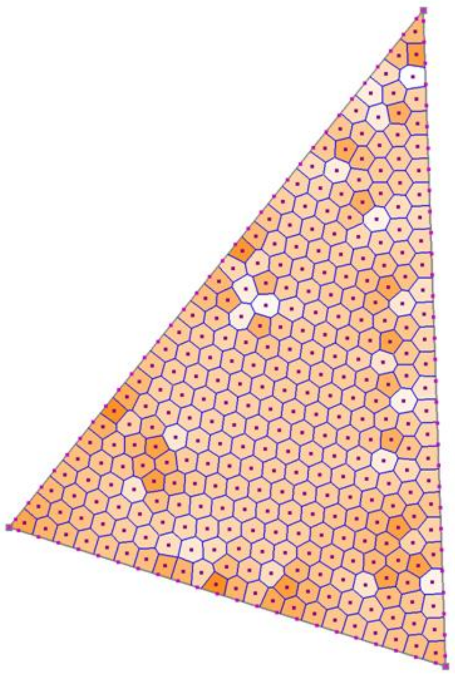
Surface Reconstruction?



Fractional transport plan
Piecewise uniform measure

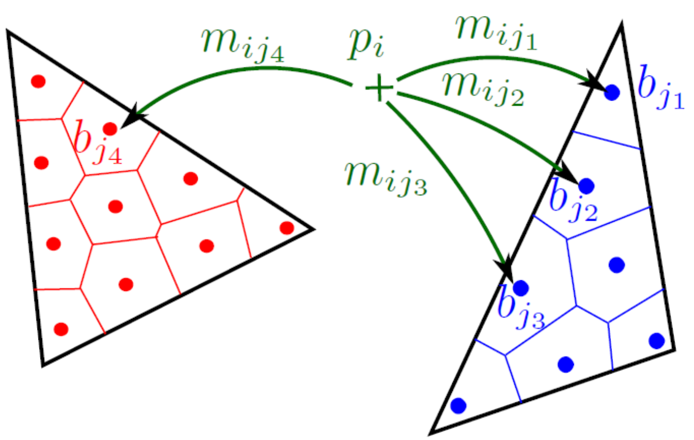


Voronoi “Bins”



Bin capacities

Solve through Linear Programming



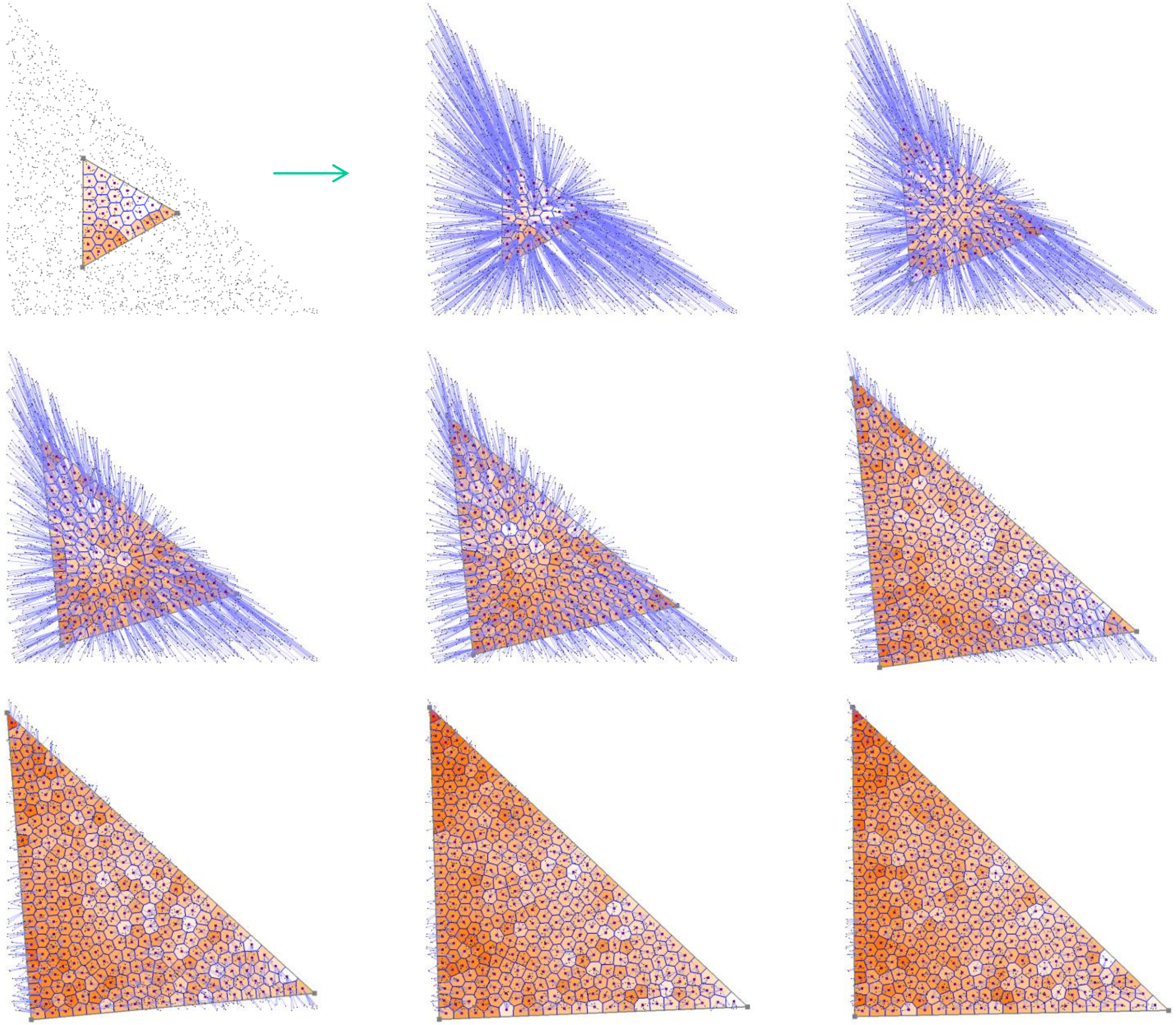
Minimize $\quad \sum_{ij} m_{ij} \|p_i - b_j\|^2$

w.r.t. the variables m_{ij} and l_j, and subject to:

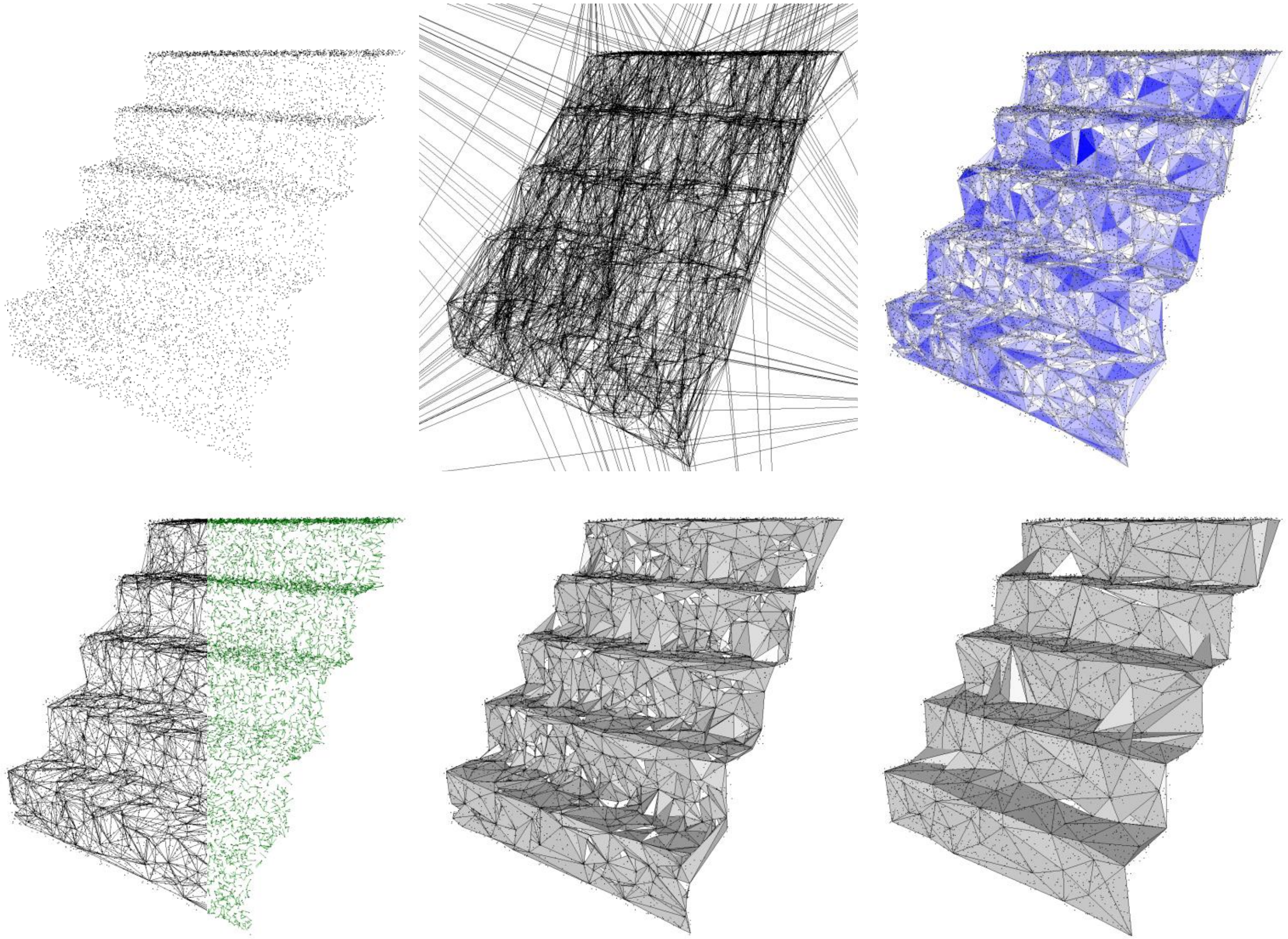
$$\begin{cases} \forall i : \sum_j m_{ij} = m_i & \text{Mass conservation} \\ \forall j : \sum_i m_{ij} = c_j \cdot l_{s(j)} & \text{Piecewise uniform} \\ \forall i,\ j : m_{ij} \geq 0,\ l_j \geq 0 & \text{Positive densities} \end{cases}$$

Feature-Preserving Surface Reconstruction and Simplification from Defect-Laden Point Sets.
Digne, Cohen-Steiner, A., Desbrun, De Goes. Journal of Mathematical Imaging and Vision.

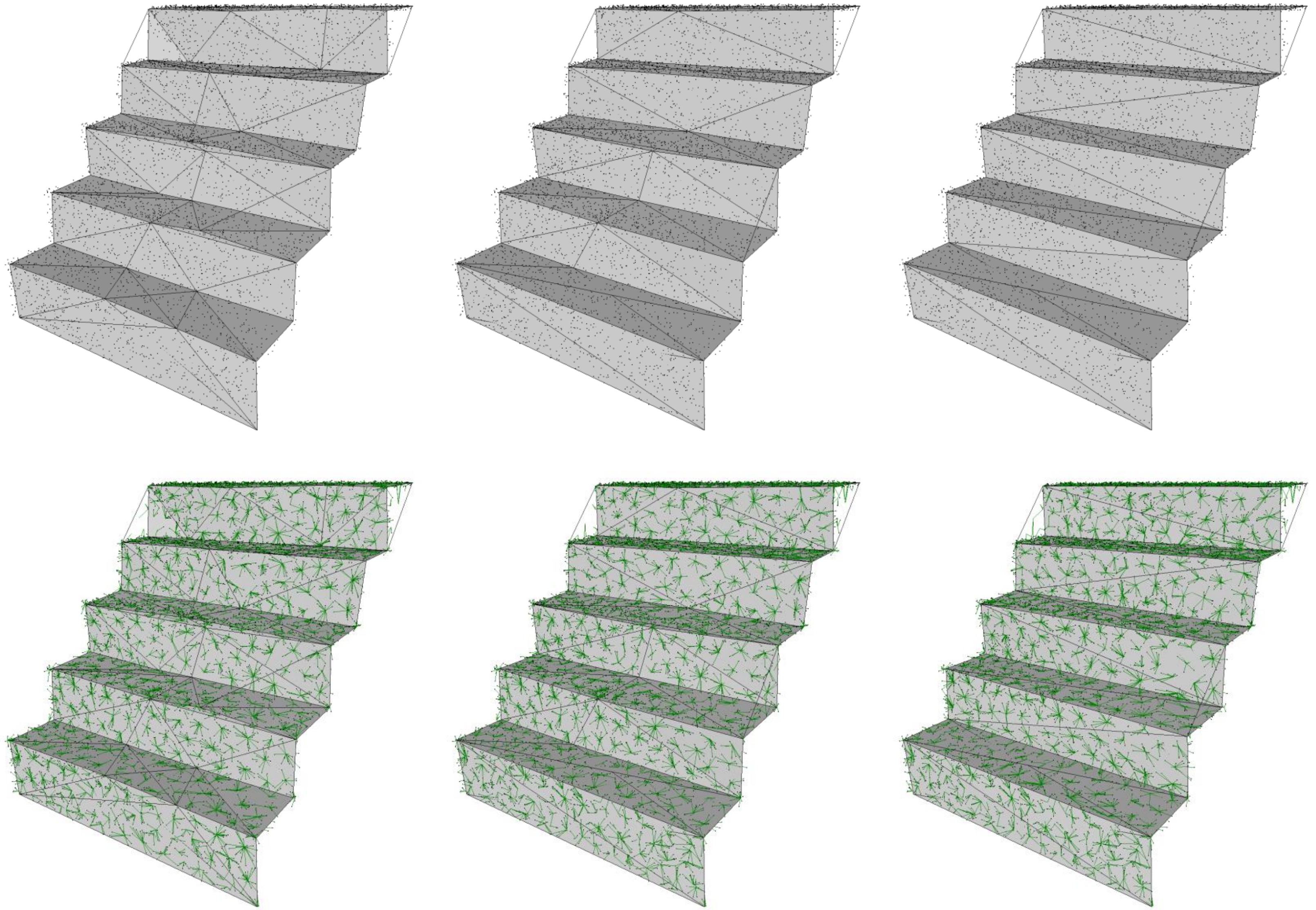
Vertex Relocation



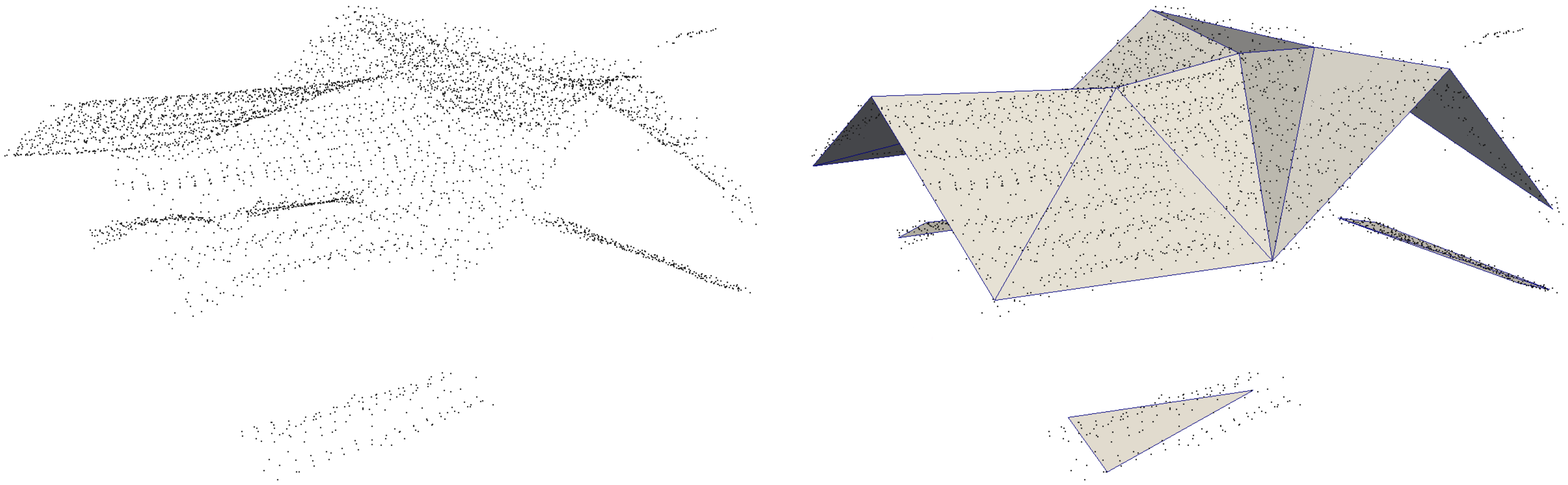
Stairs



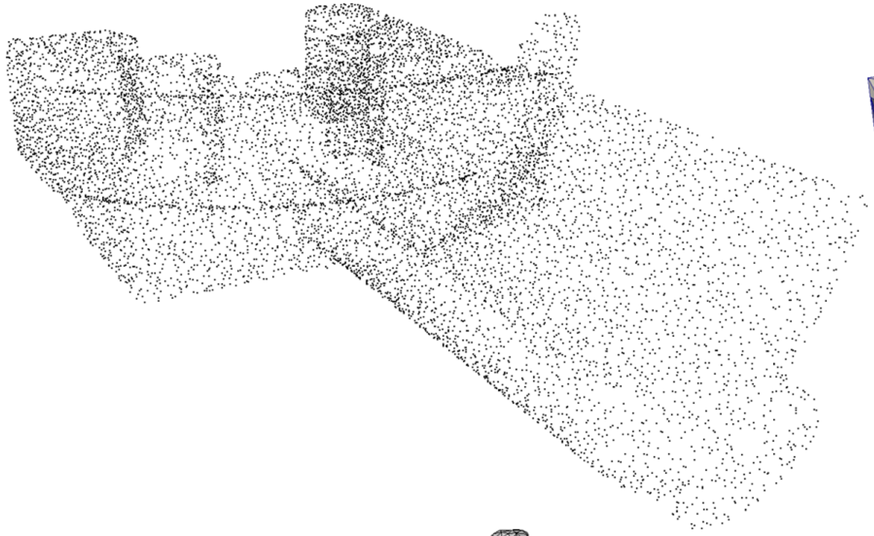
Stairs

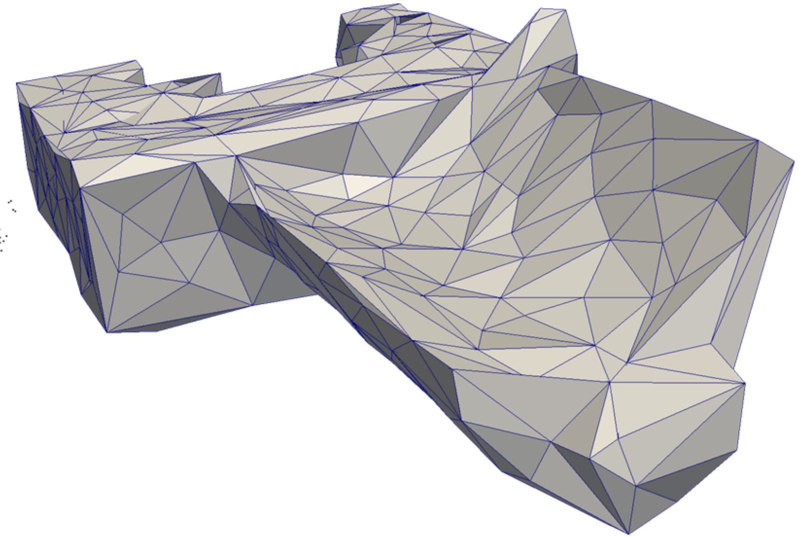


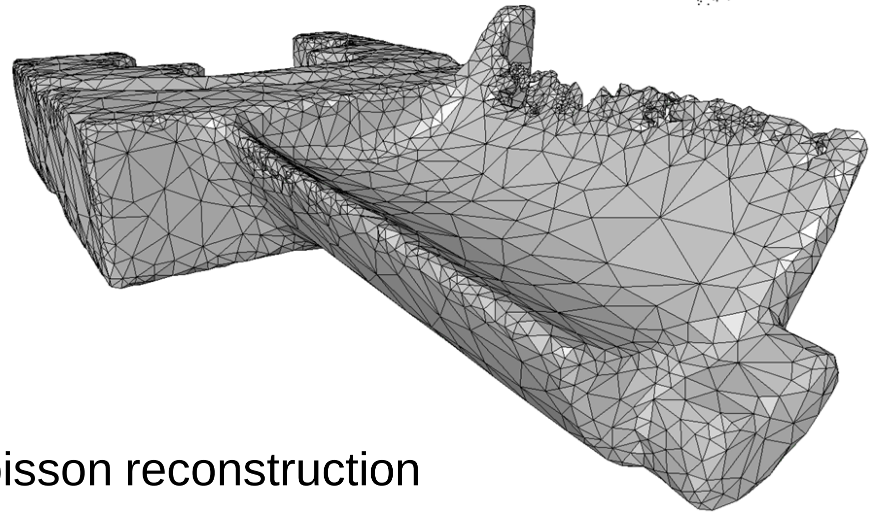
LIDAR Data (urban)



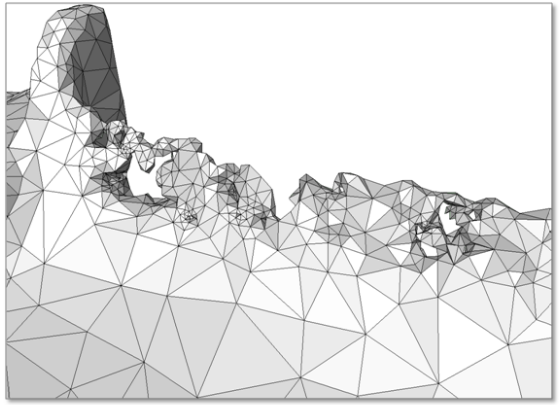
Blade





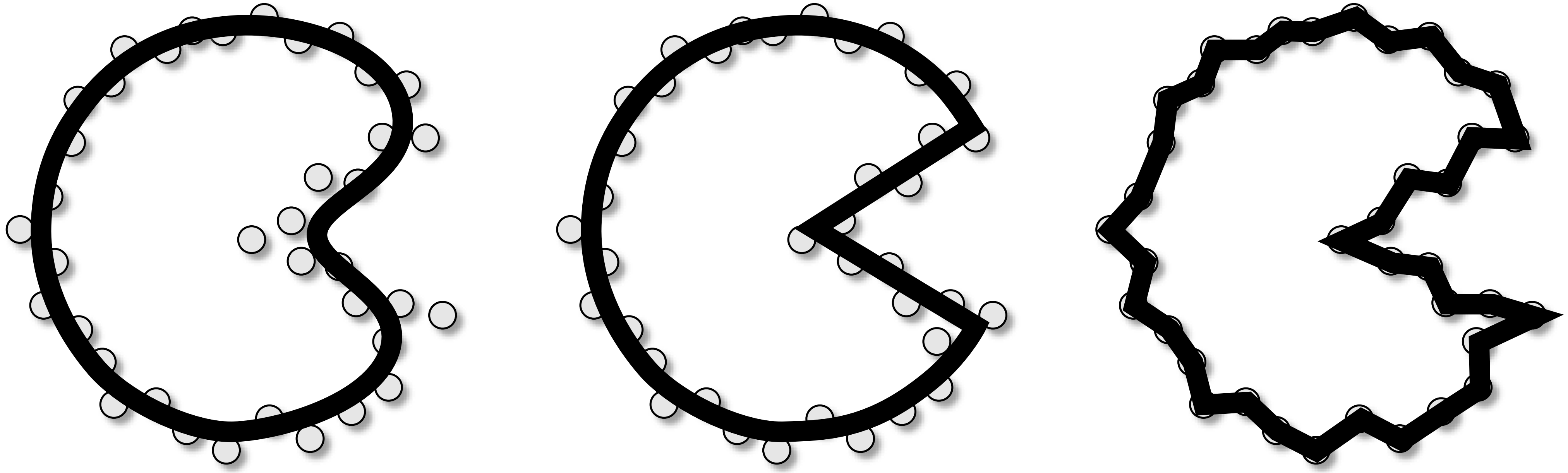


Poisson reconstruction



WHAT NEXT

Priors

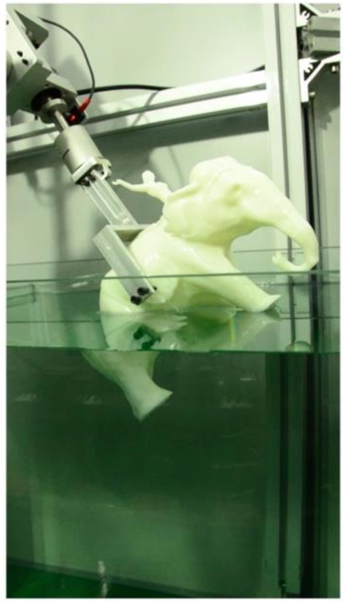


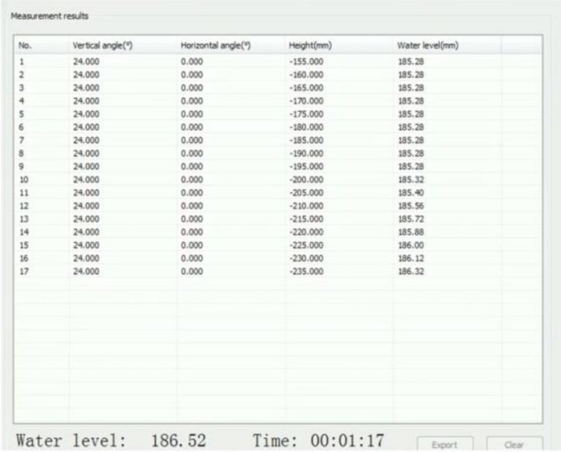
~~Smooth~~ ~~Piecewise Smooth~~ ~~"Simple"~~

Machine learning

Novel Acquisition Paradigms

- « Dip » transform

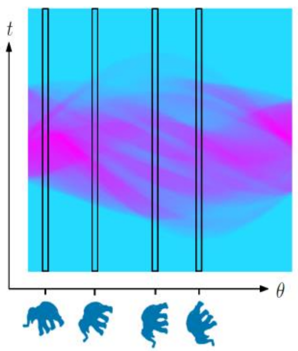


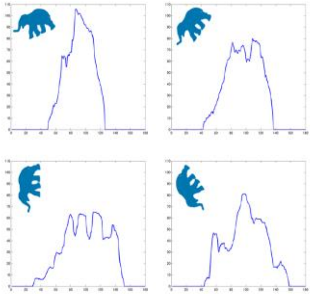


Measurement results

No.	Vertical angle(°)	Horizontal angle(°)	Height(mm)	Water level(mm)
1	24.000	0.000	-155.000	185.28
2	24.000	0.000	-160.000	185.28
3	24.000	0.000	-165.000	185.28
4	24.000	0.000	-170.000	185.28
5	24.000	0.000	-175.000	185.28
6	24.000	0.000	-180.000	185.28
7	24.000	0.000	-185.000	185.28
8	24.000	0.000	-190.000	185.28
9	24.000	0.000	-195.000	185.28
10	24.000	0.000	-200.000	185.32
11	24.000	0.000	-205.000	185.40
12	24.000	0.000	-210.000	185.56
13	24.000	0.000	-215.000	185.72
14	24.000	0.000	-220.000	185.88
15	24.000	0.000	-225.000	186.00
16	24.000	0.000	-230.000	186.12
17	24.000	0.000	-235.000	186.32

Water level: 186.52 Time: 00:01:17 Export Clear





Dip Transform for 3D Shape Reconstruction.
Aberman et al.
To appear at ACM SIGGRAPH 2017

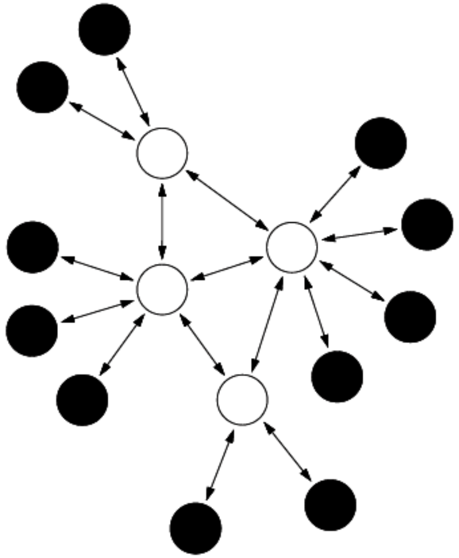
Novel Acquisition Paradigms

- Community data

Snavely, Seitz, Szeliski. *Photo tourism: Exploring photo collections in 3D*.

Novel Acquisition Paradigms

Sensor networks



Scientific challenges:

- Fusion from heterogeneous sensors
- Progressive acquisition
- Continuous update
- High level queries

3D Digitization

Societal impact:

- Cultural heritage accessible for all
- Telepresence via virtual/augmented/mixed reality
- New era of mass customization

Thank you.

Recent survey:

A Survey of Surface Reconstruction from Point Clouds. Berger, Tagliasacchi, Seversky, Alliez, Guennebaud, Levine, Sharf and Silva. Computer Graphics Forum, 2016.

Pierre Alliez

Inria Sophia Antipolis – Méditerranée

TITANE team: https://team.inria.fr/titane/

pierre.alliez@inria.fr